Cluster-type entangled coherent states: Generation and application

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We consider a type of $(M+N)$-mode entangled coherent states and propose a simple deterministic scheme to generate these states that can fly freely in space. We then exploit such free-flying states to teleport certain kinds of superpositions of multimode coherent states. We also address the issue of manipulating size and type of entangled coherent states by means of linear optics elements only.

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I. INTRODUCTION

Photons are often considered as good "flying" qubits most suitable for quantum network communication tasks thanks to their high speed and robustness against decoherence. Yet, perfect single-photon sources are demanding and photon-photon gates are hard to implement due to extremely weak direct interaction between them. An alternative elegant way to cope with these problems is to use multiphoton fields such as coherent states, which are available from standard stabilized laser sources. In fact, coherent-state coding has been widely exploited and coherent-state-based protocols for quantum information processing as well as for quantum computing have been devised by many authors (see, e.g., [1–14] and references therein).

As a rule, quantum information processing and quantum computing based on coherent-state coding require availability of efficient nonlocal resources called entangled coherent states. In fact, in the multimode case, the so-called GHZ-type entangled coherent states [4,7,9,10] $|\mathrm{GHZ}_M^\alpha\rangle \propto |\alpha,\alpha,\ldots,\alpha\rangle_{12\ldots M}+|-\alpha,-\alpha,\ldots,-\alpha\rangle_{12\ldots M}$ and W-type entangled coherent states [8,9,11,12] $|W_M^\alpha\rangle \propto |-\alpha,\alpha,\ldots,\alpha\rangle_{12\ldots M}+|\alpha,-\alpha,\ldots,\alpha\rangle_{12\ldots M}+\cdots+|\alpha,\alpha,\ldots,-\alpha\rangle_{12\ldots M}$, with $|\pm\alpha\rangle$ being coherent states with the same amplitude but opposite phases, have been introduced as natural extensions of the well-known photonic GHZ states [15] and W states [16], respectively.

In this paper we introduce yet another kind of multimode entangled coherent states which we write compactly in the form

$$|C_{M+N}^\alpha\rangle = \frac{1}{2}(|\alpha,\alpha,\ldots,\alpha\rangle_{12\ldots M}+|-\alpha,-\alpha,\ldots,-\alpha\rangle_{12\ldots M}Z_n^\alpha)\times(|\alpha,\alpha,\ldots,\alpha\rangle_{M+1M+2\ldots M+N}+|-\alpha,-\alpha,\ldots,-\alpha\rangle_{M+1M+2\ldots M+N}), \tag{1}$$

with a fixed $n\in\{M+1,M+2,\ldots,M+N\}$ and Z_n^α a formal symbol (not a real unitary operator) to be understood by the convention $Z_n^\alpha|\pm\alpha\rangle_n\to\pm|\pm\alpha\rangle_n$. The state $|C_{M+N}^\alpha\rangle$ is inequivalent to $|\mathrm{GHZ}_{M+N}^\alpha\rangle$ and $|W_{M+N}^\alpha\rangle$ in the sense that they cannot be transformed from one to another by local operations and classical communication. For $M=N=2$ we have explicitly $|C_{2+2}^\alpha\rangle=\frac{1}{2}(|\alpha,\alpha,\alpha,\alpha\rangle+|\alpha,\alpha,-\alpha,-\alpha\rangle+|-\alpha,-\alpha,\alpha,\alpha\rangle-|-\alpha,-\alpha,-\alpha,-\alpha\rangle)_{1234}$ which by its form might be thought of an extension of the linear four-photon cluster state [17,18] $|C_4\rangle=\frac{1}{2}(|HHHH\rangle+|HHVV\rangle+|VVHH\rangle-|VVVV\rangle)_{1234}$, where $|H\rangle(|V\rangle)$ denotes the state of a photon with the horizontal (vertical) polarization. Because of such a similarity in form the state $|C_{2+2}^\alpha\rangle$ was referred to as four-mode cluster-type entangled coherent state in Refs. [19–21]. Then, for $M=N=3$ the state $|C_{3+3}^\alpha\rangle=\frac{1}{2}(|\alpha,\alpha,\alpha,\alpha,\alpha,\alpha\rangle+|\alpha,\alpha,\alpha,-\alpha,-\alpha,-\alpha\rangle+|-\alpha,-\alpha,-\alpha,\alpha,\alpha,\alpha\rangle-|-\alpha,-\alpha,-\alpha,-\alpha,-\alpha,-\alpha\rangle)_{123456}$ could also be called six-mode cluster-type entangled coherent state because its form reminds one of a six-photon cluster state [22] $|C_6\rangle=\frac{1}{2}(|HHHHHH\rangle+|HHHVVV\rangle+|VVVHHH\rangle-|VVVVVV\rangle)_{123456}$ which corresponds to the qubit configuration in a certain shape (two of the three-qubit open boundary cluster states which are linked by a controlled-phase gate applied to qubits in the middle). However, the terminology "cluster-type" turns out misleading since the original cluster states introduced in Refs. [23–25] for their primary purpose of performing one-way measurement-based quantum computation are necessarily associated with a specific configuration of properly prepared qubits on which a set of controlled-phase gates act. One can show that for the case of M, $N\geq 4$ the forms of the state $|C_{M+N}^\alpha\rangle$ and a conventional cluster state $|C_{M+N}\rangle$ no longer resemble one another. To avoid any ambiguities we call our states $|C_{M+N}^\alpha\rangle$ as a whole defined by Eq. (1) an alternative type of multimode entangled coherent states to distinguish them from the known ones $|\mathrm{GHZ}_{M+N}^\alpha\rangle$ and $|W_{M+N}^\alpha\rangle$.

In the next section, Sec. II, we present a simple yet efficient scheme to generate the entangled coherent states $|C_{M+N}^\alpha\rangle$ with all the modes flying freely in space using only one π-cross-Kerr medium combined with linear optics elements. Section III deals with applications of such free-flying states. Namely, it is shown that the introduced entangled coherent states can serve as quantum channel to teleport certain multimode coherent-state superpositions. The issue of how to change the size and the type of multimode entangled coherent states is addressed in Sec. IV. Finally, a conclusion is made in Sec. V.

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II. GENERATION SCHEME

The entangled coherent states of the type defined by Eq. (1) with $M=N=2$, their nonlocal properties, and schemes of generation have been touched upon recently [19–21,26]. Based on atom-photon [19,20] or electron-photon [21] interactions within the framework of cavity QED certain four-mode entangled coherent states can be generated depending on the outcome of the measurement carried out at the end on the atom or the electron in appropriate bases. These states are confined in spatially separated cavities. However, quantum communication tasks prefer free-flying to being-confined optical fields to facilitate distribution of the fields among an optical network. Hence, generation of free-flying entangled coherent states is necessary. In Ref. [26] a scheme to generate such free-flying states is considered using multiple π-cross-Kerr media, Hadamard-type gates, and homodyne detection. Since the Hadamard-type gate is not a physical operator due to its nonunitarity, its implementation is not exact, so the scheme in Ref. [26] is an approximate one. Moreover, all the above-mentioned schemes [19–21,26] are probabilistic because postselection should be made. Here we propose a simple scheme to generate free-flying states $|C^{\alpha}_{M+N}\rangle$ of the form defined by Eq. (1). Our scheme compared with that of [26] is strict and more feasible because no Hadamard-type gates are needed and the π-cross-Kerr medium is used only once (not many times). On top of these, it is efficient in the sense that it succeeds with probability one since no measurements are involved at all.

For clarity, let us briefly describe the elements we utilize in our scheme. These include a π-cross-Kerr medium, phase shifters, and beam splitters. The π-cross-Kerr medium K_{ab} is a nonlinear medium propagating through which for a time duration t such that $t\chi=\pi$ (χ characterizes the medium nonlinearity) two coherent states $|\alpha\rangle_a|\beta\rangle_b$ become entangled in the following way (see, e.g., [27,28]):

$$K_{ab}|\alpha\rangle_a|\beta\rangle_b=\frac{1}{2}[|\alpha\rangle_a(|\beta\rangle_b+|-\beta\rangle_b)+|-\alpha\rangle_a(|\beta\rangle_b-|-\beta\rangle_b)]. \tag{2}$$

The phase shifter $P_a(T)$ is a device propagating through which a coherent state $|\alpha\rangle_a$ picks up a phase as

$$P_a(\theta)|\alpha\rangle_a=|e^{-i\theta}\alpha\rangle_a. \tag{3}$$

Also, the beam splitter $B_{ab}(\theta)$ is a device propagating through which a product of two coherent states $|\alpha\rangle_a|\beta\rangle_b$ remains a product state but with their amplitudes redistributed as

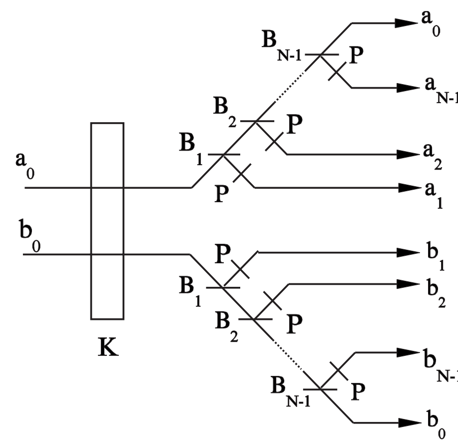

FIG. 1. Scheme for generating $2N$-mode entangled coherent states $|C^{(i)\alpha}_{N+N}\rangle$. K denotes a π-cross-Kerr medium, P a $\pi/2$ phase shifter, and B_n a beam splitter with transmittance $T_n=(N-n)/(N-n+1)$.

$$B_{ab}(T)|\alpha\rangle_a|\beta\rangle_b=|\alpha\sqrt{T}+i\beta\sqrt{R}\rangle_a|\beta\sqrt{T}+i\alpha\sqrt{R}\rangle_b, \tag{4}$$

where T is the transmittance and $R=1-T$ the reflectance. From Eqs. (3) and (4) it follows a useful transformation

$$P_b(\pi/2)B_{ab}(T)P_b(\pi/2)|\alpha\rangle_a|\beta\rangle_b=|\alpha\sqrt{T}+\beta\sqrt{R}\rangle_a|\alpha\sqrt{R}-\beta\sqrt{T}\rangle_b. \tag{5}$$

In particular, for the vacuum state of mode b transformation (5) simplifies to

$$P_b(\pi/2)B_{ab}(T)|\alpha\rangle_a|0\rangle_b=|\alpha\sqrt{T}\rangle_a|\alpha\sqrt{R}\rangle_b. \tag{6}$$

The setup of the scheme to generate the state $|C^{\alpha}_{N+N}\rangle$ (i.e., with $M=N$) is depicted in Fig. 1. First, two coherent states with the same amplitude $|\beta\rangle_{a_0}$ and $|\beta\rangle_{b_0}$ are prepared; i.e., the initial state is

$$|\Psi_i\rangle=|\beta\rangle_{a_0}|\beta\rangle_{b_0}. \tag{7}$$

Then the two coherent beams a_0 and b_0 are sent together through a π-cross-Kerr medium that makes $|\Psi_i\rangle$ to be

$$|\Phi\rangle=\frac{1}{2}[|\beta\rangle_{a_0}(|\beta\rangle_{b_0}+|-\beta\rangle_{b_0})+|-\beta\rangle_{a_0}(|\beta\rangle_{b_0}-|-\beta\rangle_{b_0})]. \tag{8}$$

After the nonlinear medium the beam a_0 is successively mixed with the vacuum on a set of $N-1$ beam splitters with transmittances T_1, $T_2,\ldots,$ and T_{N-1}. Thus, in addition to the transmitted beam a_0, there appear $N-1$ reflected beams a_1, $a_2,\ldots$ and a_{N-1}, each of which is let pass through a phase shifter with $\theta=\pi/2$. The same does the beam b_0 simultaneously. As a consequence, $2N$ beams a_0, a_1, $a_2,\ldots,a_{N-1}$, b_0, b_1, $b_2,\ldots,$ and b_{N-1} go out. Making use of transformation (6) repeatedly we have

$$|\pm\beta\rangle_{a_0(b_0)}\to|\pm\beta\sqrt{T_1T_2\ldots T_{N-1}}\rangle_{a_0(b_0)}|\pm\beta\sqrt{R_1}\rangle_{a_1(b_1)}|\pm\beta\sqrt{T_1R_2}\rangle_{a_2(b_2)}\ldots|\pm\beta\sqrt{T_1\ldots T_{N-3}R_{N-2}}\rangle_{a_{N-2}(b_{N-2})}\times|\pm\beta\sqrt{T_1\ldots T_{N-2}R_{N-1}}\rangle_{a_{N-1}(b_{N-1})}. \tag{9}$$

If we choose

$$\beta = \alpha\sqrt{N} \tag{10}$$

and $T_1=(N-1)/N$, $T_2=(N-2)/(N-1),\ldots$, $T_{N-2}=2/3$, $T_{N-1}=1/2$ (i.e.,

$$T_n = \frac{N-n}{N-n+1} \tag{11}$$

with $n=1,2,\ldots,N-1$), then transition (9) is replaced by

$$|\pm\beta\rangle_{a_0(b_0)} = |\pm\alpha\sqrt{N}\rangle_{a_0(b_0)} \to |\pm\alpha\rangle_{a_0(b_0)}|\pm\alpha\rangle_{a_1(b_1)} \times|\pm\alpha\rangle_{a_2(b_2)}\ldots|\pm\alpha\rangle_{a_{N-1}(b_{N-1})}. \tag{12}$$

This means that $|\Phi\rangle\to|\Psi_f\rangle$ with $|\Psi_f\rangle$ the final state which is of the form

$$|\Psi_f\rangle = \frac{1}{2}(|\alpha,\alpha,\ldots,\alpha\rangle_{a_0a_1\ldots a_{N-1}} + |-\alpha,-\alpha,\ldots,-\alpha\rangle_{a_0a_1\ldots a_{N-1}}Z_j^{\alpha})(|\alpha,\alpha,\ldots,\alpha\rangle_{b_0b_1\ldots b_{N-1}} + |-\alpha,-\alpha,\ldots,-\alpha\rangle_{b_0b_1\ldots b_{N-1}}), \tag{13}$$

with a fixed $j\in\{b_0,b_1,\ldots,b_{N-1}\}$. By definition (1), the generated state (13) is nothing else but the entangled coherent states $|C_{N+N}^{\alpha}\rangle$ of the $2N$ modes a_0, $a_1,\ldots$, a_{N-1}, b_0, $b_1,\ldots$, and b_{N-1}, each of which can fly freely in space. We denote this class of states by $|C_{N+N}^{(1)\alpha}\rangle$, i.e., $|\Psi_f\rangle\equiv|C_{N+N}^{(1)\alpha}\rangle$.

If instead of Eq. (7) we prepare the initial state as

$$|\Psi_i'\rangle = |\alpha\sqrt{N}\rangle_{a_0}|-\alpha\sqrt{N}\rangle_{b_0}, \tag{14}$$

$$|\Psi_i''\rangle = |-\alpha\sqrt{N}\rangle_{a_0}|\alpha\sqrt{N}\rangle_{b_0}, \tag{15}$$

or

$$|\Psi_i'''\rangle = |-\alpha\sqrt{N}\rangle_{a_0}|-\alpha\sqrt{N}\rangle_{b_0}, \tag{16}$$

then instead of Eq. (8) we have

$$|\Phi'\rangle = \frac{1}{2}[|\alpha\sqrt{N}\rangle_{a_0}(|\alpha\sqrt{N}\rangle_{b_0} + |-\alpha\sqrt{N}\rangle_{b_0}) - |-\alpha\sqrt{N}\rangle_{a_0}(|\alpha\sqrt{N}\rangle_{b_0} - |-\alpha\sqrt{N}\rangle_{b_0})], \tag{17}$$

$$|\Phi''\rangle = \frac{1}{2}[|\alpha\sqrt{N}\rangle_{a_0}(|\alpha\sqrt{N}\rangle_{b_0} - |-\alpha\sqrt{N}\rangle_{b_0}) + |-\alpha\sqrt{N}\rangle_{a_0}(|\alpha\sqrt{N}\rangle_{b_0} + |-\alpha\sqrt{N}\rangle_{b_0})], \tag{18}$$

or

$$|\Phi'''\rangle = \frac{1}{2}[-|\alpha\sqrt{N}\rangle_{a_0}(|\alpha\sqrt{N}\rangle_{b_0} - |-\alpha\sqrt{N}\rangle_{b_0}) + |-\alpha\sqrt{N}\rangle_{a_0}(|\alpha\sqrt{N}\rangle_{b_0} + |-\alpha\sqrt{N}\rangle_{b_0})], \tag{19}$$

respectively. These mean that $|\Phi'\rangle\to|\Psi_f'\rangle$, $|\Phi''\rangle\to|\Psi_f''\rangle$, and $|\Phi'''\rangle\to|\Psi_f'''\rangle$, with $|\Psi_f'\rangle$, $|\Psi_f''\rangle$, and $|\Psi_f'''\rangle$ the corresponding final states which are of the forms

$$|\Psi_f'\rangle = \frac{1}{2}(|\alpha,\alpha,\ldots,\alpha\rangle_{a_0a_1\ldots a_{N-1}} - |-\alpha,-\alpha,\ldots,-\alpha\rangle_{a_0a_1\ldots a_{N-1}}Z_j^{\alpha})(|\alpha,\alpha,\ldots,\alpha\rangle_{b_0b_1\ldots b_{N-1}} + |-\alpha,-\alpha,\ldots,-\alpha\rangle_{b_0b_1\ldots b_{N-1}}), \tag{20}$$

$$|\Psi_f''\rangle = \frac{1}{2}(|\alpha,\alpha,\ldots,\alpha\rangle_{a_0a_1\ldots a_{N-1}}Z_j^{\alpha} + |-\alpha,-\alpha,\ldots,-\alpha\rangle_{a_0a_1\ldots a_{N-1}})(|\alpha,\alpha,\ldots,\alpha\rangle_{b_0b_1\ldots b_{N-1}} + |-\alpha,-\alpha,\ldots,-\alpha\rangle_{b_0b_1\ldots b_{N-1}}), \tag{21}$$

and

$$|\Psi_f'''\rangle = \frac{1}{2}(-|\alpha,\alpha,\ldots,\alpha\rangle_{a_0a_1\ldots a_{N-1}}Z_j^{\alpha} + |-\alpha,-\alpha,\ldots,-\alpha\rangle_{a_0a_1\ldots a_{N-1}})(|\alpha,\alpha,\ldots,\alpha\rangle_{b_0b_1\ldots b_{N-1}} + |-\alpha,-\alpha,\ldots,-\alpha\rangle_{b_0b_1\ldots b_{N-1}}). \tag{22}$$

States (20)–(22) constitute other classes of $2N$-mode entangled coherent states that we respectively denote by $|C_{N+N}^{(2)\alpha}\rangle$, $|C_{N+N}^{(3)\alpha}\rangle$, and $|C_{N+N}^{(4)\alpha}\rangle$, i.e., $|\Psi_f'\rangle=|C_{N+N}^{(2)\alpha}\rangle$, $|\Psi_f''\rangle=|C_{N+N}^{(3)\alpha}\rangle$, and $|\Psi_f'''\rangle=|C_{N+N}^{(4)\alpha}\rangle$. The four classes $|C_{N+N}^{(1)\alpha}\rangle$, $|C_{N+N}^{(2)\alpha}\rangle$, $|C_{N+N}^{(3)\alpha}\rangle$, and $|C_{N+N}^{(4)\alpha}\rangle$ cannot locally be transformed from one to another since there are no exact unitary operators that can make $|-\alpha\rangle$ into $-|-\alpha\rangle$ but at the same time keep $|\alpha\rangle$ intact. They are nonorthogonal to each other and their overlappings are

$$\langle C_{N+N}^{(1)\alpha}|C_{N+N}^{(2)\alpha}\rangle = \langle C_{N+N}^{(1)\alpha}|C_{N+N}^{(3)\alpha}\rangle = \langle C_{N+N}^{(2)\alpha}|C_{N+N}^{(4)\alpha}\rangle = \langle C_{N+N}^{(3)\alpha} \times|C_{N+N}^{(4)\alpha}\rangle = e^{-2N|\alpha|^2}, \tag{23}$$

$$\langle C_{N+N}^{(1)\alpha}|C_{N+N}^{(4)\alpha}\rangle = \langle C_{N+N}^{(2)\alpha}|C_{N+N}^{(3)\alpha}\rangle = e^{-4N|\alpha|^2}. \tag{24}$$

More generally, the entangled coherent states $|C_{M+N}^{(1)\alpha}\rangle$ with any $M\neq N$ can also be generated by a slight modification in Fig. 1. Indeed, we have just to start with the product state $|\alpha\sqrt{M}\rangle_{a_0}|\alpha\sqrt{N}\rangle_{b_0}$ and use $M-1(N-1)$ beam splitters with properly chosen transmittances in the upper (lower) branch of Fig. 1. Similarly, the choices of $|\alpha\sqrt{M}\rangle_{a_0}|-\alpha\sqrt{N}\rangle_{b_0}$, $|-\alpha\sqrt{M}\rangle_{a_0}|\alpha\sqrt{N}\rangle_{b_0}$, and $|-\alpha\sqrt{M}\rangle_{a_0}|-\alpha\sqrt{N}\rangle_{b_0}$ for the initial state will generate the states $|C_{M+N}^{(2)\alpha}\rangle$, $|C_{M+N}^{(3)\alpha}\rangle$, and $|C_{M+N}^{(4)\alpha}\rangle$, respectively. The different states $|C_{M+N}^{(i)\alpha}\rangle$ overlap as

$$\langle C_{M+N}^{(1)\alpha}|C_{M+N}^{(2)\alpha}\rangle = \langle C_{M+N}^{(3)\alpha}|C_{M+N}^{(4)\alpha}\rangle = e^{-2N|\alpha|^2}, \tag{25}$$

$$\langle C_{M+N}^{(1)\alpha}|C_{M+N}^{(3)\alpha}\rangle = \langle C_{M+N}^{(2)\alpha}|C_{M+N}^{(4)\alpha}\rangle = e^{-4M|\alpha|^2}, \tag{26}$$

$$\langle C_{M+N}^{(1)\alpha}|C_{M+N}^{(4)\alpha}\rangle = \langle C_{M+N}^{(2)\alpha}|C_{M+N}^{(3)\alpha}\rangle = e^{-2(M+N)|\alpha|^2}. \tag{27}$$

III. APPLICATION

Cluster states [23–25] qualitatively differ from other kinds of entangled states such as GHZ states [15] or W states [16].

For example, in terms of the Schmidt measure [29], cluster states are much more entangled than GHZ and W states. Although cluster states were originally aimed to serve as the entire substrate for the one-way universal quantum computation, they have also found various interesting applications in quantum information processing, quantum error correction, nonlocality tests, etc. There are Bell inequalities that are maximally violated by cluster states and only partially violated by W states but not violated at all by GHZ states [30,31]. Cluster states also provide a natural implementation for a quantum version of the game named Prisoner's Dilemma [32,33]. Concerning our states $|C_{M+N}^{(i)\alpha}\rangle$, as mentioned in Sec. I, although they cannot as a whole be called cluster-type entangled coherent states in the strict sense, their resemblance in form to the cluster states in the case of $M=N=2,3$ would make them for these values of M and N to be different from the corresponding GHZ-type [4,7,9,10] and W-type [8,9,11,12] entangled coherent states in quite a similar way as cluster states differ from the GHZ and W ones. As an example, a recent investigation based on the GHZ-Mermin argument [21] shows that states $|C_{2+2}^{(i)\alpha}\rangle$ possess nonlocal properties similar to those of the cluster state $|C_4\rangle$. Yet, the concrete entanglement properties of $|C_{M+N}^{(i)\alpha}\rangle$ with $M,N\geq 3$ necessitate separate investigations. In this section, as possible applications of the free-flying entangled coherent states $|C_{M+N}^{(1)\alpha}\rangle$, $|C_{M+N}^{(2)\alpha}\rangle$, $|C_{M+N}^{(3)\alpha}\rangle$, and $|C_{M+N}^{(4)\alpha}\rangle$, we show that they can be used for teleportation purposes.

For simplicity, let us first consider $M=N=2$ in which case the concerned entangled coherent states have the explicit expressions

$$|C_{2+2}^{(1)\alpha}\rangle = \frac{1}{2}(|\alpha,\alpha,\alpha,\alpha\rangle + |\alpha,-\alpha,\alpha,-\alpha\rangle + |-\alpha,\alpha,-\alpha,\alpha\rangle - |-\alpha,-\alpha,-\alpha,-\alpha\rangle)_{1234}, \tag{28}$$

$$|C_{2+2}^{(2)\alpha}\rangle = \frac{1}{2}(|\alpha,\alpha,\alpha,\alpha\rangle + |\alpha,-\alpha,\alpha,-\alpha\rangle - |-\alpha,\alpha,-\alpha,\alpha\rangle + |-\alpha,-\alpha,-\alpha,-\alpha\rangle)_{1234}, \tag{29}$$

$$|C_{2+2}^{(3)\alpha}\rangle = \frac{1}{2}(|\alpha,\alpha,\alpha,\alpha\rangle - |\alpha,-\alpha,\alpha,-\alpha\rangle + |-\alpha,\alpha,-\alpha,\alpha\rangle + |-\alpha,-\alpha,-\alpha,-\alpha\rangle)_{1234}, \tag{30}$$

$$|C_{2+2}^{(4)\alpha}\rangle = \frac{1}{2}(-|\alpha,\alpha,\alpha,\alpha\rangle + |\alpha,-\alpha,\alpha,-\alpha\rangle + |-\alpha,\alpha,-\alpha,\alpha\rangle + |-\alpha,-\alpha,-\alpha,-\alpha\rangle)_{1234}, \tag{31}$$

where we have made a relabeling $a_0\to 1$, $a_1\to 3$, $b_0\to 2$, and $b_1\to 4$ for convenience. We now show that the above states can serve as quantum channel to teleport a class of the following two-mode three-component entangled coherent states

$$|\varphi\rangle_{56} = x|\alpha,\alpha\rangle_{56} + y|\alpha,-\alpha\rangle_{56} + z|-\alpha,\alpha\rangle_{56}, \tag{32}$$

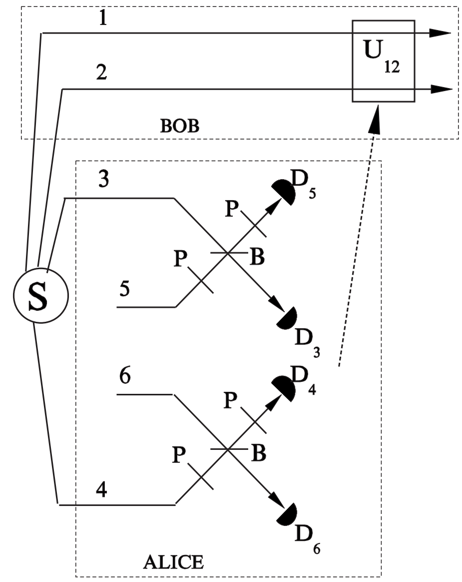

FIG. 2. Scheme for teleporting one of the two-mode coherent-state superpositions $\{|\varphi\rangle_{56},|\varphi'\rangle_{56},|\varphi''\rangle_{56},|\varphi'''\rangle_{56}\}$, Eqs. (32)–(35), using one of the four-mode $\{|C_{2+2}^{(1)\alpha}\rangle,|C_{2+2}^{(2)\alpha}\rangle,|C_{2+2}^{(3)\alpha}\rangle,|C_{2+2}^{(4)\alpha}\rangle\}$, Eqs. (28)–(31), as the quantum channel. S denotes an entanglement source, P a $\pi/2$ phase shifter, B a beam splitter with transmittance $T=1/2$, and U_{12} the recovery operator. The dashed arrow indicates the Alice-to-Bob classical communication which in this case costs 2 bits because there are four situations (i), (ii), (iii), and (iv) to be discriminated.

$$|\varphi'\rangle_{56} = x|\alpha,\alpha\rangle_{56} + y|\alpha,-\alpha\rangle_{56} + z|-\alpha,-\alpha\rangle_{56}, \tag{33}$$

$$|\varphi''\rangle_{56} = x|\alpha,\alpha\rangle_{56} + y|-\alpha,\alpha\rangle_{56} + z|-\alpha,-\alpha\rangle_{56}, \tag{34}$$

$$|\varphi'''\rangle_{56} = x|\alpha,-\alpha\rangle_{56} + y|-\alpha,\alpha\rangle_{56} + z|-\alpha,-\alpha\rangle_{56}, \tag{35}$$

with x, y, and z arbitrary unknown complex coefficients satisfying the normalization condition. Protocols to teleport two-mode two- and four-component entangled coherent states by a three-mode entangled coherent state and a pair of two-mode entangled coherent states as the quantum channel were proposed in Refs. [4,10] and Refs. [13,14], respectively. Here we study teleportation of two-mode three-component entangled coherent states by a single four-mode entangled coherent state $|C_{2+2}^{(i)\alpha}\rangle$. In connection with states (32)–(35) we note that two-qubit three-component states $x|00\rangle+y|01\rangle+z|10\rangle$ and $x|01\rangle+y|10\rangle+z|11\rangle$ are found to be robust, while

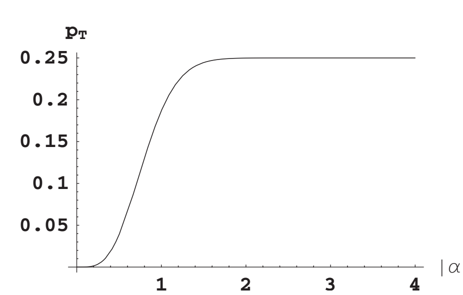

FIG. 3. The total success probability p_T, Eq. (41), as a function of $|\alpha|$, for the teleportation process shown in Fig. 2.

TABLE I. Success probability p_i and recovery operator U_{12} for teleportation of $|\varphi\rangle_{56}$, $|\varphi'\rangle_{56}$, $|\varphi''\rangle_{56}$, and $|\varphi'''\rangle_{56}$ via the quantum channel served by $|C_{2+2}^{(1)\alpha}\rangle_{1234}$, $|C_{2+2}^{(2)\alpha}\rangle_{1234}$, $|C_{2+2}^{(3)\alpha}\rangle_{1234}$, and $|C_{2+2}^{(4)\alpha}\rangle_{1234}$, respectively, in dependence of the detected photon numbers n_3, n_4, n_5, and n_6.

n_3	n_4	n_5	n_6	p_i	U_{12}
0	0	Odd	Odd	p_1	$P_1(\pi)\otimes P_2(\pi)$
Nonzero even	Nonzero even	0	0	p_2	$I_1\otimes I_2$
Odd	0	0	Nonzero even	p_3	$I_1\otimes P_2(\pi)$
0	Odd	Nonzero even	0	p_3	$P_1(\pi)\otimes I_2$

$x|00\rangle+y|10\rangle+z|11\rangle$ and $x|00\rangle+y|01\rangle+z|11\rangle$ fragile against decoherence [34].

If Alice is given the state $|\varphi\rangle_{56}$ she can teleport it to a distant Bob by sharing with him the state $|C_{2+2}^{(1)\alpha}\rangle$ of which modes 1,2 belong to Bob and modes 3,4 to Alice. The teleportation scheme is depicted in Fig. 2. Alice first lets mode 5 (6) go through a $\pi/2$ phase shifter $P_{5(6)}(\pi/2)$, then inputs mode 3 and 5 (4 and 6) on a 50:50 beam splitter $B_{35(46)}(1/2)$. For the output modes, Alice gain lets mode 5 (6) go through another $\pi/2$ phase shifter before all the modes are registered by a corresponding photodetector D_i $(i=3,4,5,6)$. Let n_i be the photon number counted by photodetector D_i. Then the matrix element $\mathcal{M}={}_{3456}\langle n_3n_4n_5n_6|\Omega\rangle_{123456}$ with $|\Omega\rangle_{123456}=P_6(\pi/2)B_{46}(1/2)P_6(\pi/2)P_5(\pi/2)B_{35}(1/2)\times P_5(\pi/2)|C_{2+2}^{(1)\alpha}\rangle_{1234}|\varphi\rangle_{56}$ is

$$\begin{aligned}\mathcal{M}=\frac{1}{2}\{&-\delta_{n_30}\delta_{n_40}g_{n_5}(\gamma)g_{n_6}(\gamma)[(-1)^{n_5+n_6}x|-\alpha,-\alpha\rangle\\&-(-1)^{n_5}y|-\alpha,\alpha\rangle-(-1)^{n_6}z|\alpha,-\alpha\rangle]_{12}\\&+g_{n_3}(\gamma)g_{n_4}(\gamma)\delta_{n_50}\delta_{n_60}[x|\alpha,\alpha\rangle+(-1)^{n_4}y|\alpha,-\alpha\rangle\\&+(-1)^{n_3}z|-\alpha,\alpha\rangle]_{12}+g_{n_3}(\gamma)\delta_{n_40}\delta_{n_50}g_{n_6}(\gamma)[(-1)^{n_6}x|\alpha,\\&-\alpha\rangle+y|\alpha,\alpha\rangle-(-1)^{n_3+n_6}z|-\alpha,-\alpha\rangle]_{12}\\&+\delta_{n_30}g_{n_4}(\gamma)g_{n_5}(\gamma)\delta_{n_60}[(-1)^{n_5}x|-\alpha,\alpha\rangle-(-1)^{n_4+n_5}y|\\&-\alpha,-\alpha\rangle+z|\alpha,\alpha\rangle]_{12}\},\end{aligned}\tag{36}$$

where $\gamma=\alpha\sqrt{2}$, δ_{mn} is the Kronecker symbol, and $g_n(\gamma)=\langle n|\gamma\rangle=\exp(-|\gamma|^2/2)\gamma^n/\sqrt{n!}$. A closer look at Eq. (36) reveals that the teleportation only succeeds in the following four situations:

(i) $n_3=n_4=0$, while n_5 and n_6 are both odd, occurring with a probability

$$p_1=\frac{1}{4}e^{-4|\alpha|^2}\sinh^2(2|\alpha|^2),\tag{37}$$

in which case Bob should apply on his modes the recovery operator $U_{12}=P_1(\pi)\otimes P_2(\pi)$;

(ii) n_3 and n_4 are both nonzero even numbers, while $n_5=n_6=0$, occurring with a probability

$$p_2=e^{-4|\alpha|^2}\sinh^4(|\alpha|^2),\tag{38}$$

in which case $U_{12}=I_1\otimes I_2$ with I_i the identity operator;

(iii) $n_4=n_5=0$, while n_3 is odd and n_6 is a nonzero even number, occurring with a probability

$$p_3=e^{-4|\alpha|^2}\cosh(|\alpha|^2)\sinh^3(|\alpha|^2),\tag{39}$$

in which case $U_{12}=I_1\otimes P_2(\pi)$;

(iv) $n_3=n_6=0$, while n_4 is odd and n_5 is a nonzero even number, occurring with a probability

$$p_4=p_3,\tag{40}$$

in which case $U_{12}=P_1(\pi)\otimes I_2$.

Thus, the total success probability is

$$p_T=p_1+p_2+2p_3=e^{-2|\alpha|^2}\sinh^2(|\alpha|^2),\tag{41}$$

which is almost 1/4 for $|\alpha|\geq 2$, as visualized from Fig. 3.

Likewise, $|C_{2+2}^{(1)\alpha}\rangle_{1234}$ can also serve as quantum channel to teleport $|\varphi'\rangle_{56}$, $|\varphi''\rangle_{56}$, and $|\varphi'''\rangle_{56}$. Furthermore, we find out that not only $|C_{2+2}^{(1)\alpha}\rangle_{1234}$, but also $|C_{2+2}^{(2)\alpha}\rangle_{1234}$, $|C_{2+2}^{(3)\alpha}\rangle_{1234}$, and $|C_{2+2}^{(4)\alpha}\rangle_{1234}$ can be applied to teleport any state among the class of states (32)–(35), using the same execution scheme as in Fig. 2. We summarize all the details in Tables I–IV.

Generally, $|C_{N+N}^{(i)\alpha}\rangle$ are useful for teleportation of certain superpositions of N-mode coherent states, while $|C_{M+N}^{(i)\alpha}\rangle$ with $M>N(M<N)$ apply for controlled teleportation of certain superpositions of N-mode (M-mode) coherent states under the control of $M-N(N-M)$ supervisors. There are many pos-

TABLE II. Same as in Table I but for teleportation of $|\varphi'\rangle_{56}$, $|\varphi\rangle_{56}$, $|\varphi'''\rangle_{56}$, and $|\varphi''\rangle_{56}$ via $|C_{2+2}^{(1)\alpha}\rangle_{1234}$, $|C_{2+2}^{(2)\alpha}\rangle_{1234}$, $|C_{2+2}^{(3)\alpha}\rangle_{1234}$, and $|C_{2+2}^{(4)\alpha}\rangle_{1234}$, respectively.

n_3	n_4	n_5	n_6	p_i	U_{12}
0	0	Nonzero even	Odd	p_3	$P_1(\pi)\otimes P_2(\pi)$
Odd	Nonzero even	0	0	p_3	$I_1\otimes I_2$
Nonzero even	0	0	Nonzero even	p_2	$I_1\otimes P_2(\pi)$
0	Odd	Odd	0	p_1	$P_1(\pi)\otimes I_2$

TABLE III. Same as in Table I but for teleportation of $|\varphi''\rangle_{56}$, $|\varphi'''\rangle_{56}$, $|\varphi\rangle_{56}$, and $|\varphi'\rangle_{56}$ via $|C_{2+2}^{(1)\alpha}\rangle_{1234}$, $|C_{2+2}^{(2)\alpha}\rangle_{1234}$, $|C_{2+2}^{(3)\alpha}\rangle_{1234}$, and $|C_{2+2}^{(4)\alpha}\rangle_{1234}$, respectively.

n_3	n_4	n_5	n_6	p_i	U_{12}
0	0	Odd	Nonzero even	p_3	$P_1(\pi)\otimes P_2(\pi)$
Nonzero even	Odd	0	0	p_3	$I_1\otimes I_2$
Odd	0	0	Odd	p_1	$I_1\otimes P_2(\pi)$
0	Nonzero even	Nonzero even	0	p_2	$P_1(\pi)\otimes I_2$

sibilities that we are unable to deal with all at a time. As another illustration, we proceed a step further by considering $M=N=3$ in which case $|C_{3+3}^{(i)\alpha}\rangle$ can be used at least to teleport a family of three-mode two-component coherent states. For concreteness let us take $|C_{3+3}^{(1)\alpha}\rangle$ whose explicit expression is

$$|C_{3+3}^{(1)\alpha}\rangle=\frac{1}{2}(|\alpha,\alpha,\alpha,\alpha,\alpha,\alpha\rangle+|\alpha,\alpha,-\alpha,\alpha,-\alpha,-\alpha\rangle+|-\alpha,-\alpha,\alpha,-\alpha,\alpha,\alpha\rangle-|-\alpha,-\alpha,-\alpha,-\alpha,-\alpha,-\alpha\rangle)_{123456}, \tag{42}$$

where a relabeling $a_0\to 1$, $a_1\to 2$, $b_0\to 3$, $a_2\to 4$, $b_1\to 5$, and $b_2\to 6$ has been made for convenience. We have realized that, among others, a superposition of three-mode entangled coherent state of the form

$$|\psi\rangle_{789}=x|\alpha,\alpha,-\alpha\rangle_{789}+y|-\alpha,-\alpha,\alpha\rangle_{789}, \tag{43}$$

with arbitrary unknown complex normalization coefficients x and y, can be teleported via the state $|C_{3+3}^{(1)\alpha}\rangle$ of which Bob holds modes 1, 2, and 3 while modes 4, 5, and 6 are with Alice. For that purpose, Alice manipulates modes 4 and 7 (5 and 8, 6 and 9) in the same way as she did in Fig. 2 with modes 3 and 5 (or 4 and 6). In the $M=N=3$ case there are six photodetectors and the detailed results are collected in Table V.

The total success probability is

$$\mathcal{P}_T=2(\mathcal{P}_1+\mathcal{P}_4)+6(\mathcal{P}_2+\mathcal{P}_3)=\frac{1}{2}(1-e^{-2|\alpha|^2})^3, \tag{44}$$

where

$$\mathcal{P}_1=2e^{-6|\alpha|^2}\sinh^6(|\alpha|^2), \tag{45}$$

$$\mathcal{P}_2=2e^{-6|\alpha|^2}\cosh^2(|\alpha|^2)\sinh^4(|\alpha|^2), \tag{46}$$

$$\mathcal{P}_3=2e^{-6|\alpha|^2}\cosh(|\alpha|^2)\sinh^5(|\alpha|^2), \tag{47}$$

and

$$\mathcal{P}_4=\frac{1}{4}e^{-6|\alpha|^2}\sinh^3(2|\alpha|^2). \tag{48}$$

For $|\alpha|\geq 2$ the total success probability $\mathcal{P}_T$ is approaching 1/2.

IV. MANIPULATION OF ENTANGLEMENT SIZE AND TYPE

Depending on an assigned task an entangled state of appropriate size and type will be engineered. For instance, as has been illustrated in Ref. [35], to split quantum information of an arbitrary two-qubit state one needs quintpartite cluster states while quadpartite ones are suitable only for a particular two-qubit state. In Sec. II we have generated free-flying entangled coherent states $|C_{M+N}^{(i)\alpha}\rangle$ and applied them to teleportation purposes in Sec. III. At this point questions may be asked regarding manipulation of the size and the type of entangled coherent states employing only linear optics elements.

One question is the following: "Given a state $|C_{M+N}^{(i)\alpha}\rangle$, can we enlarge or reduce its size?" For a possible enlargement we send each mode of the state through a 50:50 beam splitter and let the reflected mode pass through a $\pi/2$ phase shifter. In this way we shall obtain the state $|C_{2M+2N}^{(i)\alpha/\sqrt{2}}\rangle$, which has doubled number of modes but the amplitude of each mode is reduced by $\sqrt{2}$. The amplitude reduction would not cause a serious problem since correlation rather than amplitude value is a figure of merit. More generally, if each mode of the state $|C_{M+N}^{(i)\alpha}\rangle$ is sent through a collection of $Q-1$ proper beam splitters like in a branch of Fig. 1, then the resulting state will be $|C_{QM+QN}^{(i)\alpha/\sqrt{Q}}\rangle$; i.e., the size is greatly enlarged. Transparently, if we reverse the process then $|C_{QM+QN}^{(i)\alpha/\sqrt{Q}}\rangle$ will become

TABLE IV. Same as in Table I but for teleportation of $|\varphi'''\rangle_{56}$, $|\varphi''\rangle_{56}$, $|\varphi'\rangle_{56}$, and $|\varphi\rangle_{56}$ via $|C_{2+2}^{(1)\alpha}\rangle_{1234}$, $|C_{2+2}^{(2)\alpha}\rangle_{1234}$, $|C_{2+2}^{(3)\alpha}\rangle_{1234}$, and $|C_{2+2}^{(4)\alpha}\rangle_{1234}$, respectively.

n_3	n_4	n_5	n_6	p_i	U_{12}
0	0	Nonzero even	Nonzero even	p_2	$P_1(\pi)\otimes P_2(\pi)$
Odd	Odd	0	0	p_1	$I_1\otimes I_2$
Nonzero even	0	0	Odd	p_3	$I_1\otimes P_2(\pi)$
0	Nonzero even	Odd	0	p_3	$P_1(\pi)\otimes I_2$

TABLE V. Success probability $\mathcal{P}_i$ and recovery operator U_{123} for teleportation of $|\psi\rangle_{789}$, Eq. (43), via the quantum channel served by $|C_{3+3}^{(1)\alpha}\rangle_{123456}$, Eq. (42), in dependence of the detected photon numbers n_4, $n_5,\ldots$ and n_9 registered by the six photodetectors.

n_4	n_5	n_6	n_7	n_8	n_9	$\mathcal{P}_i$	U_{123}
0	Nonzero even	0	Nonzero even	0	Nonzero even	$\mathcal{P}_1$	$P_1(\pi)\otimes P_2(\pi)\otimes P_3(\pi)$
0	Odd	0	Nonzero even	0	Odd	$\mathcal{P}_2$	$P_1(\pi)\otimes P_2(\pi)\otimes P_3(\pi)$
0	Nonzero even	0	Odd	0	Odd	$\mathcal{P}_2$	$P_1(\pi)\otimes P_2(\pi)\otimes P_3(\pi)$
0	Odd	0	Odd	0	Nonzero even	$\mathcal{P}_2$	$P_1(\pi)\otimes P_2(\pi)\otimes P_3(\pi)$
0	0	Nonzero even	Nonzero even	Odd	0	$\mathcal{P}_3$	$P_1(\pi)\otimes P_2(\pi)\otimes I_3$
0	0	Nonzero even	Odd	Nonzero even	0	$\mathcal{P}_3$	$P_1(\pi)\otimes P_2(\pi)\otimes I_3$
0	0	Odd	Nonzero even	Nonzero even	0	$\mathcal{P}_3$	$P_1(\pi)\otimes P_2(\pi)\otimes I_3$
0	0	Odd	Odd	Odd	0	$\mathcal{P}_4$	$P_1(\pi)\otimes P_2(\pi)\otimes I_3$
Nonzero even	0	Nonzero even	0	Nonzero even	0	$\mathcal{P}_1$	$I_1\otimes I_2\otimes I_3$
Nonzero even	0	Odd	0	Odd	0	$\mathcal{P}_2$	$I_1\otimes I_2\otimes I_3$
Odd	0	Nonzero even	0	Odd	0	$\mathcal{P}_2$	$I_1\otimes I_2\otimes I_3$
Odd	0	Odd	0	Nonzero even	0	$\mathcal{P}_2$	$I_1\otimes I_2\otimes I_3$
Nonzero even	Nonzero even	0	0	0	Odd	$\mathcal{P}_3$	$I_1\otimes I_2\otimes P_3(\pi)$
Nonzero even	Odd	0	0	0	Nonzero even	$\mathcal{P}_3$	$I_1\otimes I_2\otimes P_3(\pi)$
Odd	Nonzero even	0	0	0	Nonzero even	$\mathcal{P}_3$	$I_1\otimes I_2\otimes P_3(\pi)$
Odd	Odd	0	0	0	Odd	$\mathcal{P}_4$	$I_1\otimes I_2\otimes P_3(\pi)$

$|C_{M+N}^{(i)\alpha}\rangle$; i.e., the size is reduced with an increased modal amplitude.

Another question is the following: "Can we go back and forth between states $|C_{M+N}^{(i)\alpha}\rangle$ and GHZ-type states?" Only one way is possible and it is not deterministic. For example, if we want to obtain GHZ-type states $|\mathrm{GHZ}_N^{\pm\alpha}\rangle \propto|\alpha,\alpha,\ldots,\alpha\rangle_{M+1M+2\ldots M+N}\pm|-\alpha,-\alpha,\ldots,-\alpha\rangle_{M+1M+2\ldots M+N}$ (normalization coefficients are omitted) from the state $|C_{M+N}^{(1)\alpha}\rangle$, we have to measure the M modes $1,2,\ldots,M$ of it. The measurement of a mode m is performed by superimposing this mode with an auxiliary mode $|-i\alpha\rangle_{m'}$ on a 50:50 beam splitter followed by detecting both the modes by photodetectors D_m and $D_{m'}$. If D_m clicks and $D_{m'}$ does not, $|\alpha\rangle_m$ is measured. If D_m does not click and $D_{m'}$ does, $|-\alpha\rangle_m$ is measured. If both D_m and $D_{m'}$ do not click, we cannot discriminate between $|\alpha\rangle_m$ and $|-\alpha\rangle_m$. The modes $1,2,\ldots,M$ should be measured in the following manner. First we measure mode 1. If D_1 clicks and $D_{1'}$ does not, we obtain $|\mathrm{GHZ}_N^{+\alpha}\rangle$ together with $|\alpha,\alpha,\ldots,\alpha\rangle_{23\ldots M}$ as a by-product. If D_1 does not click and $D_{1'}$ does, we obtain $|\mathrm{GHZ}_N^{-\alpha}\rangle$ together with $|-\alpha,-\alpha,\ldots,-\alpha\rangle_{23\ldots M}$ as a by-product. If both D_1 and $D_{1'}$ do not click, we proceed to measure mode 2. If D_2 clicks and $D_{2'}$ does not, we obtain $|\mathrm{GHZ}_N^{+\alpha}\rangle$ together with $|\alpha,\alpha,\ldots,\alpha\rangle_{34\ldots M}$ as a by-product. If D_2 does not click and $D_{2'}$ does, we obtain $|\mathrm{GHZ}_N^{-\alpha}\rangle$ together with $|-\alpha,-\alpha,\ldots,-\alpha\rangle_{34\ldots M}$ as a by-product. If both D_2 and $D_{2'}$ do not click, we proceed to measure mode 3. This process is continued until we finish the measurement of mode M. We only fail in the worst situation when all the photodetectors are silent that occurs with a negligible probability. Similarly, if we want to obtain $|\mathrm{GHZ}_M^{\pm\alpha}\rangle$ from $|C_{M+N}^{(1)\alpha}\rangle$, we have to measure the N modes $M+1$, $M+2,\ldots$, $M+N$ in the same manner. However, for going the other way around, i.e., transforming GHZ-type entangled coherent states to our states $|C_{M+N}^{(i)\alpha}\rangle$, it is not possible with linear optics elements.

V. CONCLUSION

We have introduced an alternative type of $(M+N)$-mode entangled coherent states $|C_{M+N}^{(i)\alpha}\rangle$ defined by Eqs. (1) and (20)–(22). A scheme has been proposed to generate such states with all the modes free-flying. Compared to previous schemes, the advantages of ours is that it is both much simpler and deterministic since no measurements are necessary. The only challenging point is the use of a π-cross-Kerr medium. However, with respect to electromagnetically-induced-transparency (EIT) based modern techniques (see, e.g., discussions in Sec. 4 of Ref. [36]) our scheme can be regarded as a feasible one. By the way, nonlinear optics elements cannot be avoided in problems employing coherent-state coding. In this sense, our scheme can be treated as optimum because the use of nonlinear optics is kept minimum (Kerr medium is used only once, not many times as in [26]). Possible applications of free-flying states $|C_{N+N}^{(i)\alpha}\rangle$ have been found. Namely, we have proposed schemes using $|C_{2+2}^{(i)\alpha}\rangle$ as quantum channel to teleport a family of two-mode three-component coherent states and $|C_{3+3}^{(i)\alpha}\rangle$ to teleport a family of three-mode two-component coherent states. We have also addressed the issue of manipulating entangled coherent states by showing that, with only linear optics elements, size of the entangled coherent states $|C_{M+N}^{(i)\alpha}\rangle$ can be enlarged or reduced and from them we can probabilistically obtain certain GHZ-type entangled coherent states. Undoubtedly, the entangled coherent states $|C_{M+N}^{(i)\alpha}\rangle$ will find wider applications in quantum-information processing and quantum computing. Further studies are in progress.

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