Quantum discord between relatively accelerated observers

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We calculate the quantum discord between two free modes of a scalar field, which start in a maximally entangled state and then undergo a relative constant acceleration. In a regime where there is no distillable entanglement due to the Unruh effect, we show that there is a finite amount of quantum discord, which is a measure of purely quantum correlations in a state over and above quantum entanglement. Even in the limit of infinite acceleration of the observer detecting one of the modes, we provide evidence for a nonzero amount of purely quantum correlations. We discuss our result in the context of secure quantum communications involving eavesdroppers in noninertial frames.

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I. INTRODUCTION

The theory of relativity and quantum theory, together with information theory, may be said to form the cornerstones of theoretical physics [1]. The last of these two are being, over the last decade, amalgamated into the field of quantum information science that seeks to compute and process information limited by the laws of quantum mechanics [2]. The enterprise of incorporating the principles of the theory of relativity into quantum information is, in comparison, nascent. Nonetheless, there have been several studies at the intersection of relativity theory and quantum information science, particularly, in the study of Bell's inequalities [3–7], quantum entropy [8,9], quantum entanglement [10–14], teleportation [15], and beyond [16]. There have also been studies involving the entanglement in fermionic fields [17] and continuous-variable systems in noninertial frames [18]. These have shown that entanglement between some degrees of freedom can be transferred to others and that the notion of entanglement is observer dependant.

In addition to the investigations into fundamental nature of quantum entanglement in a curved space time, there have been several proposals for detecting relativistic effects in laboratory systems like cavity QED [19], ion traps [20], and atom dots in Bose-Einstein condensates [21]. These effects of detecting acceleration radiation are a consequence of the Unruh effect [22]. A result from quantum field theory, it states that uniformly accelerated observers (that is, with constant proper acceleration) in Minkowski space time associate a thermal bath to the vacuum state of the inertial observers. For the inertial observer, the Minkowski coordinates (T,Z) are appropriate, while for a uniformly accelerating observer, Rindler coordinates (τ,ξ) are more apt. Minkowski space time is invariant under the boosts, and this motivates the hyperbolic coordinate transformations

$$T=\frac{1}{a}e^{a\xi}\sinh a\tau,\quad Z=\frac{1}{a}e^{a\xi}\cosh a\tau,\quad |Z|<T,\qquad (1)$$

$$T=-\frac{1}{a}e^{a\xi}\sinh a\tau,\quad Z=\frac{1}{a}e^{a\xi}\cosh a\tau,\quad |Z|>T.\qquad (2)$$

These two transformations lead to two sets of Rindler coordinates called the right and left Rindler wedges, respectively, which together form a complete set of solutions of the Klein-Gordon equation in Minkowski space time.

The solutions of the Klein-Gordon equation in Minkowski space time are related to those in the Rindler wedges via a Bogoliubov transformation [22]. These transform the vacuum of the inertial observer into a two-mode squeezed state for the accelerating observer; the two modes residing in the two Rindler wedges. If we probe only one of the wedges, as we are constrained to due to causality, the other mode is traced over, leaving us with a mixed state of free bosons at a temperature proportional to the acceleration. Additionally, if one starts with a pure entangled state of two free modes of a scalar field shared between two observers Alice and Bob, and one of them, say, Bob accelerates, the result is a mixed state, whose entanglement, as measured by the logarithmic negativity, is degraded from the point of view of Rob (accelerating Bob) [13], while there is no change from the point of view of Alice.

Our endeavor in this paper will be to explore the above phenomenon from the perspective of quantum discord [23–25]. It is a measure of purely quantum correlations, and we show that although the quantum discord suffers some degradation, there is a finite amount of quantum discord between Alice and Rob at accelerations at which the distillable entanglement has gone to zero. The use of quantum discord is first motivated by the fact that noninertial observers inevitably encounter mixed states, for which there is a lack of universally accepted easily computable measures of entanglement. Quantum discord is ideally suited for application to mixed states. Second, quantum discord is a measure of purely quantum correlations, over and above entanglement, although for pure states, they coincide. Finally, the quantum discord has been presented as a possible resource for certain quantum advantages [26], and the presence of nonzero amounts of quantum discord as perceived by the nonintertial observer might allow him to achieve nontrivial quantum advantage beyond a point where the distillable entanglement

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touches zero. We will also show that a "symmetrized" form of quantum discord called the measurement induced disturbance (MID) measure [27,28] and defined as the difference between the entropy of a quantum state and that obtained by measuring both the subsystems in their reduced eigenbases has a finite value at accelerations, at which the logarithmic negativity is zero. This is comparatively easier to calculate than the quantum discord and is an upper bound on it. What both these measures however show is that starting with an initially entangled state shared between Alice and Bob, there will persist quantum correlations between them when Bob accelerates, beyond accelerations at which the distillable entanglement has fallen to zero.

Finally, we view our results in the context of a cryptographic protocol called the private quantum channel protocol. It deals with the amount of encryption necessary to securely transmit a quantum state via a channel susceptible to an eavesdropper Eve. It is known that if parties involved in the communication, Alice and Bob as well as Eve are in the same inertial frame, two bits of classical resources are needed to securely transmit a qubit state. If Eve resides in a relatively accelerating noninertial frame, less than two bits provide complete security, in fact just one bit in the limit of infinite acceleration [29]. This is one of the fascinating ramifications of the Unruh effect in quantum cryptography, and we will look at private quantum channel capacity from the viewpoint of quantum discord.

II. MINKOWSKI TO RINDLER MODES

For concreteness, we start with the maximally entangled state between Alice and Bob of two Minkowski modes s and k

$$|\Psi\rangle^{\mathcal{M}} = \frac{1}{\sqrt{2}}(|0_s\rangle^{\mathcal{M}}|0_k\rangle^{\mathcal{M}} + |1_s\rangle^{\mathcal{M}}|1_k\rangle^{\mathcal{M}}). \tag{3}$$

When Bob accelerates with respect to Alice with a constant acceleration, the Minkowski vacuum can be expressed as a two-mode squeezed state of the Rindler vacuum [22]

$$|0_k\rangle^{\mathcal{M}} = \frac{1}{\cosh r}\sum_{n=0}^{\infty} \tanh^n r|n_k\rangle_1|n_k\rangle_2, \tag{4}$$

with

$$\tanh r = e^{-\pi|k|c/a} \equiv t, \tag{5}$$

and $|n_k\rangle_1$ and $|n_k\rangle_2$ refer to the two modes, corresponding to the left and right Rindler wedges. An excitation in the Minkowski mode can be easily represented as

$$|1_k\rangle^{\mathcal{M}} = \frac{1}{\cosh r}\sum_{n=0}^{\infty} \tanh^n r\sqrt{n+1}|(n+1)_k\rangle_1|n_k\rangle_2. \tag{6}$$

As only one of the modes is accessible to Rob due to the causality constraint, modes in one of the Rindler wedges (say mode 2) need to be traced over. Using the above expressions, the maximally entangled state in Eq. (3) is now transformed into

$$\rho_{AR} = \frac{1}{2\cosh^2 r}\sum_{n=0}^{\infty} \tanh^n r\rho_n, \tag{7}$$

where

$$\rho_n = |0,n\rangle\langle 0,n| + \frac{\sqrt{n+1}}{\cosh r}|0,n\rangle\langle 1,n+1| + \frac{\sqrt{n+1}}{\cosh r}|1,n+1\rangle\langle 0,n| + \frac{n+1}{\cosh^2 r}|1,n+1\rangle\langle 1,n+1|,$$

with $|m,n\rangle \equiv |m_s\rangle^{\mathcal{M}}|n_k\rangle_1$. The entanglement in the state (7) shared by Alice and Rob has been calculated in Ref. [13]. Our aim in this paper will be to calculate the quantum discord in this state.

III. QUANTUM DISCORD

Quantum discord aims at capturing all quantum correlations in a state, including entanglement. The quantum mutual information is generally taken to be the measure of total correlations, classical and quantum, in a quantum state. For two systems A and R, it is defined as

$$I(A:R) = H(A) + H(R) - H(A,R), \tag{8}$$

where $H(\cdot)$ stands for the von Neumann entropy $H(\rho) \equiv -\mathrm{Tr}(\rho \log \rho)$. In our paper, all logarithms are taken to base 2. For a classical probability distribution, Bayes' rule leads to an equivalent definition of the mutual information as $I(A:R)=H(R)-H(R|A)$, where the conditional entropy $H(R|A)$ is an average of the Shannon entropies of R, conditioned on the alternatives of A. It captures the ignorance in R once the state of A has been determined. For a quantum system, this depends on the measurements that are made on A. If we restrict to projective measurements described by a complete set of projectors $\{\Pi_i\}$, corresponding to the measurement outcome i, the state of R after the measurement is given by

$$\rho_{R|i} = \mathrm{Tr}_A(\Pi_i\rho_{AR}\Pi_i)/p_i, \quad p_i = \mathrm{Tr}_{A,R}(\Pi_i\rho_{AR}\Pi_i). \tag{9}$$

A quantum analog of the conditional entropy can then be defined as $\tilde{H}_{\{\Pi_i\}}(R|A) \equiv \Sigma_i p_i H(\rho_{R|i})$, and an alternative version of the quantum mutual information can now be defined as

$$\mathcal{J}_{\{\Pi_i\}}(A:R) = H(R) - \tilde{H}_{\{\Pi_i\}}(R|A). \tag{10}$$

The above quantity depends on the chosen set of measurements $\{\Pi_i\}$. To capture all the classical correlations present in ρ_{AR}, we maximize $\mathcal{J}_{\{\Pi_i\}}(A:R)$ over all $\{\Pi_i\}$, arriving at a measurement-independent quantity $\mathcal{J}(A:R)=\max_{\{\Pi_i\}}[H(R)-\tilde{H}_{\{\Pi_i\}}(R|A)] \equiv H(R)-\tilde{H}(R|A)$, where $\tilde{H}(R|A)=\min_{\{\Pi_i\}}\tilde{H}_{\{\Pi_i\}}(R|A)$. The quantum discord is finally defined as

$$\mathcal{D}(A:R) = I(A:R) - \mathcal{J}(A:R) = H(A) - H(A:R) + \min_{\{\Pi_i\}}\tilde{H}_{\{\Pi_i\}}(R|A). \tag{11}$$

As a first step towards the calculation of quantum discord, we begin by rewriting the state ρ_{AR} in a more conducive form, as

$$\rho_{AR} = \frac{1-t^2}{2}(|0\rangle\langle 0| \otimes M_{00} + |1\rangle\langle 1| \otimes M_{11} + |0\rangle\langle 1| \otimes M_{01} + |1\rangle\langle 0| \otimes M_{10}), \quad (12)$$

where

$$M_{00} = \sum_{n=0}^{\infty} t^{2n}|n\rangle\langle n|,$$

$$M_{11} = (1-t^2)\sum_{n=0}^{\infty}(n+1)t^{2n}|n+1\rangle\langle n+1|,$$

$$M_{01} = \sqrt{1-t^2}\sum_{n=0}^{\infty}\sqrt{n+1}t^{2n}|n\rangle\langle n+1|,$$

$$M_{10} = M_{01}^{\dagger}. \quad (13)$$

This form of the state suggests a natural bipartite split across which to calculate the quantum discord. We have, in effect, a $2\times\infty$ dimensional system, and we will make our measurement on the two-dimensional subsystem, which in our case, will be Alice's side. It is now easy to obtain the reduced state of the measured subsystem as

$$\rho_A = \mathrm{Tr}_R(\rho_{AR}) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (14)$$

whereby $H(A)=1$. The spectrum of the complete state ρ_{AR} is given by

$$\boldsymbol{\lambda}(\rho_{AR}) = \left\{\frac{1-t^2}{2}t^{2n}[1+(n+1)(1-t^2)]\right\}_{n=0}^{\infty}, \quad (15)$$

whereby

$$H(A{:}R) = -\frac{1-t^2}{2}\sum_{n=0}^{\infty}t^{2n}[1+(n+1)(1-t^2)] \times\log\left\{\frac{1-t^2}{2}t^{2n}[1+(n+1)(1-t^2)]\right\}. \quad (16)$$

The evaluation of the quantum conditional entropy requires a minimization over all one-qubit projective measurements, which are of the form

$$\Pi_{\pm} = \frac{I_1 \pm \boldsymbol{x}\,.\,\boldsymbol{\sigma}}{2} \quad (17)$$

with $\boldsymbol{x}\,.\,\boldsymbol{x}=x_1^2+x_2^2+x_3^2=1$, and I_1 is the one-qubit 2×2 identity matrix. The postmeasurement state is then given by

$$\rho_{R|\pm} = \frac{1-t^2}{4p_{\pm}}[(1\pm x_3)M_{00} + (1\mp x_3)M_{11} \pm (x_1+ix_2)M_{10} \pm (x_1-ix_2)M_{01}], \quad (18)$$

with outcome probabilities

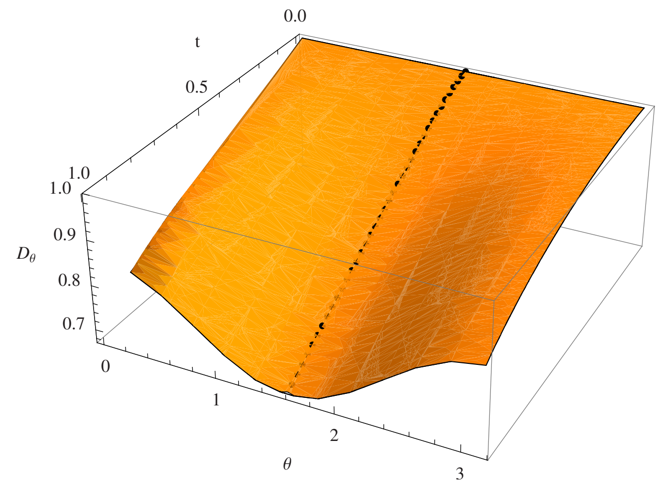

FIG. 1. (Color online) The plot of the quantum discord $\mathcal{D}_\theta$ [Eq. (20)], as a function of acceleration parameter $t=\tanh r$ and θ. In black dots are shown the minima for different values of t, which can be seen to be attained for $\theta=\pi/2$. They correspond to the solid green line in Fig. 2.

$$p_{\pm} = \frac{1-t^2}{4}\{(1\pm x_3)\mathrm{Tr}[M_{00}] + (1\mp x_3)\mathrm{Tr}[M_{11}]\} = \frac{1}{2}.$$

The density matrices $\rho_{R|\pm}$ are tridiagonal, whose eigenvalues can be obtained easily numerically, in particular, by using the parametrization $x_1=\sin\theta\cos\phi$, $x_2=\sin\theta\sin\phi$, and $x_3=\cos\theta$. It is immediately realized that the eigenvalues of these states that are used to calculate the conditional quantum entropy are independent of ϕ. This is because the initial state in Eq. (12) is azimuthally invariant, and the final state whose spectrum is to be evaluated reduces to

$$\rho_{R|\pm} = \frac{1-t^2}{2}[(1\pm\cos\theta)M_{00} + (1\mp\cos\theta)M_{11} \pm \sin\theta M_{10} \pm \sin\theta M_{01}], \quad (19)$$

having spectra $\boldsymbol{\lambda}_{\pm}$. Then, following Eq. (11), the expression for quantum discord in the state ρ_{AR}, as a function of the parameter θ, is given by

$$\mathcal{D}_\theta = 1 + \frac{1-t^2}{2}\sum_{n=0}^{\infty}t^{2n}[1+(n+1)(1-t^2)] \times\log\left\{\frac{1-t^2}{2}t^{2n}[1+(n+1)(1-t^2)]\right\} - \frac{1}{2}\sum_{i=\pm}\mathrm{Tr}(\boldsymbol{\lambda}_i\log\boldsymbol{\lambda}_i), \quad (20)$$

and is plotted in Fig. 1 as a function of θ and t. Realizing that the minimum is obtained for $\theta=\pi/2$, we obtained the final value of the quantum discord for the state ρ_{AR} as $\mathcal{D}=\mathcal{D}_{\theta=\pi/2}$. This value is plotted in Fig. 2. This is the main result of our paper. To put our result in perspective, we also plot the logarithmic negativity of the same state [13]. This shows that in the range of accelerations, where the state has no distillable entanglement, as shown by the vanishing loga-

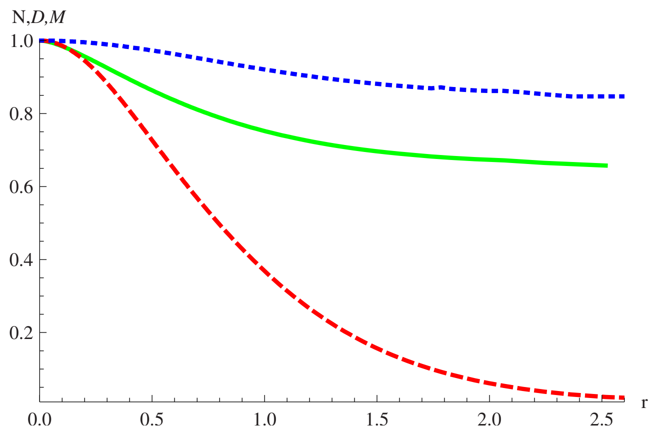

FIG. 2. (Color online) The solid green line is the quantum discord in the state ρ_{AR} for a measurement made on Alice's side. The red dashed line is the logarithmic negativity in the same state, as in Ref. [13]. The blue dotted line is the MID measure for the same state ρ_{AR}.

rithmic negativity, the state indeed has finite quantum discord.

IV. MID MEASURE

In the calculation of quantum discord, as per Eq. (11), one maximizes over one-dimensional projective measurements on one of the subsystems, in our case Alice. For the MID measure, one performs measurements on *both* the subsystems, with the measurements being given by projectors onto the eigenvectors of the reduced subsystems. This can be thought of as a bidirectional form of discord, which actually depends on the party making the measurement [30]. The MID measure of quantum correlations for a quantum state ρ_{AR} is given by [27]

$$\mathcal{M}(\rho_{AR}) \coloneqq \mathcal{I}(\rho_{AR}) - \mathcal{I}(\mathcal{P}(\rho_{AR})), \tag{21}$$

where

$$\mathcal{P}(\rho_{AR}) \coloneqq \sum_{i=1}^{M}\sum_{j=1}^{N} (\Pi_i^A \otimes \Pi_j^R)\rho_{AR}(\Pi_i^A \otimes \Pi_j^R). \tag{22}$$

Here $\{\Pi_i^A\},\{\Pi_j^R\}$ denote rank one projections onto the eigenbases of ρ_A and ρ_R, respectively. $\mathcal{I}(\sigma)$ is the quantum mutual information, which is considered to the measure of total, classical and quantum, correlations in the quantum state σ. Since no optimizations are involved in this measure, it is much easier to calculate in practice than the quantum discord. The measurement induced by the spectral resolution leaves the entropy of the reduced states invariant and is, in a certain sense, the least disturbing. Actually, this choice of measurement even leaves the reduced states invariant [27]. For pure states, both the quantum discord and the MID measure reduce to the von Neumann entropy of the reduced density matrix, which is a measure of bipartite entanglement.

Starting from the expression of ρ_{AR} in Eq. (12), we have

$$\rho_R = \mathrm{Tr}_A(\rho_{AR}) = \frac{1-t^2}{2}(M_{00}+M_{11}), \tag{23}$$

which, being diagonal, leads to

$$\{\Pi_j^R\} = \{E_j\} \text{ where } [E_j]_{kl} = \delta_{kj}\delta_{lj}, \quad j,k,l = 1,\ldots,\infty.$$

From Eq. (14),

$$\{\Pi_j^A\} = \{E_j\} \text{ where } [E_j]_{kl} = \delta_{kj}\delta_{lj}, \quad j,k,l = 1,2.$$

Given these, $\mathcal{P}(\rho_{AR}) = \mathrm{diag}(\rho_{AR})$ and,

$$\begin{aligned} H(\mathcal{P}(\rho_{AR})) &= -\frac{1-t^2}{2}\sum_{n=0}^{\infty} t^{2n}\log\left(\frac{1-t^2}{2}t^{2n}\right) - \frac{(1-t^2)^2}{2} \\ &\quad\times \sum_{n=0}^{\infty}(n+1)t^{2n}\log\left[(n+1)\frac{(1-t^2)^2}{2}t^{2n}\right] \\ &= 1 - \frac{3t^2}{1-t^2}\log(t) - \frac{3}{2}\log(1-t^2) - \frac{(1-t^2)^2}{2}\mathcal{S}, \end{aligned} \tag{24}$$

where $\mathcal{S} = \sum_{n=0}^{\infty} t^{2n}(n+1)\log(n+1)$. The MID measure can now be calculated as

$$\mathcal{M}(\rho_{AR}) = H(\mathcal{P}(\rho_{AR})) - H(\rho_{AR}), \tag{25}$$

the latter of which can be obtained from Eq. (15). A plot of this measure is shown in Fig. 2, as the blue dotted line.

V. CONCLUDING DISCUSSIONS

We have shown the existence of purely quantum correlations between two initially entangled free modes of a scalar field, when the party detecting one of the modes undergoes a constant acceleration, while the other is inertial. In this regime, there is no distillable entanglement between them as a consequence of the Unruh effect. The Unruh effect has interesting consequences in quantum information processing and cryptography. For instance, it is known that as a consequence of this effect, asymptotically half the shared amount of classical resources are necessary to securely transmit a quantum state from Alice to Bob when Eve lives in a uniformly accelerating frame, as opposed to an inertial one [29]. The primary reason is that states appear more mixed to the accelerating Eve, making them less distinguishable, which is the intent of secure transmission. Here we show that there is a finite amount of quantum discord in a bipartite state, where one of the parties undergoes uniform acceleration, in the limit of infinite acceleration. As quantum discord captures nonclassical correlations beyond entanglement, it might be possible to use these correlations to attain nontrivial quantum advantage. One such scenario might be the capacity of a private quantum channel, where the eavesdropper Eve is uniformly accelerated with respect to Alice and Bob. It is known that the capacity of the so-called Unruh channel is equal to the entanglement-assisted quantum capacity for the channel to Eve's environment, which can be presented in terms of polylogarithmic functions and their derivatives [31]. This is valid for all accelerations, including at the limit of infinite acceleration. By that point, however, there is no distillable entanglement left, as shown in Fig. 2. We have here the evidence for the presence of purely quantum correlations in the form of quantum discord that might explain the resource behind this advantage provided by the Unruh channel.

Finally, it is intriguing to note that this study about quantum discord can raise interesting questions about the entanglement of these states. We mention one in brief. The entanglement of formation can, in principle, be larger than the quantum discord for quantum states [32]. As the entanglement cost is the regularized version of the entanglement of formation in the limit of a large number of copies, this could imply that although the accelerated state may asymptotically have no distillable entanglement, it might still have a nonzero entanglement cost, by virtue of it having a nonzero quantum discord. Further investigations are necessary to answer these questions in a more complete manner.

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