

Classical and quantum correlations under decoherence

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Recently some authors have pointed out that there exist nonclassical correlations which are more general, and possibly more fundamental, than entanglement. For these general quantum correlations and their classical counterparts, under the action of decoherence, we identify three general types of dynamics that include a peculiar sudden change in their decay rates. We show that, under suitable conditions, the classical correlation is unaffected by decoherence. Such dynamic behavior suggests an operational measure of both classical and quantum correlations that can be computed without any extremization procedure.

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It is largely accepted that quantum mutual information is the information-theoretic measure of the total correlation in a bipartite quantum state. Groisman *et al.* [1], inspired by Landauer's erasure principle [2], gave an operational definition of correlations based on the amount of noise required to destroy them. From this definition, they proved that the total amount of correlation in any bipartite quantum state (ρ_{AB}) is equal to the quantum mutual information [$\mathcal{I}(\rho_{A:B})=S(\rho_A)+S(\rho_B)-S(\rho_{AB})$], where $S(\rho)=-\text{Tr}(\rho \log_2 \rho)$ is the von Neumann entropy and $\rho_{A(B)}=\text{Tr}_{B(A)}(\rho_{AB})$ is the reduced density operator of the partition $A(B)$. Another argument in favor of the claim that quantum mutual information is a measure of the total correlation in a bipartite quantum state was given by Schumacher and Westmoreland [3]. They showed that, if Alice and Bob share a correlated composite quantum system that is used as the key for a "one-time pad cryptographic system," the maximum amount of information that Alice can send securely to Bob is the quantum mutual information of the shared correlated state.

We are interested here in the dynamics of both quantum and classical correlations under the action of noisy environments. For these purposes, it is reasonable to assume that the total correlation contained in a bipartite quantum state may be separated as $\mathcal{I}(\rho_{A:B})=\mathcal{Q}(\rho_{AB})+\mathcal{C}(\rho_{AB})$, owing to the distinct nature of quantum (\mathcal{Q}) and classical (\mathcal{C}) correlations [1,4–6]. Some proposals for characterization and quantification of \mathcal{Q} and \mathcal{C} in a composite quantum state have appeared in the last few years [1,4,5,7–9]. The quantum correlation, $\mathcal{Q}(\rho_{AB})$, between partitions A and B of a composite state can be quantified by the so-called quantum discord, $\mathcal{D}(\rho_{AB})$, introduced by Ollivier and Zurek [7]. Such a quantum correlation is more general than entanglement in the sense that separable mixed states can have a nonclassical correlation that leads to a nonzero discord. It measures general nonclassical correlations, including entanglement. For separable

mixed states (unentangled states) with nonzero discord, this quantum correlation provides a speedup, in performing some tasks, over the best known classical counterpart, as was shown theoretically [10] and experimentally [11] in a non-universal model of quantum computation. Therefore, such a nonclassical correlation might have a significant role in quantum information protocols. For pure states, we have a special situation where the quantum correlation is equal to the entropy of entanglement and also to the classical correlation. In other words, $\mathcal{Q}(\rho_{AB})=\mathcal{C}(\rho_{AB})=\mathcal{I}(\rho_{A:B})/2$ [1,4]. In this case, the total amount of quantum correlation is captured by an entanglement measure. On the other hand, for mixed states, the entanglement is only a part of this more general nonclassical correlation, $\mathcal{Q}(\rho_{AB})$ [7,11,10]. A quantum composite state may also have a classical correlation, $\mathcal{C}(\rho_{AB})$, which for bipartite quantum states can be quantified via the measure proposed by Henderson and one of us [4]. Since we assume that the total correlation is given by the quantum mutual information and if we adopt the definition of classical correlation given in [4], $\mathcal{Q}(\rho_{AB})$ turns out to be identical to the definition of quantum discord in Ref. [7]; in other words, $\mathcal{Q}(\rho_{AB})=\mathcal{D}(\rho_{AB})=\mathcal{I}(\rho_{A:B})-\mathcal{C}(\rho_{AB})$, as already noted in Ref. [12].

We have identified three different kinds of dynamic behavior of \mathcal{C} and \mathcal{Q} under decoherence, which depend on the "geometry" of the initial composite state and on the noise channel: (i) \mathcal{C} remains constant and \mathcal{Q} decays monotonically over time; (ii) \mathcal{C} suffers a sudden change in behavior, decaying monotonically until a specific parametrized time, p_{SC} (to be defined below), and remaining constant thereafter, while \mathcal{Q} has an abrupt change in its rate of decay at p_{SC} , becoming greater than \mathcal{C} within certain parametrized time interval; and (iii) \mathcal{C} and \mathcal{Q} decay monotonically. For two-qubit states with maximally mixed marginals we show which conditions lead to the different types of dynamic behavior for certain noise channels (i.e., phase flip, bit flip, and bit-phase flip). We also recognize a symmetry among these channels and provide a necessary condition for \mathcal{C} to remain constant under decoherence, which enables us to define an operational measure for both classical and quantum correlations.

Let us start with the definition of classical correlation [4],

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$$\mathcal{C}(\rho_{AB}) \equiv \max_{\{\Pi_j\}} [S(\rho_A) - S_{\{\Pi_j\}}(\rho_{A|B})], \quad (1)$$

where the maximum is taken over the set of projective measurements $\{\Pi_j\}$ [13] on subsystem B [14], $S_{\{\Pi_j\}}(\rho_{A|B}) = \sum_j q_j S(\rho_A^j)$ is the conditional entropy of subsystem A , given the knowledge (measure) of the state of subsystem B , $\rho_A^j = \text{Tr}_B(\Pi_j \rho_{AB} \Pi_j) / q_j$, and $q_j = \text{Tr}_{AB}(\rho_{AB} \Pi_j)$.

We consider the scenario of two qubits under local decoherence channels. The evolved state of such a system under local environments may be described as a completely positive trace preserving map, $\varepsilon(\cdot)$, which, written in the operator-sum representation, is given by [15,16]

$$\varepsilon(\rho_{AB}) = \sum_{i,j} \Gamma_i^{(A)} \Gamma_j^{(B)} \rho_{AB} \Gamma_i^{(B)\dagger} \Gamma_j^{(A)\dagger},$$

where $\Gamma_i^{(k)}$ ($k=A,B$) are the Kraus operators that describe the noise channels A and B .

For simplicity, let us consider a class of states with maximally mixed marginals ($\rho_{A(B)} = \mathbf{1}_{A(B)}/2$), described by

$$\rho_{AB} = \frac{1}{4} \left(\mathbf{1}_{AB} + \sum_{i=1}^3 c_i \sigma_i^A \otimes \sigma_i^B \right), \quad (2)$$

where σ_i^k is the standard Pauli operator in direction i acting on the subspace $k=A,B$, $c_i \in \mathfrak{R}$ such that $0 \leq |c_i| \leq 1$ for $i=1,2,3$, and $\mathbf{1}_{A(B)}$ is the identity operator in subspace $A(B)$. The state in Eq. (2) represents a considerable class of states including the Werner ($|c_1|=|c_2|=|c_3|=c$) and Bell ($|c_1|=|c_2|=|c_3|=1$) basis states.

Phase flip channel. This is a quantum noise process with loss of quantum information without loss of energy. For this channel, the Kraus operators are given by [15,16] $\Gamma_0^{(A)} = \text{diag}(\sqrt{1-p_A}/2, \sqrt{1-p_A}/2) \otimes \mathbf{1}_B$, $\Gamma_1^{(A)} = \text{diag}(\sqrt{p_A}/2, -\sqrt{p_A}/2) \otimes \mathbf{1}_B$, $\Gamma_0^{(B)} = \mathbf{1}_A \otimes \text{diag}(\sqrt{1-p_B}/2, \sqrt{1-p_B}/2)$, and $\Gamma_1^{(B)} = \mathbf{1}_A \otimes \text{diag}(\sqrt{p_B}/2, -\sqrt{p_B}/2)$, written in the subsystem basis $\{|0\rangle_k, |1\rangle_k\}$, $k=A,B$. We are using $p_{A(B)}$ ($0 \leq p_{A(B)} \leq 1$) as parametrized time in channel $A(B)$. We consider here the symmetric situation in which the decoherence rate is equal in both channels, so $p_A = p_B \equiv p$.

The description of the dynamical evolution of the system under the action of a decoherence channel using the parametrized time p is more general than that using a specific functional dependence on time t in the sense that it accounts for a large range of physical scenarios. For example, for the phase damping channel (the phase damping and phase flip channels are the same quantum operation [15]), we have $p = 1 - \exp(-\gamma t)$, where γ is the phase damping rate [17].

The density operator in Eq. (2) under the multimode noise channel, $\varepsilon(\rho_{AB})$, has the eigenvalue spectrum,

$$\begin{aligned} \lambda_1 &= \frac{1}{4}[1 - \alpha - \beta - \gamma], & \lambda_2 &= \frac{1}{4}[1 - \alpha + \beta + \gamma], \\ \lambda_3 &= \frac{1}{4}[1 + \alpha - \beta + \gamma], & \lambda_4 &= \frac{1}{4}[1 + \alpha + \beta - \gamma], \end{aligned} \quad (3)$$

with $\alpha = (1-p)^2 c_1$, $\beta = (1-p)^2 c_2$, $\gamma = c_3$, and the von Neumann entropies of the marginal states remain constant under phase flip for any p , $S[\text{Tr}_{A(B)} \varepsilon(\rho_{AB})] = 1$. To compute the classical correlation [Eq. (1)] under phase flip, we take the complete set of orthonormal

projectors $\{\Pi_j = |\Theta_j\rangle\langle\Theta_j|, j = \parallel, \perp\}$, where $|\Theta_\parallel\rangle \equiv \cos(\theta)|0\rangle + e^{i\phi} \sin(\theta)|1\rangle$ and $|\Theta_\perp\rangle \equiv e^{-i\phi} \sin(\theta)|0\rangle - \cos(\theta)|1\rangle$. Then the reduced measured density operator of subsystem A under phase flip, $\tilde{\rho}_A^j = \text{Tr}_B[\Pi_j \varepsilon(\rho_{AB}) \Pi_j] / q_j$, will have the following eigenvalue spectrum,

$$\begin{aligned} \xi_{1,2}^{(j)} &= \frac{1}{4} \{ 2 \pm [2\gamma^2 + \alpha^2 + \beta^2 + (2\gamma^2 - \alpha^2 - \beta^2) \cos(4\theta) + 2(\alpha^2 \\ &\quad - \beta^2) \cos(2\phi) \sin^2(2\theta)]^{1/2} \}, \end{aligned} \quad (4)$$

and $q_j = 1/2$ for $j = \parallel, \perp$. From Eq. (1), it follows that

$$\mathcal{C}[\varepsilon(\rho_{AB})] = 1 - \min_{\theta, \phi} [S(\tilde{\rho}_A^\parallel)] \quad (5)$$

since $\xi_{1,2}^{(\parallel)} = \xi_{1,2}^{(\perp)}$ and hence $S(\tilde{\rho}_A^\parallel) = S(\tilde{\rho}_A^\perp)$. The classical correlation and the quantum correlation under phase flip may be written as

$$\mathcal{C}[\varepsilon(\rho_{AB})] = \sum_{k=1}^2 \frac{1 + (-1)^k \chi}{2} \log_2 [1 + (-1)^k \chi], \quad (6a)$$

$$\mathcal{Q}[\varepsilon(\rho_{AB})] = 2 + \sum_{k=1}^4 \lambda_k \log_2 \lambda_k - \mathcal{C}[\varepsilon(\rho_{AB})], \quad (6b)$$

respectively, where $\chi = \max(|\alpha|, |\beta|, |\gamma|)$, which depends on the relation between the coefficients c_i of state in Eq. (2) and on the parametrized time p .

(i) If $|c_3| \geq |c_1|, |c_2|$ in Eq. (2), the minimum in Eq. (5) is obtained by $\theta = \phi = 0$. The classical and the quantum correlations under phase flip will be given in Eqs. (6a) and (6b), respectively, with $\chi = |c_3|$. In this case, the classical correlation $\mathcal{C}[\varepsilon(\rho_{AB})]$ is constant (it does not depend on the parametrized time p) and equal to the mutual information of the completely decohered state ($p=1$), $\mathcal{C}(\rho_{AB}) = \mathcal{C}[\varepsilon(\rho_{AB})] = \mathcal{I}[\varepsilon(\rho_{A:B})|_{p=1}]$, while the quantum correlation [Eq. (6b)] decays monotonically.

(ii) If $|c_1| \geq |c_2|, |c_3|$ or $|c_2| \geq |c_1|, |c_3|$ and $|c_3| \neq 0$, we have a peculiar dynamics with a sudden change in behavior. \mathcal{C} decays monotonically until a specific parametrized time, $p_{SC} = 1 - \sqrt{|c_3| / \max(|c_1|, |c_2|)}$, and from then on \mathcal{C} remains constant. For $p < p_{SC}$, the minimum in Eq. (5) is achieved when $\theta = \pi/4$, $\phi = 0$ (if $|c_1| \geq |c_2|$) or $\phi = \pi/2$ (if $|c_1| < |c_2|$), and $\chi = (1-p)^2 \max(|c_1|, |c_2|)$. Thus, \mathcal{C} decays monotonically. On the other hand, for $p \geq p_{SC}$, the choice $\theta = \phi = 0$ leads to the minimum in Eq. (5) and $\chi = |c_3|$. Then \mathcal{C} suddenly becomes constant at $p = p_{SC}$, $\mathcal{C}[\varepsilon(\rho_{AB})|_{p \geq p_{SC}}] = \mathcal{I}[\varepsilon(\rho_{A:B})|_{p=1}]$, and the decay rate of \mathcal{Q} changes suddenly at $p = p_{SC}$. In Fig. 1, we depict this peculiar behavior for a given choice of parameters and, in Fig. 2, we show the values of the sudden change parametrized time, p_{SC} , as a function of c_1 and c_2 .

(iii) Finally, if $|c_3| = 0$, we have a monotonic decay of both correlations \mathcal{C} and \mathcal{Q} .

The dynamic behavior of correlations under the phase flip channel described in Fig. 1 is quite general. Such a sudden change in behavior occurs also when we consider the bit flip and the bit-phase flip channels [of course under other conditions on the c_k 's in state (2)]. Moreover, these results contradict the early conjecture that $\mathcal{C} \geq \mathcal{Q}$ for any quantum state

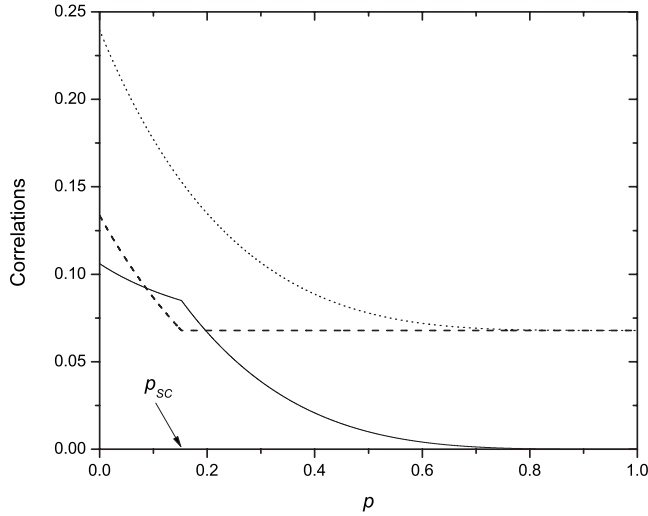


FIG. 1. Classical $\mathcal{C}[\varepsilon(\rho_{AB})]$ (dashed line), quantum $\mathcal{Q}[\varepsilon(\rho_{AB})]$ (solid line), and total $\mathcal{I}[\varepsilon(\rho_{A:B})]$ (dotted line) correlations under phase flip. We have set, in this figure, $c_1=0.06$, $c_2=0.42$, and $c_3=0.30$. For this state the sudden change occurs at $p_{SC}=0.15$ and \mathcal{Q} is greater than \mathcal{C} for $0.09 \leq p \leq 0.20$. At $p=0.09$ and $p=0.20$, we have $\mathcal{Q}[\varepsilon(\rho_{AB})]=\mathcal{C}[\varepsilon(\rho_{AB})]=\mathcal{I}[\varepsilon(\rho_{A:B})]/2$, as happens for pure states.

[1,4,18]. Here, we have shown that the quantum correlation may be greater than the classical one for some states, for example, $\varepsilon(\rho_{A:B})|_{p=p_{SC}}$.

It is worth mentioning that this peculiar sudden change in behavior is a different phenomenon from entanglement sudden death [16,19,20]. Indeed, it seems that these correlations do not present sudden death [21].

Bit flip channel. The Kraus operators are [15,16] $\Gamma_0^{(A)} = \text{diag}(\sqrt{1-p/2}, \sqrt{1-p/2}) \otimes \mathbf{1}_B$, $\Gamma_1^{(A)} = \sqrt{p/2} \sigma_x^{(A)} \otimes \mathbf{1}_B$, $\Gamma_0^{(B)} = \mathbf{1}_A \otimes \text{diag}(\sqrt{1-p/2}, \sqrt{1-p/2})$, and $\Gamma_1^{(B)} = \mathbf{1}_A \otimes \sqrt{p/2} \sigma_x^{(B)}$. The eigenvalue spectrum of $\varepsilon(\rho_{AB})$ is given by Eq. (3), where the variables now take the form $\alpha=c_1$, $\beta=(1-p)^2 c_2$, and $\gamma=(1-p)^2 c_3$. The correlations can again be written as Eqs. (6a) and (6b). The dynamic behavior of \mathcal{C} and \mathcal{Q} under bit flip is symmetrical to that for the phase flip channel (just

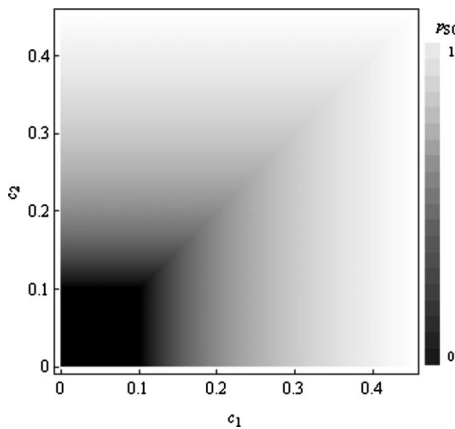


FIG. 2. Sudden change parametrized time, p_{SC} as a function of c_1 and c_2 , for $c_3=0.1$, under a phase flip channel. In the regions where $p_{SC}=0$ or $p_{SC}=1$ there is no sudden change.

exchanging c_1 and c_3). Type (i) dynamics is obtained when $|c_1| \geq |c_2|, |c_3|$. Type (ii) occurs for $|c_3| \geq |c_1|, |c_2|$ or $|c_2| \geq |c_1|, |c_3|$ and $|c_1| \neq 0$, with a sudden change in behavior of \mathcal{C} and \mathcal{Q} at $p_{SC} = 1 - \sqrt{|c_1|/\max(|c_2|, |c_3|)}$. Finally, if $|c_1| = 0$, we have type (iii) dynamics.

Bit-phase flip channel. Now, the Kraus operators are [15,16] $\Gamma_0^{(A)} = \text{diag}(\sqrt{1-p/2}, \sqrt{1-p/2}) \otimes \mathbf{1}_B$, $\Gamma_1^{(A)} = \sqrt{p/2} \sigma_y^{(A)} \otimes \mathbf{1}_B$, $\Gamma_0^{(B)} = \mathbf{1}_A \otimes \text{diag}(\sqrt{1-p/2}, \sqrt{1-p/2})$, and $\Gamma_1^{(B)} = \mathbf{1}_A \otimes \sqrt{p/2} \sigma_y^{(B)}$. The variables in Eq. (3) turn out to be $\alpha=(1-p)^2 c_1$, $\beta=c_2$, and $\gamma=(1-p)^2 c_3$. \mathcal{C} and \mathcal{Q} under bit-phase flip can again be written as Eqs. (6a) and (6b), respectively. Once more, the conditions for the various types of dynamics are obtained by swapping c_2 and c_3 in the phase flip channel. For type (ii) dynamics, we now have $p_{SC} = 1 - \sqrt{|c_2|/\max(|c_1|, |c_3|)}$.

Necessary conditions for \mathcal{C} to remain constant under decoherence are the following:

$$[\Pi_j, \Gamma_k^{(B)}] = 0, \quad \forall j, k. \quad (7)$$

These relations depend on the angles θ and ϕ that define the minimum in Eq. (5). For the channels mentioned above, $\Gamma_0^{(B)} \propto \mathbf{1}_B$ and $\Gamma_1^{(B)} \propto \sigma_i^{(B)}$ with $i=1$ for the bit flip, $i=2$ for the bit-phase flip, and $i=3$ for the phase flip. Hence, condition (7) will be satisfied when the projective measurements that reach the minimum in Eq. (5), Π_j , are performed on eigenstates of $\sigma_i^{(B)}$ [22]. On the other hand, the angles θ and ϕ that define the minimum in Eq. (5) depend on the geometry of the initial state. When the larger component of state in Eq. (2) is in the direction 1, 2, or 3, \mathcal{C} remains constant under bit flip, bit-phase flip, or phase flip, respectively.

The fact that, for a given state, the classical correlation can remain unaffected by a suitable choice of noise channel, ε , immediately suggests an operational way (without any extremization procedure) of computing classical and quantum correlations. It could be done as follows: depending on the state geometry, we send its component parts through local channels that preserve its classical correlation, so that the quantum correlation will be given simply by the difference between the state mutual information $\mathcal{I}(\rho_{A:B})$ and the completely decohered mutual information, $\mathcal{I}[\varepsilon(\rho_{A:B})|_{p=1}]$,

$$\mathcal{Q}(\rho_{AB}) \equiv \mathcal{I}(\rho_{A:B}) - \mathcal{I}[\varepsilon(\rho_{A:B})|_{p=1}],$$

since $\mathcal{I}(\rho_{A:B}) = \mathcal{Q}(\rho_{AB}) + \mathcal{C}(\rho_{AB})$ and

$$\mathcal{C}(\rho_{AB}) = \mathcal{I}[\varepsilon(\rho_{A:B})|_{p=1}].$$

A suitable channel for the class of states described by Eq. (2) is chosen which satisfies condition (7) as discussed above.

A problem to be addressed before such a measure can be used for a general state is to establish a protocol to find the map (if this map exists) which leaves the classical correlation unaffected [23]. This suggests an interesting research program to develop an operational way of investigating the role of quantum and classical correlations in many scenarios, such as quantum phase transitions [24], nonequilibrium thermodynamics [25], etc.

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