

Casimir-Polder forces on moving atoms

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Polarizable atoms and molecules experience the Casimir-Polder force near magnetoelectric bodies, a force that is induced by quantum fluctuations of the electromagnetic field and the matter. Atoms and molecules in relative motion to a magnetoelectric surface experience an additional velocity-dependent force. We present a full quantum-mechanical treatment of this force and identify a generalized Doppler effect, the time delay between photon emission and reabsorption, and the Röntgen interaction as its three sources. For ground-state atoms, the force is very small and always decelerating, hence commonly known as quantum friction. For atoms and molecules in electronically excited states, on the contrary, both decelerating and accelerating forces can occur depending on the magnitude of the atomic transition frequency relative to the surface-plasmon frequency.

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I. INTRODUCTION

The ground-state fluctuations of the electromagnetic field lead to several inherently quantum effects such as the spontaneous decay of excited atoms and molecules as well as dispersion forces [1]. Forces between isolated atoms that are mediated by the quantum vacuum are known as van der Waals forces [2], while forces between macroscopic bodies are referred to as Casimir forces [3]. The third type of dispersion (in a sense an interpolation between these two extreme cases) is the Casimir-Polder (CP) force exerted on single atoms near macroscopic bodies [4].

For a two-level atom with transition frequency ω_A and electric-dipole moment d located at a distance z_A away from a perfectly conducting plate, the short-distance (nonretarded) [4] and long-distance (retarded) [5] limits of the CP potential take the well-known forms

$$U_{\text{nr}}(z_A) = -\frac{d^2}{48\pi\epsilon_0 z_A^3}, \quad U_{\text{r}}(z_A) = -\frac{cd^2}{16\pi^2\epsilon_0\omega_A z_A^4}. \quad (1)$$

These potentials, acting on atoms at rest, lead to conservative forces perpendicular to the plate's surface. CP forces (as well as all other dispersion forces) play important roles as limiting factors in efforts to miniaturize atom-optical devices [6] and have been measured at distances as small as 6 μm [7].

Casimir-Polder forces are well understood far beyond the aforementioned special case of a perfectly conducting plate, with magnetoelectric materials of arbitrary shape [8] and finite temperature being investigated theoretically [9,10] as well as experimentally [11]. While most theoretical investigations are based on Lifshitz's macroscopic treatment [12] or a linear-response description [13], full quantum theories based on electromagnetic-field quantization in magnetoelectrics have also been developed [8]. In the latter approach, the operator-valued Lorentz force

$$\hat{\mathbf{F}} = \int d^3r [\hat{\rho}_A(\mathbf{r})\hat{\mathbf{E}}(\mathbf{r}) + \hat{\mathbf{j}}_A(\mathbf{r}) \times \hat{\mathbf{B}}(\mathbf{r})] \quad (2)$$

on the atomic charge and current densities due to the body-assisted electromagnetic fields is computed. In the long-wavelength approximation, it leads to the well-known expression

$$\hat{\mathbf{F}} = \nabla_A[\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{r}_A)] + \frac{d}{dt}[\hat{\mathbf{d}} \times \hat{\mathbf{B}}(\mathbf{r}_A)] \quad (3)$$

with $\hat{\mathbf{d}}$ denoting the atomic electric-dipole moment operator.

Intuitively, dispersion forces can be understood as dipole-dipole forces generated by spontaneous polarization due to the electric-field fluctuations. Therefore, CP forces on atoms at rest act either toward (attractively) or away (repulsively) from the macroscopic body. For atoms in motion, retardation delays the dipole response, and a force component emerges along the direction of motion. In most cases, this force component acts against the motion and is thus the origin of quantum friction.

Quantum friction forces have traditionally been studied within a linear-response formalism [14–17]. Evaluating the correlated quantum fluctuations of moving atom and dielectrics, the friction force on ground-state atoms can be obtained in this way. However, the predicted forces are typically very small. For the stationary case it is well known that CP forces can be resonantly enhanced for excited atoms [8,18]. For such nonequilibrium situations, linear-response methods cannot be applied and a more detailed investigation of the atom-field dynamics becomes necessary.

In this paper, we develop a full quantum theory of the velocity-dependent CP force. In particular, we will show that for atoms and molecules in electronically excited states, both decelerating and accelerating forces can occur depending on the relative magnitude of the frequencies of the atomic transition and the surface plasmon. The paper is organized as follows. After briefly reviewing the formalism of macroscopic quantum electrodynamics in Sec. II, we study the atom-field dynamics in Sec. III A before investigating the resulting force in Sec. III B and applying our results to the

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quantum friction scenario in Sec. III C. We illustrate the theory with representative examples in Sec. IV, followed by a summary in Sec. V.

II. BASIC FORMULAS

Let us assume an arbitrary arrangement of dispersing and absorbing magnetoelectric bodies, characterized by their complex-valued Kramers-Kronig consistent permittivity $\varepsilon(\mathbf{r}, \omega)$ and permeability $\mu(\mathbf{r}, \omega)$. The Hamiltonian of the quantum electromagnetic field and the bodies can be given as (for a recent review, see Ref. [19])

$$\hat{H}_F = \sum_{\lambda=e,m} \int d^3r \int_0^\infty d\omega \hbar \omega \hat{\mathbf{f}}_\lambda^\dagger(\mathbf{r}, \omega) \cdot \hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega), \quad (4)$$

where the fundamental variables $\hat{\mathbf{f}}_\lambda^\dagger(\mathbf{r}, \omega)$ and $\hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega)$ are creation and annihilation operators for the elementary electric ($\lambda=e$) and magnetic ($\lambda=m$) excitations of the system; they obey the bosonic commutation relations

$$[\hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega), \hat{\mathbf{f}}_\lambda^\dagger(\mathbf{r}', \omega')] = \delta_{\lambda\lambda'} \delta(\mathbf{r} - \mathbf{r}') \delta(\omega - \omega'). \quad (5)$$

The electric and magnetic fields can be expanded in terms of the fundamental variables according to

$$\begin{aligned} \hat{\mathbf{E}}(\mathbf{r}) &= \int_0^\infty d\omega \hat{\mathbf{E}}(\mathbf{r}, \omega) + \text{H.c.} \\ &= \sum_{\lambda=e,m} \int d^3r' \int_0^\infty d\omega \mathbf{G}_\lambda(\mathbf{r}, \mathbf{r}', \omega) \cdot \hat{\mathbf{f}}_\lambda(\mathbf{r}', \omega) + \text{H.c.}, \end{aligned} \quad (6)$$

$$\begin{aligned} \hat{\mathbf{B}}(\mathbf{r}) &= \int_0^\infty d\omega \hat{\mathbf{B}}(\mathbf{r}, \omega) + \text{H.c.} \\ &= \sum_{\lambda=e,m} \int d^3r' \int_0^\infty \frac{d\omega}{i\omega} \nabla \times \mathbf{G}_\lambda(\mathbf{r}, \mathbf{r}', \omega) \cdot \hat{\mathbf{f}}_\lambda(\mathbf{r}', \omega) + \text{H.c.}, \end{aligned} \quad (7)$$

with coefficients

$$\mathbf{G}_e(\mathbf{r}, \mathbf{r}', \omega) = i \frac{\omega^2}{c^2} \sqrt{\frac{\hbar}{\pi \varepsilon_0}} \text{Im} \varepsilon(\mathbf{r}', \omega) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega), \quad (8)$$

$$\mathbf{G}_m(\mathbf{r}, \mathbf{r}', \omega) = i \frac{\omega}{c} \sqrt{\frac{\hbar}{\pi \varepsilon_0}} \frac{\text{Im} \mu(\mathbf{r}', \omega)}{|\mu(\mathbf{r}', \omega)|^2} [\nabla' \times \mathbf{G}(\mathbf{r}', \mathbf{r}, \omega)]^\top. \quad (9)$$

Here, \mathbf{G} is the classical Green tensor as uniquely defined by the inhomogeneous Helmholtz equation,

$$\left[\nabla \times \frac{1}{\mu(\mathbf{r}, \omega)} \nabla \times - \frac{\omega^2}{c^2} \varepsilon(\mathbf{r}, \omega) \right] \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}'), \quad (10)$$

together with the boundary condition

$$\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) \rightarrow \mathbf{0} \quad \text{for } |\mathbf{r} - \mathbf{r}'| \rightarrow \infty. \quad (11)$$

The Green tensor is an analytic function in the upper half of the complex frequency plane and it has the following useful properties:

$$\mathbf{G}(\mathbf{r}, \mathbf{r}', -\omega^*) = \mathbf{G}^*(\mathbf{r}, \mathbf{r}', \omega), \quad (12)$$

$$\mathbf{G}(\mathbf{r}', \mathbf{r}, \omega) = \mathbf{G}^\top(\mathbf{r}, \mathbf{r}', \omega), \quad (13)$$

$$\sum_{\lambda=e,m} \int d^3s \mathbf{G}_\lambda(\mathbf{r}, \mathbf{s}, \omega) \cdot \mathbf{G}_\lambda^{\top*}(\mathbf{r}', \mathbf{s}, \omega) \quad (14)$$

$$= \frac{\hbar \mu_0}{\pi} \omega^2 \text{Im} \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega). \quad (15)$$

The Hamiltonian describing the internal dynamics of an atom with eigenenergies E_n and eigenstates $|n\rangle$ can be given as

$$\hat{H}_A = \sum_k E_k \hat{A}_{kk} \quad (16)$$

($\hat{A}_{kk} = |k\rangle\langle k|$ are the atomic flip operators). Throughout this paper, we will assume that the center-of-mass motion is sufficiently slow so that it separates from the internal dynamics in the spirit of a Born-Oppenheimer approximation. The interaction of the atom with the body-assisted electromagnetic field is then adequately described by the atom-field coupling Hamiltonian for given center-of-mass position \mathbf{r}_A and velocity \mathbf{v}_A , which in the multipolar coupling scheme and electric-dipole approximation reads

$$\begin{aligned} \hat{H}_{AF} &= -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{r}_A) - \hat{\mathbf{d}} \cdot \mathbf{v}_A \times \hat{\mathbf{B}}(\mathbf{r}_A) \\ &= -\sum_{kl} \mathbf{d}_{kl} \cdot \hat{\mathbf{E}}(\mathbf{r}_A) \hat{A}_{kl} - \sum_{kl} \mathbf{d}_{kl} \cdot \mathbf{v}_A \times \hat{\mathbf{B}}(\mathbf{r}_A) \hat{A}_{kl} \end{aligned} \quad (17)$$

($\mathbf{d}_{kl} = \langle k | \hat{\mathbf{d}} | l \rangle$). The first term is the familiar electric-dipole interaction, while the second term is the Röntgen interaction associated with the center-of-mass motion. Combining Eqs. (4), (16), and (17), the total Hamiltonian of the atom-body-field system reads

$$\hat{H} = \hat{H}_A + \hat{H}_F + \hat{H}_{AF}. \quad (18)$$

Finally, the total Lorentz force on the atomic charge and current distribution can in electric-dipole approximation be given as

$$\begin{aligned} \hat{\mathbf{F}} &= \nabla_A [\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{r}_A) + \hat{\mathbf{d}} \cdot \mathbf{v}_A \times \hat{\mathbf{B}}(\mathbf{r}_A)] + \frac{d}{dt} [\hat{\mathbf{d}} \times \hat{\mathbf{B}}(\mathbf{r}_A)] \\ &= \nabla_A \sum_{kl} [\mathbf{d}_{kl} \cdot \hat{\mathbf{E}}(\mathbf{r}_A) \hat{A}_{kl} + \mathbf{d}_{kl} \cdot \mathbf{v}_A \times \hat{\mathbf{B}}(\mathbf{r}_A) \hat{A}_{kl}] \\ &\quad + \frac{d}{dt} \sum_{kl} [\mathbf{d}_{kl} \times \hat{\mathbf{B}}(\mathbf{r}_A) \hat{A}_{kl}]. \end{aligned} \quad (19)$$

III. CASIMIR-POLDER FORCE ON A MOVING ATOM

The Casimir-Polder force on an atom is the quantum average of the Lorentz force (19) with the body-assisted field being in its ground state. To evaluate this expression, we first need to solve the coupled atom-field dynamics.

A. Atom-field dynamics

Using Hamiltonian (18), the Heisenberg equations of motion of the atomic and field operators are found to be

$$\begin{aligned} \hat{A}_{mn} &= i\omega_{mn}\hat{A}_{mn} + \frac{i}{\hbar} \sum_k (\mathbf{d}_{nk}\hat{A}_{mk} - \mathbf{d}_{km}\hat{A}_{kn}) \cdot \hat{\mathbf{E}}(\mathbf{r}_A) \\ &+ \frac{i}{\hbar} \sum_k (\mathbf{d}_{nk}\hat{A}_{mk} - \mathbf{d}_{km}\hat{A}_{kn}) \cdot \mathbf{v}_A \times \hat{\mathbf{B}}(\mathbf{r}_A), \end{aligned} \quad (20)$$

where $\omega_{mn} = (E_m - E_n)/\hbar$, and

$$\begin{aligned} \hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega) &= -i\omega \hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega) + \frac{i}{\hbar} \sum_{k,l} \mathbf{G}_\lambda^* \mathbf{T}(\mathbf{r}, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kl} \hat{A}_{kl} \\ &+ \frac{1}{\hbar\omega} \sum_{k,l} [\mathbf{G}_\lambda^* \mathbf{T}(\mathbf{r}, \mathbf{r}_A, \omega) \times \tilde{\nabla}'] \times \mathbf{v}_A \cdot \mathbf{d}_{kl} \hat{A}_{kl} \end{aligned} \quad (21)$$

(by convention, ∇ and ∇' only act on the first or second argument of the Green tensor, respectively). The latter equation is formally solved by

$$\hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega, t) = \hat{\mathbf{f}}_{\lambda,f}(\mathbf{r}, \omega, t) + \hat{\mathbf{f}}_{\lambda,s}(\mathbf{r}, \omega, t), \quad (22)$$

where

$$\hat{\mathbf{f}}_{\lambda,f}(\mathbf{r}, \omega, t) = e^{-i\omega t} \hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega), \quad (23)$$

$$\begin{aligned} \hat{\mathbf{f}}_{\lambda,s}(\mathbf{r}, \omega, t) &= \frac{i}{\hbar} \sum_{k,l} \int_0^t d\tau e^{-i\omega(t-\tau)} \mathbf{G}_\lambda^* \mathbf{T}[\mathbf{r}, \mathbf{r}_A(\tau), \omega] \cdot \mathbf{d}_{kl} \hat{A}_{kl}(\tau) \\ &- \frac{1}{\hbar\omega} \sum_{k,l} \int_0^t d\tau e^{-i\omega(t-\tau)} \\ &\times \{ \mathbf{G}_\lambda^* \mathbf{T}[\mathbf{r}, \mathbf{r}_A(\tau), \omega] \times \tilde{\nabla}' \} \times \mathbf{v}_A \cdot \mathbf{d}_{kl} \hat{A}_{kl}(\tau) \end{aligned} \quad (24)$$

determine the free and source parts of the electromagnetic field.

We assume that the atom moves with uniform nonrelativistic speed ($v_A \ll c$) and we are seeking a solution to the system of Eqs. (20) and (21) within linear order of \mathbf{v}_A . We may hence write

$$\mathbf{r}_A(\tau) = \mathbf{r}_A(t) - (t - \tau)\mathbf{v}_A, \quad (25)$$

and after substituting Eqs. (22)–(24) into Eq. (6), using the integral relation (14), and applying a linear Taylor expansion in \mathbf{v}_A , the time-dependent frequency components of the electric field are given by

$$\hat{\mathbf{E}}(\mathbf{r}, \omega, t) = \hat{\mathbf{E}}_f(\mathbf{r}, \omega, t) + \hat{\mathbf{E}}_s(\mathbf{r}, \omega, t) \quad (26)$$

with

$$\hat{\mathbf{E}}_f(\mathbf{r}, \omega, t) = e^{-i\omega t} \hat{\mathbf{E}}(\mathbf{r}, \omega), \quad (27)$$

$$\begin{aligned} \hat{\mathbf{E}}_s(\mathbf{r}, \omega, t) &= \frac{i\mu_0}{\pi} \omega^2 \int_0^t d\tau e^{-i\omega(t-\tau)} \sum_{k,l} \text{Im} \mathbf{G}(\mathbf{r}, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kl} \hat{A}_{kl}(\tau) \\ &- \frac{i\mu_0}{\pi} \omega^2 \int_0^t d\tau (t - \tau) e^{-i\omega(t-\tau)} \\ &\times \sum_{k,l} \text{Im} \mathbf{G}(\mathbf{r}, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kl} (\tilde{\nabla}' \cdot \mathbf{v}_A) \hat{A}_{kl}(\tau) \\ &- \frac{\mu_0}{\pi} \omega \int_0^t d\tau e^{-i\omega(t-\tau)} \\ &\times \sum_{k,l} \text{Im} [\mathbf{G}(\mathbf{r}, \mathbf{r}_A, \omega) \times \tilde{\nabla}'] \times \mathbf{v}_A \cdot \mathbf{d}_{kl} \hat{A}_{kl}(\tau) \end{aligned} \quad (28)$$

$[\mathbf{r}_A = \mathbf{r}_A(t)]$. The magnetic field (7) only enters the equations of motion in conjunction with a factor \mathbf{v}_A , so we only require its zero-order expansion in the velocity,

$$\hat{\mathbf{B}}(\mathbf{r}, \omega, t) = \hat{\mathbf{B}}_f(\mathbf{r}, \omega, t) + \hat{\mathbf{B}}_s(\mathbf{r}, \omega, t) \quad (29)$$

with

$$\hat{\mathbf{B}}_f(\mathbf{r}, \omega, t) = e^{-i\omega t} \hat{\mathbf{B}}(\mathbf{r}, \omega), \quad (30)$$

$$\begin{aligned} \hat{\mathbf{B}}_s(\mathbf{r}, \omega, t) &= \frac{\mu_0}{\pi} \omega \int_0^t d\tau e^{-i\omega(t-\tau)} \\ &\times \sum_{k,l} \nabla \times \text{Im} \mathbf{G}(\mathbf{r}, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kl} \hat{A}_{kl}(\tau). \end{aligned} \quad (31)$$

We can next substitute our solutions (27)–(31) for the time-independent electromagnetic fields into the equation of motion (20) for the atomic flip operators. Noting that the total field operators $\hat{\mathbf{E}}(\mathbf{r}, \omega, t)$ and $\hat{\mathbf{B}}(\mathbf{r}, \omega, t)$ commute with the atomic flip operators at equal times, we arrange all products such that creation operators $\hat{\mathbf{f}}_\lambda^\dagger(\mathbf{r}, \omega)$ are always at the left and annihilation operators $\hat{\mathbf{f}}_\lambda(\mathbf{r}, \omega)$ are always at the right. Assuming the field to be initially prepared in its vacuum state and taking expectation values, all contributions from the source fields vanish. For weak atom-field coupling, the time integrals can be evaluated with the aid of the Markov approximation,

$$\begin{aligned} \int_0^t d\tau e^{-i\omega(t-\tau)} \langle \hat{A}_{ij}(t) \hat{A}_{kl}(\tau) \rangle &\simeq \langle \hat{A}_{ij}(t) \hat{A}_{kl}(t) \rangle \int_{-\infty}^t d\tau e^{-i(\omega - \tilde{\omega}_{lk})(t-\tau)} \\ &= \langle \hat{A}_{ij}(t) \rangle \delta_{jk} \left[\pi \delta(\omega - \tilde{\omega}_{lk}) - i\mathcal{P} \frac{1}{\omega - \tilde{\omega}_{lk}} \right] \end{aligned} \quad (32)$$

(here \mathcal{P} represents the principal value); similarly we have

$$\int_0^t d\tau(t-\tau)e^{-i\omega(t-\tau)}\langle\hat{A}_{ij}(t)\hat{A}_{kl}(\tau)\rangle \simeq \langle\hat{A}_{il}(t)\rangle\delta_{jk}\frac{d}{d\omega}\left[\mathcal{P}\frac{1}{\omega-\tilde{\omega}_{lk}}+i\pi\delta(\omega-\tilde{\omega}_{lk})\right], \quad (33)$$

where the shifted atomic transition frequencies

$$\tilde{\omega}_{mn} = \omega_{mn} + \delta\omega_m - \delta\omega_n \quad (34)$$

have yet to be determined.

For a nondegenerate atom, the resulting equations of motion for the off-diagonal atomic flip operators decouple from each other as well as from the diagonal ones. In addition, we consider an atom whose internal Hamiltonian (16) is time-reversal invariant, so that we may assume real dipole-matrix elements. After a lengthy but straightforward calculation, we finally obtain the following equations of motion for the internal atomic density matrix elements $\sigma_{mn} = \langle\hat{A}_{nm}\rangle$:

$$\dot{p}_n = -\Gamma_n p_n + \sum_k \Gamma_{kn} p_k, \quad (35)$$

$$\dot{\sigma}_{mn} = [-i\tilde{\omega}_{mn} - (\Gamma_m + \Gamma_n)/2]\sigma_{mn} \quad \text{for } m \neq n, \quad (36)$$

where we have introduced the probabilities $p_n = \sigma_{nn}$. The equations of motion for a moving atom have exactly the same form as for an atom at rest: the population of the diagonal density matrix elements is governed by spontaneous decay, while the off-diagonal ones undergo damped oscillations. However, the respective transition rates

$$\Gamma_n = \sum_k \Gamma_n^k \quad (37)$$

and frequency shifts

$$\delta\omega_n = \sum_k \delta\omega_n^k \quad (38)$$

are affected by the atomic motion,

$$\delta\omega_n^k = \delta\omega_n^k(\mathbf{r}_A) + \delta\omega_n^k(\mathbf{r}_A, \mathbf{v}_A), \quad (39)$$

$$\delta\omega_n^k(\mathbf{r}_A) = \frac{\mu_0}{\pi\hbar}\mathcal{P}\int_0^\infty d\omega \frac{\omega^2 \mathbf{d}_{nk} \cdot \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}}{\tilde{\omega}_{nk} - \omega}, \quad (40)$$

$$\delta\omega_n^k(\mathbf{r}_A, \mathbf{v}_A) = \frac{\mu_0}{2\hbar}\Theta(\tilde{\omega}_{nk})(\mathbf{v}_A \cdot \nabla_A) \times [\omega^2 \mathbf{d}_{nk} \cdot \text{Im } \mathbf{G}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}]'_{\omega=\tilde{\omega}_{nk}}, \quad (41)$$

$$\Gamma_n^k = \Gamma_n^k(\mathbf{r}_A) + \Gamma_n^k(\mathbf{r}_A, \mathbf{v}_A), \quad (42)$$

$$\Gamma_n^k(\mathbf{r}_A) = \frac{2\mu_0}{\hbar}\Theta(\tilde{\omega}_{nk})\tilde{\omega}_{nk}^2 \mathbf{d}_{nk} \cdot \text{Im } \mathbf{G}(\mathbf{r}_A, \mathbf{r}_A, \tilde{\omega}_{nk}) \cdot \mathbf{d}_{kn}, \quad (43)$$

$$\Gamma_n^k(\mathbf{r}_A, \mathbf{v}_A) = -\frac{\mu_0}{\pi\hbar}(\mathbf{v}_A \cdot \nabla_A)\mathcal{P}\int_0^\infty d\omega \times \frac{[\omega^2 \mathbf{d}_{nk} \cdot \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}]'}{\tilde{\omega}_{nk} - \omega} \quad (44)$$

(the primes indicate derivatives with respect to ω). Here, we have decomposed the Green tensor into its bulk (free-space) and scattering parts according to

$$\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \mathbf{G}^{(0)}(\mathbf{r}, \mathbf{r}', \omega) + \mathbf{G}^{(1)}(\mathbf{r}, \mathbf{r}', \omega) \quad (45)$$

and have discarded the Lamb-shift contribution due to $\mathbf{G}^{(0)}$ from the frequency shift $\delta\omega_n^k(\mathbf{r}_A)$ (as the free-space Lamb shift is assumed to be already included in the bare transition frequencies ω_{mn}). We have further exploited the symmetry (13) of the Green tensor which implies $\nabla' \mapsto \frac{1}{2}\nabla_A$, showing that the translationally invariant bulk Green tensor does not contribute to the velocity-dependent shifts and rates.

It is worth noting that for real dipole-matrix elements, the contributions from the Röntgen interaction exactly cancel. As a result, the velocity dependence of these quantities is entirely due to the fact that the moving atom emits and receives the electromagnetic field at different positions. The velocity-dependent contributions are proportional to the total derivative of the scattering Green tensor along the direction of motion. As a consequence, the decay rates and frequency shifts are unaffected by uniform motion in a direction along which the environment is translationally invariant (e.g., motion parallel to a plate or a cylinder). It is worth pointing out that such a vanishing of velocity-dependent frequency shifts does not necessarily imply that the velocity-dependent part of the CP force must also be zero. One should bear in mind that all of the above have only been shown within linear order in the velocity.

B. Casimir-Polder force

Having solved the coupled atom-field dynamics, we can now evaluate the quantum average of the Lorentz force (19). We restrict our attention to the pure dispersion force by again assuming the field to be initially prepared in its ground state. The atom may initially be in an arbitrary incoherent superposition of internal energy eigenstates. For an atom at rest, it has been found that the third term in Eq. (19), which involves a total time derivative, does not contribute to the force on atoms in incoherent internal states. We have explicitly checked that the same is true here, so that we only need to consider the force

$$\mathbf{F} = \nabla_A \langle \hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{r}_A) + \hat{\mathbf{d}} \cdot \mathbf{v}_A \times \hat{\mathbf{B}}(\mathbf{r}_A) \rangle = \nabla_A \sum_{kl} [\mathbf{d}_{kl} \cdot \langle \hat{\mathbf{E}}(\mathbf{r}_A) \hat{A}_{kl} \rangle + \mathbf{d}_{kl} \cdot \mathbf{v}_A \times \langle \hat{\mathbf{B}}(\mathbf{r}_A) \hat{A}_{kl} \rangle]. \quad (46)$$

We begin by substituting the time-dependent electromagnetic fields (26)–(31) where again we retain only terms up to linear order in \mathbf{v}_A and we arrange all products such that the contributions from the free fields vanish. The source fields give rise to intra-atomic correlation functions. By virtue of

the quantum regression theorem, Eq. (36) implies that the relevant correlation functions are of the form

$$\langle \hat{A}_{nk}(t) \hat{A}_{ln}(\tau) \rangle = \delta_{kl} e^{i\Omega_{nk}(t-\tau)} \sigma_{nn}(\tau) \quad (47)$$

with

$$\Omega_{nk} = \tilde{\omega}_{nk} + i(\Gamma_n + \Gamma_k)/2. \quad (48)$$

We evaluate time integrals in the spirit of the Markov approximation

$$\begin{aligned} \int_0^t d\tau e^{-i\omega(t-\tau)} \langle \hat{A}_{nk}(t) \hat{A}_{ln}(\tau) \rangle &\simeq \sigma_{nn}(t) \delta_{kl} \int_{-\infty}^t d\tau e^{-i(\omega - \Omega_{nk})(t-\tau)} \\ &= -\frac{\sigma_{nn}(t) i \delta_{kl}}{\omega - \Omega_{nk}} \end{aligned} \quad (49)$$

and similarly

$$\int_0^t d\tau(t-\tau) e^{-i\omega(t-\tau)} \langle \hat{A}_{nk}(t) \hat{A}_{ln}(\tau) \rangle \simeq -\frac{\sigma_{nn}(t) \delta_{kl}}{(\omega - \Omega_{nk})^2}. \quad (50)$$

Again assuming real dipole-matrix elements, the resulting expression for the CP force can be written in the form

$$\mathbf{F}(t) = \sum_n p_n(t) \mathbf{F}_n \quad (51)$$

with

$$\begin{aligned} \mathbf{F}_n &= \frac{\mu_0}{\pi} \sum_k \int_0^\infty d\omega \omega^2 \frac{\nabla \mathbf{d}_{nk} \cdot \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}}{\omega - \Omega_{nk}} + \frac{i\mu_0}{\pi} \sum_k \int_0^\infty d\omega \omega^2 \frac{\nabla(\mathbf{v}_A \cdot \nabla') \mathbf{d}_{nk} \cdot \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}}{(\omega - \Omega_{nk})^2} \\ &\quad + \frac{i\mu_0}{\pi} \sum_k \int_0^\infty d\omega \omega \frac{(\nabla' - \nabla) \mathbf{d}_{nk} \cdot \mathbf{v}_A \times [\nabla \times \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega)] \cdot \mathbf{d}_{kn}}{\omega - \Omega_{nk}} + \text{c.c.} \end{aligned} \quad (52)$$

Note that the two contributions from the Röntgen interaction have been collected in a single term as given on the second line of the above equation by making use of the symmetry (13) of the Green tensor. In addition, the (vanishing) contributions from the free-space Green tensor have been discarded.

Next, let us separate the forces \mathbf{F}_n into their position- and velocity-dependent parts. The shifted and broadened atomic transition frequencies Ω_{nk} are velocity dependent, so that the first term in Eq. (52) also contributes to the velocity-dependent part of the force. Again retaining only terms up to linear order in the velocity, we find

$$\mathbf{F}_n = \mathbf{F}_n(\mathbf{r}_A) + \mathbf{F}_n(\mathbf{r}_A, \mathbf{v}_A), \quad (53)$$

with

$$\begin{aligned} \mathbf{F}_n(\mathbf{r}_A) &= \frac{\mu_0}{2\pi} \sum_k \int_0^\infty d\omega \omega^2 \\ &\quad \times \frac{\nabla_A \mathbf{d}_{mk} \cdot \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}}{\omega - \Omega_{nk}} + \text{c.c.} \end{aligned} \quad (54)$$

and

$$\begin{aligned} \mathbf{F}_n(\mathbf{r}_A, \mathbf{v}_A) &= \frac{\mu_0}{\pi} \sum_k \int_0^\infty d\omega \omega^2 \frac{\Omega_{nk}(\mathbf{v}_A) \nabla \mathbf{d}_{nk} \cdot \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}}{(\omega - \Omega_{nk})^2} \\ &\quad + \frac{i\mu_0}{\pi} \sum_k \int_0^\infty d\omega \omega^2 \frac{\nabla(\mathbf{v}_A \cdot \nabla') \mathbf{d}_{nk} \cdot \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}}{(\omega - \Omega_{nk})^2} \\ &\quad + \frac{i\mu_0}{\pi} \sum_k \int_0^\infty d\omega \omega \frac{(\nabla' - \nabla) \mathbf{d}_{nk} \cdot \mathbf{v}_A \times [\nabla \times \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega)] \cdot \mathbf{d}_{kn}}{\omega - \Omega_{nk}} + \text{c.c.}, \end{aligned} \quad (55)$$

where $\Omega_{nk} \equiv \Omega_{nk}(\mathbf{r}_A)$ and $\Omega_{nk}(\mathbf{v}_A) \equiv \Omega_{nk}(\mathbf{r}_A, \mathbf{v}_A)$. The velocity-independent force (54) is just the well-known CP

force on an atom at rest. We will in the following restrict our attention to the velocity-dependent force (55), which consists

of three terms: The first generalized Doppler term is due to the velocity dependence of the atomic transition frequencies. The second delay term is associated with the time interval between emission and reabsorption of the electromagnetic field. The third Röntgen term is due to the coupling of the current density associated with the moving atom to the magnetic field.

In close analogy to the case of an atom at rest, the force can be separated into its resonant and nonresonant parts using contour-integral techniques. Writing $\text{Im } \mathbf{G} = (\mathbf{G} - \mathbf{G}^*) / (2i)$, using the property (12) of the Green tensor, and employing Cauchy's theorem to transform integrals along the real axis to integrals along the positive imaginary axis plus contributions from the poles, one can show that

$$\int_0^\infty d\omega \frac{\omega \text{Im } \mathbf{G}^{(1)}(\mathbf{r}, \mathbf{r}', \omega)}{\omega - \Omega_{nk}} = \int_0^\infty d\xi \frac{\xi^2 \mathbf{G}^{(1)}(\mathbf{r}, \mathbf{r}', i\xi)}{\xi^2 + \Omega_{nk}^2} + \pi \Omega_{nk} \mathbf{G}^{(1)}(\mathbf{r}, \mathbf{r}', \Omega_{nk}) \Theta(\tilde{\omega}_{nk}), \quad (56)$$

$$\int_0^\infty d\omega \frac{\omega^2 \text{Im } \mathbf{G}^{(1)}(\mathbf{r}, \mathbf{r}', \omega)}{(\omega - \Omega_{nk})^2} = - \int_0^\infty d\xi \frac{\xi^2 (\Omega_{nk}^2 - \xi^2) \mathbf{G}^{(1)}(\mathbf{r}, \mathbf{r}', i\xi)}{(\xi^2 + \Omega_{nk}^2)^2} + \pi [\omega^2 \mathbf{G}^{(1)}(\mathbf{r}, \mathbf{r}', \omega)]'_{\omega=\Omega_{nk}} \Theta(\tilde{\omega}_{nk}). \quad (57)$$

Substituting these results into Eq. (55), one finds

$$\mathbf{F}_n(\mathbf{r}_A, \mathbf{v}_A) = \mathbf{F}_n^{\text{nr}}(\mathbf{r}_A, \mathbf{v}_A) + \mathbf{F}_n^{\text{r}}(\mathbf{r}_A, \mathbf{v}_A) \quad (58)$$

with

$$\begin{aligned} \mathbf{F}_n^{\text{nr}}(\mathbf{r}_A, \mathbf{v}_A) = & - \frac{\hbar \mu_0}{2\pi} \int_0^\infty d\xi \xi^2 \nabla \text{tr}\{[\boldsymbol{\alpha}_n(\mathbf{v}_A, i\xi) + \boldsymbol{\alpha}_n(\mathbf{v}_A, -i\xi)] \cdot \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, i\xi)\} \\ & - \frac{i\hbar \mu_0}{2\pi} \int_0^\infty d\xi \xi^2 \nabla (\mathbf{v}_A \cdot \nabla') \text{Tr}\{[\boldsymbol{\alpha}'_n(i\xi) + \boldsymbol{\alpha}'_n(-i\xi)] \cdot \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, i\xi)\} \\ & + \frac{\hbar \mu_0}{2\pi} \int_0^\infty d\xi \xi (\nabla' - \nabla) \text{Tr}\{[\boldsymbol{\alpha}_n(i\xi) - \boldsymbol{\alpha}_n(-i\xi)] \cdot \mathbf{v}_A \times [\nabla \times \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, i\xi)]\} \end{aligned} \quad (59)$$

and

$$\begin{aligned} \mathbf{F}_n^{\text{r}}(\mathbf{r}_A, \mathbf{v}_A) = & \mu_0 \sum_k \Theta(\tilde{\omega}_{nk}) \Omega_{nk}(\mathbf{v}_A) [\omega^2 \nabla \mathbf{d}_{nk} \cdot \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}]'_{\omega=\Omega_{nk}} \\ & + i\mu_0 \sum_k \Theta(\tilde{\omega}_{nk}) [\omega^2 \nabla (\mathbf{v}_A \cdot \nabla') \mathbf{d}_{nk} \cdot \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}]'_{\omega=\Omega_{nk}} \\ & + i\mu_0 \sum_k \Theta(\tilde{\omega}_{nk}) \Omega_{nk} (\nabla' - \nabla) \mathbf{d}_{nk} \cdot \mathbf{v}_A \times [\nabla \times \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \Omega_{nk})] \cdot \mathbf{d}_{kn} + \text{c.c.} \end{aligned} \quad (60)$$

Here,

$$\boldsymbol{\alpha}_n(\omega) = \frac{1}{\hbar} \sum_k \left[\frac{\mathbf{d}_{kn} \mathbf{d}_{nk}}{\omega - \Omega_{nk}^*} - \frac{\mathbf{d}_{nk} \mathbf{d}_{kn}}{\omega + \Omega_{nk}} \right] \quad (61)$$

is the polarizability for an atom at rest and

$$\boldsymbol{\alpha}_n(\mathbf{v}_A, \omega) = \frac{1}{\hbar} \sum_k \left[\frac{\Omega_{nk}^*(\mathbf{v}_A) \mathbf{d}_{kn} \mathbf{d}_{nk}}{(\omega - \Omega_{nk}^*)^2} + \frac{\Omega_{nk}(\mathbf{v}_A) \mathbf{d}_{nk} \mathbf{d}_{kn}}{(\omega + \Omega_{nk})^2} \right] \quad (62)$$

is the correction to this polarizability for a moving atom within linear order of the atomic velocity.

It is instructive to consider the perturbative limit $\Omega_{nk} \rightarrow \omega_{nk}$ (i.e., $\delta\omega_{n/k}, \Gamma_{n/k} \rightarrow 0$). The resonant force can be represented by its zero-order approximation in $\delta\omega_{n/k}$ and $\Gamma_{n/k}$ which reads

$$\begin{aligned}
\mathbf{F}_n^r(\mathbf{r}_A, \mathbf{v}_A) = & 2\mu_0 \sum_k \Theta(\omega_{nk}) [\delta\omega_n(\mathbf{v}_A) - \delta\omega_k(\mathbf{v}_A)] [\omega^2 \nabla \mathbf{d}_{nk} \cdot \text{Re } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}]'_{\omega=\omega_{nk}} \\
& - \mu_0 \sum_k \Theta(\omega_{nk}) [\Gamma_n(\mathbf{v}_A) + \Gamma_k(\mathbf{v}_A)] [\omega^2 \nabla \mathbf{d}_{nk} \cdot \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}]'_{\omega=\omega_{nk}} \\
& - 2\mu_0 \sum_k \Theta(\omega_{nk}) [\omega^2 \nabla (\mathbf{v}_A \cdot \nabla') \mathbf{d}_{nk} \cdot \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) \cdot \mathbf{d}_{kn}]'_{\omega=\omega_{nk}} \\
& - 2\mu_0 \sum_k \Theta(\omega_{nk}) \omega_{nk} (\nabla' - \nabla) \mathbf{d}_{nk} \cdot \mathbf{v}_A \times [\nabla \times \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega_{nk})] \cdot \mathbf{d}_{kn}. \tag{63}
\end{aligned}$$

The nonresonant velocity-dependent force vanishes to zeroth order in the frequency shifts and decay rates in contrast to the force observed for an atom at rest. The leading nonvanishing contribution is linear in these quantities and it reads

$$\begin{aligned}
\mathbf{F}_n^{\text{nr}}(\mathbf{r}_A, \mathbf{v}_A) = & -\frac{2\mu_0}{\pi} \sum_k \int_0^\infty d\xi \xi^2 \frac{[\delta\omega_n(\mathbf{v}_A) - \delta\omega_k(\mathbf{v}_A)] (\omega_{kn}^2 - \xi^2)}{(\omega_{kn}^2 + \xi^2)^2} \nabla \mathbf{d}_{nk} \cdot \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, i\xi) \cdot \mathbf{d}_{kn} \\
& + \frac{2\mu_0}{\pi} \sum_k \int_0^\infty d\xi \xi^2 \frac{\omega_{kn} (\Gamma_n + \Gamma_k) (\omega_{kn}^2 - 3\xi^2)}{(\omega_{kn}^2 + \xi^2)^3} \nabla (\mathbf{v}_A \cdot \nabla') \mathbf{d}_{nk} \cdot \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, i\xi) \cdot \mathbf{d}_{kn} \\
& - \frac{2\mu_0}{\pi} \sum_k \int_0^\infty d\xi \xi^2 \frac{\omega_{kn} (\Gamma_n + \Gamma_k)}{(\omega_{kn}^2 + \xi^2)^2} (\nabla' - \nabla) \mathbf{d}_{nk} \cdot \mathbf{v}_A \times [\nabla \times \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, i\xi)] \cdot \mathbf{d}_{kn}. \tag{64}
\end{aligned}$$

For an isotropic atom, these results reduce to

$$\begin{aligned}
\mathbf{F}_n^r(\mathbf{r}_A, \mathbf{v}_A) = & \frac{2\mu_0}{3} \sum_k \Theta(\omega_{nk}) |\mathbf{d}_{nk}|^2 [\delta\omega_n(\mathbf{v}_A) - \delta\omega_k(\mathbf{v}_A)] [\omega^2 \nabla \text{Tr Re } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega)]'_{\omega=\omega_{nk}} \\
& - \frac{\mu_0}{3} \sum_k \Theta(\omega_{nk}) |\mathbf{d}_{nk}|^2 [\Gamma_n(\mathbf{v}_A) + \Gamma_k(\mathbf{v}_A)] [\omega^2 \nabla \text{Tr Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega)]'_{\omega=\omega_{nk}} \\
& - \frac{2\mu_0}{3} \sum_k \Theta(\omega_{nk}) |\mathbf{d}_{nk}|^2 [\omega^2 \nabla (\mathbf{v}_A \cdot \nabla') \text{Tr Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega)]'_{\omega=\omega_{nk}} \\
& - \frac{2\mu_0}{3} \sum_k \Theta(\omega_{nk}) \omega_{nk} |\mathbf{d}_{nk}|^2 (\nabla' - \nabla) \text{Tr} \{ \mathbf{v}_A \times [\nabla \times \text{Im } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega_{nk})] \} \tag{65}
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{F}_n^{\text{nr}}(\mathbf{r}_A, \mathbf{v}_A) = & -\frac{2\mu_0}{3\pi} \sum_k |\mathbf{d}_{nk}|^2 \int_0^\infty d\xi \xi^2 \frac{[\delta\omega_n(\mathbf{v}_A) - \delta\omega_k(\mathbf{v}_A)] (\omega_{kn}^2 - \xi^2)}{(\omega_{kn}^2 + \xi^2)^2} \nabla \text{Tr } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, i\xi) \\
& + \frac{2\mu_0}{3\pi} \sum_k |\mathbf{d}_{nk}|^2 \int_0^\infty d\xi \xi^2 \frac{\omega_{kn} (\Gamma_n + \Gamma_k) (\omega_{kn}^2 - 3\xi^2)}{(\omega_{kn}^2 + \xi^2)^3} \nabla (\mathbf{v}_A \cdot \nabla') \text{Tr } \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, i\xi) \\
& - \frac{2\mu_0}{3\pi} \sum_k |\mathbf{d}_{nk}|^2 \int_0^\infty d\xi \xi^2 \frac{\omega_{kn} (\Gamma_n + \Gamma_k)}{(\omega_{kn}^2 + \xi^2)^2} (\nabla' - \nabla) \text{Tr} \{ \mathbf{v}_A \times [\nabla \times \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, i\xi)] \}. \tag{66}
\end{aligned}$$

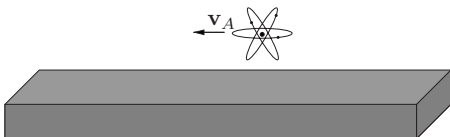


FIG. 1. Atom moving with a velocity \mathbf{v} near a planar surface.

C. Motion parallel to a planar interface

Up until this point, all results are valid for arbitrary geometries. In order to gain physical insight, we restrict ourselves to the generic quantum friction scenario of an atom moving parallel $[\mathbf{v}_A = \mathbf{v}_\parallel = (v_x, v_y, 0)^T]$ to a homogeneous dielectric or metal of permittivity $\varepsilon(\omega)$ whose plane surface defines the (x, y) plane (see Fig. 1). The Weyl expansion of the Green tensor,

$$\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \int \frac{d^2 k_{\parallel}}{(2\pi)^2} e^{i\mathbf{k}_{\parallel} \cdot (\boldsymbol{\rho} - \boldsymbol{\rho}')} \mathbf{G}(\mathbf{k}_{\parallel}, z, z', \omega), \quad (67)$$

with $\mathbf{r} = (\boldsymbol{\rho}, z)$ can then be used to calculate explicit expressions for the terms that contribute to the velocity-dependent force. The relevant Weyl components $\mathbf{G}(\mathbf{k}_{\parallel}, z, z', \omega)$ of the Green tensor for $z, z' > 0$ are given by [$G_{ij} \equiv G_{ij}(\mathbf{k}_{\parallel}, z, z', \omega)$]

$$G_{xx} = \frac{i}{2k_z} e^{ik_z(z+z')} \left[r_s \frac{k_y^2}{k_{\parallel}^2} - r_p \frac{k_x^2 k_y^2}{k_{\parallel}^2 k_{\parallel}^2} \right], \quad (68)$$

$$G_{xy} = \frac{i}{2k_z} e^{ik_z(z+z')} \left[-r_s \frac{k_x k_y}{k_{\parallel}^2} - r_p \frac{k_x^2 k_y}{k_{\parallel}^2 k_{\parallel}^2} \right], \quad (69)$$

$$G_{xz} = -\frac{i}{2k_z} e^{ik_z(z+z')} r_p \frac{k_z k_x}{k^2}, \quad (70)$$

$$G_{zz} = \frac{i}{2k_z} e^{ik_z(z+z')} r_p \frac{k_{\parallel}^2}{k^2}, \quad (71)$$

with

$$r_s = \frac{k_z - k_{1z}}{k_z + k_{1z}}, \quad r_p = \frac{\varepsilon(\omega)k_z - k_{1z}}{\varepsilon(\omega)k_z + k_{1z}} \quad (72)$$

being the Fresnel reflection coefficients of the surface for s - and p -polarized waves [$k^2 = \omega^2/c^2$, $k_1^2 = \varepsilon(\omega)\omega^2/c^2$, $k_{(1)z}^2 = k_{(1)}^2 - k_{\parallel}^2$]. The other components of the Green's tensor can be obtained by using the reciprocity condition $\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \mathbf{G}^T(\mathbf{r}', \mathbf{r}, \omega)$, which translates into $\mathbf{G}(\mathbf{k}_{\parallel}, z, z', \omega) = \mathbf{G}^T(-\mathbf{k}_{\parallel}, z', z, \omega)$, and the replacement rules $G_{yy} = G_{xx}$ ($k_x \leftrightarrow k_y$), $G_{yz} = G_{xz}$ ($k_x \leftrightarrow k_y$).

For the assumed motion parallel to the surface, the velocity-dependent shifts and rates vanish $\delta\omega_n^k(\mathbf{r}_A, \mathbf{v}_A) = \Gamma_n^k(\mathbf{r}_A, \mathbf{v}_A) = 0$ (cf. the remark at the end of Sec. III A) and so do the generalized Doppler contributions to the resonant force (65) (first two terms) and the nonresonant force (66) (first term). To calculate the delay and Röntgen contributions, we require second derivatives of the Green's tensor as given above. It is useful to note that derivatives vanish unless they contain an even number for each of the Cartesian indices (x, y, z). For example, terms such as $\partial_x \partial_y G_{xx}$ or $\partial_y \partial_y G_{xz}$ will not contribute, whereas terms such as $\partial_x \partial_y G_{xy}$ or $\partial_y \partial_y G_{zz}$ will. For simplicity, we restrict our attention to the nonretarded or near-field limit, where the dominant contribution to the Green's tensor is due to evanescent waves with $k_{1z} \approx k_z \approx ik_{\parallel}$. With this replacement, we have

$$r_s = 0, \quad r_p = \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1} \quad (73)$$

and Eqs. (67)–(71) lead to

$$\nabla(\mathbf{v}_{\parallel} \cdot \nabla') \text{Tr} \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega) = \frac{3c^2 \mathbf{v}_{\parallel}}{16\pi\omega^2 z_A^5} \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1}, \quad (74)$$

$$(\nabla' - \nabla) \text{Tr} \{ \mathbf{v}_{\parallel} \times [\nabla \times \mathbf{G}^{(1)}(\mathbf{r}_A, \mathbf{r}_A, \omega)] \} = \mathbf{0}. \quad (75)$$

In the near-field limit, the Röntgen contribution hence also vanishes and quantum friction is entirely due to the delay effect.

Substituting Eqs. (76) and (75) into Eqs. (65) and (66), we find the friction forces,

$$\mathbf{F}_n^r(\mathbf{r}_A, \mathbf{v}_{\parallel}) = -\frac{\mathbf{v}_{\parallel}}{4\pi\varepsilon_0 z_A^5} \sum_k \Theta(\omega_{nk}) |\mathbf{d}_{nk}|^2 \left[\frac{\text{Im} \varepsilon(\omega)}{|\varepsilon(\omega) + 1|^2} \right]_{\omega=\omega_{nk}}, \quad (76)$$

and

$$\mathbf{F}_n^{\text{nr}}(\mathbf{r}_A, \mathbf{v}_{\parallel}) = -\frac{\mathbf{v}_{\parallel}}{8\pi\varepsilon_0 z_A^5} \sum_k |\mathbf{d}_{nk}|^2 \omega_{kn} (\Gamma_n + \Gamma_k) \times \int_0^{\infty} d\xi \frac{\omega_{kn}^2 - 3\xi^2}{(\omega_{kn}^2 + \xi^2)^3} \frac{\varepsilon(i\xi) - 1}{\varepsilon(i\xi) + 1}. \quad (77)$$

If we further assume a single-resonance Drude-Lorentz model for the permittivity,

$$\varepsilon(\omega) = 1 + \frac{\omega_p^2}{\omega_T^2 - \omega^2 - 2i\gamma\omega}, \quad (78)$$

with plasma frequency ω_p , transverse resonance frequency ω_T , and linewidth γ , we find that for a weakly absorbing medium ($\gamma \ll \omega_{p,T}$) the resonant and nonresonant forces are given by

$$\mathbf{F}_n^r(\mathbf{r}_A, \mathbf{v}_{\parallel}) = \frac{\mathbf{v}_{\parallel}}{8\pi\varepsilon_0 z_A^5} \sum_k \Theta(\omega_{nk}) |\mathbf{d}_{nk}|^2 \frac{\gamma \omega_p^2 (\omega_S^2 + 3\omega_{nk}^2)}{(\omega_{nk}^2 - \omega_S^2)^3} \quad (79)$$

and

$$\mathbf{F}_n^{\text{nr}}(\mathbf{r}_A, \mathbf{v}_{\parallel}) = -\frac{\mathbf{v}_{\parallel}}{32\pi\varepsilon_0 z_A^5} \sum_k |\mathbf{d}_{nk}|^2 \frac{\text{sgn}(\omega_{kn}) (\Gamma_n + \Gamma_k) \omega_p^2}{\omega_S (|\omega_{kn}| + \omega_S)^3} \quad (80)$$

($\omega_S = \sqrt{\omega_T^2 + \omega_p^2/2}$ is the surface-plasmon frequency).

Let us discuss our results. We first note that in a quantum friction scenario of an atom moving parallel to a plane surface, a generalized Doppler effect does not contribute to the velocity-dependent force; this will be different for an atom moving perpendicularly toward the surface. In the near-field limit, the magnetic Röntgen coupling becomes negligible as well; it will become relevant for larger distances. Near-field quantum friction forces are hence dominantly caused by a delay effect.

For a ground-state atom, only a nonresonant force component (80) is present. With both ω_{k0} and Γ_k being positive quantities, $\mathbf{F}_n^{\text{nr}}(\mathbf{r}_A, \mathbf{v}_{\parallel})$ is strictly antiparallel to the velocity and hence presents a genuine friction force. Note that this force is proportional to the rates of spontaneous decay Γ_k , the absorption parameters of the atom. In the near-field limit, these decay rates are given by [20–22]

$$\begin{aligned}\Gamma_n &= \sum_k \Gamma_{nk} = \sum_k \Theta(\omega_{nk}) \frac{|\mathbf{d}_{nk}|^2}{6\pi\hbar\epsilon_0 z_A^3} \frac{\text{Im } \varepsilon(\omega_{nk})}{|\varepsilon(\omega_{nk}) + 1|^2} \\ &= \sum_k \Theta(\omega_{nk}) \frac{|\mathbf{d}_{nk}|^2}{12\pi\hbar\epsilon_0 z_A^3} \frac{\gamma\omega_{nk}\omega_p^2}{(\omega_{nk}^2 - \omega_S^2)^2}.\end{aligned}\quad (81)$$

Inserting this into Eq. (80) yields a friction force that is extremely short ranged and falls off as z_A^{-8} . This is in contrast to previous theories [14] that predict a z_A^{-5} scaling, resulting from a disregard of the distance dependence of the spontaneous decay rate.

For an excited atom, resonant forces arise as a consequence of possible transitions to lower lying atomic energy levels. They will dominate the velocity-dependent force in particular if one of them is near resonant with the surface-plasmon frequency ω_S . Depending on whether the respective atomic transition frequency ω_{nk} is smaller or greater than the surface-plasmon frequency, the velocity-dependent resonant force will either be a decelerating friction force antiparallel to the velocity or it may be a quantum acceleration force parallel to the velocity. This can be qualitatively understood from an energy consideration. The energy $\hbar\omega_{nk}$ emitted during a downward transition of the atom is resonantly absorbed by the surface and leads to the excitation of a surface plasmon with energy $\hbar\omega_S$. The energy difference between these two reservoirs leads to a change in the atom's kinetic energy. If the emitted energy is smaller than the absorbed one, the atom has to decelerate; if the emitted energy is greater than the absorbed one, the atom will accelerate. Note also that the off-resonant contribution (80) consists of strictly accelerating downward contributions as well as strictly decelerating upward contributions which, as before, have a z_A^{-8} scaling and can be safely neglected with regard to the z_A^{-5} scaling of the resonant forces. It is known from previous studies [23] that the rate of spontaneous decay increases for atoms in motion. This mechanism leads to a more rapid dissipation of the internal energy initially stored in the atom, restricting the lifetime of resonant forces; it needs to be taken into account in a more quantitative analysis of energy conservation.

IV. EXAMPLES

In order to illustrate the effect of velocity-dependent forces on atoms, we present a selection of numerical examples. We will concentrate on forces that are dominated by a single atomic transition between a ground state $|0\rangle$ and an excited state $|1\rangle$ with frequency ω_A and (isotropic) dipole-matrix element d . In this case, the nonresonant ground-state force (80) reduces to the simpler expression

$$\mathbf{F}_0(\mathbf{r}_A, \mathbf{v}_{\parallel}) = -\frac{\mathbf{v}_{\parallel} d^2}{32\pi\epsilon_0 z_A^5} \frac{\Gamma\omega_p^2}{\omega_S(\omega_A + \omega_S)^3}, \quad (82)$$

where the decay rate (81) now reads

$$\Gamma = \frac{d^2}{12\pi\hbar\epsilon_0 z_A^3} \frac{\gamma\omega_A\omega_p^2}{(\omega_A^2 - \omega_S^2)^2}. \quad (83)$$

The excited-state force is dominated by the resonant force component, $\mathbf{F}_1(\mathbf{r}_A, \mathbf{v}_{\parallel}) = \mathbf{F}_1^{\text{nr}}(\mathbf{r}_A, \mathbf{v}_{\parallel}) + \mathbf{F}_1^{\text{r}}(\mathbf{r}_A, \mathbf{v}_{\parallel}) \approx \mathbf{F}_1^{\text{r}}(\mathbf{r}_A, \mathbf{v}_{\parallel})$,

$$\mathbf{F}_1(\mathbf{r}_A, \mathbf{v}_{\parallel}) = \frac{\mathbf{v}_{\parallel} d^2}{8\pi\epsilon_0 z_A^5} \frac{\gamma\omega_p^2(\omega_S^2 + 3\omega_A^2)}{(\omega_A^2 - \omega_S^2)^3}. \quad (84)$$

As a first example, we consider a ground-state ^{87}Rb atom moving parallel to a gold surface. We consider the lowest electronic transition $D_2(5^2S_{1/2} \rightarrow 5^2P_{3/2})$ with wavelength $\lambda_A = 780 \text{ nm}$ ($\omega_A = 2.41 \times 10^{15} \text{ rad s}^{-1}$) [24] and dipole moment $d = 4.23ea_0 = 3.58 \times 10^{-29} \text{ C m}$ [25]. The permittivity of gold may be characterized by a plasma frequency $\omega_p = 1.37 \times 10^{16} \text{ rad s}^{-1}$ and an absorption parameter $\gamma = 4.12 \times 10^{13} \text{ rad s}^{-1}$ [22]. Note that the transverse resonance frequency vanishes for metals, $\omega_T = 0$, so that the surface-plasmon resonance is located at $\omega_S = \omega_p/\sqrt{2}$. With these parameters, we find a deceleration of the rubidium atom as ($m_{^{87}\text{Rb}} = 1.44 \times 10^{-25} \text{ kg}$)

$$\mathbf{a}_{\parallel} = -\mathbf{v}_{\parallel} (9.6 \text{ s}^{-1}) \left[\frac{1 \text{ nm}}{z_A} \right]^8. \quad (85)$$

The force is extremely short ranged and is negligible for any reasonable values of the velocity and atom-surface distance.

In contrast, for an excited rubidium atom with the same data as above, the deceleration becomes

$$\mathbf{a}_{\parallel} = -\mathbf{v}_{\parallel} (5.0 \times 10^4 \text{ s}^{-1}) \left[\frac{1 \text{ nm}}{z_A} \right]^5. \quad (86)$$

In comparison to the ground-state force, excited-state quantum friction is strongly enhanced and has a much longer range. For an atomic velocity of $v = 200 \text{ ms}^{-1}$, the deceleration at an atom-surface distance $z_A = 10 \text{ nm}$ can be as large as $a = -100 \text{ ms}^{-2}$. Even at $z_A = 100 \text{ nm}$ the deceleration is still $a = -10^{-3} \text{ ms}^{-2}$.

Results for other atoms and metallic surfaces can be easily obtained by noting that in most cases, the relevant atomic transition frequency is much smaller than the surface-plasmon frequency of the metal, hence $\omega_A \ll \omega_S$. Under this approximation, the excited-state force (84) and the decay rate (83) read as

$$\mathbf{F}_1(\mathbf{r}_A, \mathbf{v}_{\parallel}) \approx -\frac{\mathbf{v}_{\parallel} d^2}{2\pi\epsilon_0 z_A^5} \frac{\gamma}{\omega_p^2} \quad (87)$$

and

$$\Gamma \approx \frac{d^2 \omega_A}{3\pi\hbar\epsilon_0 z_A^3} \frac{\gamma}{\omega_p^2}. \quad (88)$$

Typical values for the material parameter ω_p^2/γ are tabulated in Ref. [22]. Note that unless the excitation is maintained by continuous repumping, the excited-state force only acts during a time interval $\Delta t \approx \Gamma^{-1}$. The relative velocity change during this time is approximately

$$\frac{\Delta v}{v} \approx \frac{F_1}{\Gamma m v} \approx -\frac{3\hbar}{m_A z_A^2} \frac{1}{2\omega_A}. \quad (89)$$

In this limit ($\omega_A \ll \omega_S$), the relative change in velocity is independent of the strength of the atomic dipole transition and all material parameters.

Upon inspection of the excited-state force (84) one notices that this force can be resonantly enhanced if an atomic

transition matches the frequency of a surface-plasmon resonance. An example of such a close match has been pointed out in Refs. [26,27] and involves a sapphire substrate with its principal surface plasmon at $\lambda_S=12.21 \mu\text{m}$ ($\omega_S=1.54 \times 10^{14} \text{ rad s}^{-1}$) and the $6D_{3/2} \rightarrow 7P_{1/2}$ transition in ^{133}Cs with a wavelength of $\lambda_A=12.15 \mu\text{m}$ ($\omega_A=1.55 \times 10^{14} \text{ rad s}^{-1}$). Near this plasmon resonance, the permittivity of sapphire is well approximated by [27]

$$\varepsilon_{\text{sapphire}}(\omega) = \eta + \frac{\eta\omega_p^2}{\omega_T^2 - \omega^2 - 2i\gamma\omega} \quad (90)$$

with $\eta=2.71$, $\omega_p=0.84\omega_S=1.29 \times 10^{14} \text{ rad s}^{-1}$, $\omega_T=0.70\omega_S=1.08 \times 10^{14} \text{ rad s}^{-1}$, and $\gamma=7.5 \times 10^{-3}\omega_S=1.16 \times 10^{12} \text{ rad s}^{-1}$; note that $\omega_S=\sqrt{\omega_T^2+\omega_p^2}/(\eta+1)$. With this model and introducing the atom-plasmon detuning $\delta=\omega_A-\omega_S$, we find that the force (76) and the decay rate (81) in the vicinity of the surface-plasmon resonance may be given as ($|\delta|, \gamma \ll \omega_S$)

$$\mathbf{F}_1(\mathbf{r}_A, \mathbf{v}_{\parallel}) \approx \frac{\mathbf{v}_{\parallel} d^2}{4\pi\varepsilon_0 z_A^5} \frac{\eta}{(\eta+1)^2} \frac{\omega_p^2}{\omega_S} \frac{\gamma\delta}{(\delta^2 + \gamma^2)^2} \quad (91)$$

and

$$\Gamma \approx \frac{d^2}{12\pi\hbar\varepsilon_0 z_A^3} \frac{\eta}{(\eta+1)^2} \frac{\omega_p^2}{\omega_S} \frac{\gamma}{\delta^2 + \gamma^2}. \quad (92)$$

With the dipole moment of the above-mentioned transition being $d=5.85 \times 10^{-29} \text{ C m}$ [28], one finds ($m^{133}\text{Cs}=2.21 \times 10^{-25} \text{ kg}$)

$$\mathbf{a}_{\parallel} = +\mathbf{v}_{\parallel} (7.1 \times 10^{11} \text{ s}^{-1}) \left[\frac{1 \text{ nm}}{z_A} \right]^5. \quad (93)$$

Compared with the result (86) for the excited-state force near a metal, we find a significantly enhanced force. Note also that, because $\omega_A > \omega_S$ (i.e., $\delta > 0$), the force is accelerating rather than decelerating. As a numerical example, for a particle velocity of $v=100 \text{ ms}^{-1}$ and an atom-surface distance of $z_A=100 \text{ nm}$, one would observe an acceleration of $a=7 \times 10^4 \text{ ms}^{-2}$. As before, without continuous repumping this force acts only for a very short time, leading to a net relative change in velocity

$$\frac{\Delta v}{v} \approx \frac{3\hbar}{m z_A^2} \frac{\delta}{\delta^2 + \gamma^2}. \quad (94)$$

V. SUMMARY

We have shown that atoms or molecules in relative motion with respect to an electric surface experience velocity-dependent CP forces. Solving the coupled atom-field dynamics for a slowly moving atom, we have found an expression for the linearized velocity-dependent force on an atom in an arbitrary incoherent internal quantum state moving near an arbitrary arrangement of magnetoelectric bodies. In general, three effects contribute to the velocity-dependent Casimir-Polder force: a generalized Doppler effect due to the velocity dependence of the atomic transition frequencies; the delay between the emission and reabsorption of photons by the atom; and the Röntgen interaction, i.e., the coupling of the current density associated with the atomic motion to the magnetic field.

In order to illustrate the general theory, we have studied the near-field force on an atom that moves parallel to a planar dielectric or metallic surface. Due to the translational invariance of the system, the Doppler term does not contribute in this case. Furthermore, the delay term dominates over the Röntgen term. For a ground-state atom the force is a genuine friction force, i.e., a force antiparallel to its velocity. It is proportional to the atomic linewidth and hence very small. In contrast, excited-state atoms can be either decelerated or accelerated depending on the relative magnitude of their transition frequency with respect to the characteristic frequency of the substrate material. For metals, the force is always decelerating while for dielectric substrates with sufficiently small surface-plasmon frequency, acceleration of excited-state atoms can be realized.

In addition, the force on such atoms is strongly enhanced when atom and substrate are near resonant. Much stronger enhancement can be achieved when the atom moves through resonator structures, in close analogy to the stationary case [29].

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