PHYSICAL REVIEW A **80**, 042305 (2009)

Entanglement generation between distant atoms by Lyapunov control

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(Received 15 June 2009; published 6 October 2009)

We show how to apply Lyapunov control design to the problem of entanglement creation between two atoms in distant cavities connected by optical fibers. The Lyapunov control design is optimal in the sense that the distance from the target state decreases monotonically and exponentially, and the concurrence increases accordingly. This method is far more robust than simple geometric schemes.

DOI: 10.1103/PhysRevA.80.042305 PACS number(s): 03.67.Bg, 42.50.Dv

I. INTRODUCTION

Atoms, or their artificial counterparts, quantum dots, in cavities or traps have great potential for applications in quantum communication, metrology, and information processing. Since entanglement is a crucial resource in quantum computation and communication, the preparation of maximally entangled states is a crucial task. Nonlocal interactions between two physical qubits are required to generate entanglement and there have been numerous proposals to effect such interactions, especially for atoms trapped in distant cavities [1–14], and similar schemes are conceivable for artificial atoms such as quantum dots. Some of these proposals make use of continuous feedback in open quantum systems [14] but most are based on Hamiltonian systems, and in most cases only simple geometric control schemes are employed to create the maximally entangled state. These methods have the advantage of simplicity but unfortunately often suffer from robustness issues.

In this paper, we explore an alternative control design inspired by Lyapunov functions [15–24] for robust entanglement generation. Lyapunov control is a form of local optimal control with numerous variants (see, e.g., [25] and references therein), which has the advantage of being sufficiently simple to be amenable to rigorous analysis. Therefore, its convergence, robustness, and stability properties have been well studied, and it has been shown to be highly effective for systems that satisfy certain sufficient conditions, roughly equivalent to the controllability of the linearized system [20,22]. Unfortunately, this appears to be a strong requirement that is not satisfied by many physical systems. However, in certain cases, in particular for systems such as the two-atom model proposed by Mancini and Bose [13], we can circumvent these restrictions by considering the dynamics on a subspace and successfully apply Lyapunov control to create maximally entangled states from certain initial product states in a robust fashion.

The paper is organized as follows. In Sec. II we briefly review the distant-atom model and the geometric control scheme proposed in [13] to generate entanglement. In Sec. III we briefly review Lyapunov control and show how to apply it to the problem of steering the system from certain product states to one of the four Bell states in a robust fashion. We will consider two control paradigms: one is to control the local Hamiltonian which is easier to implement experimentally; the other is to control the nonlocal interaction Hamiltonian, which might be possible for certain systems.

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II. TWO-DISTANT-ATOM MODEL AND GEOMETRIC CONTROL

We consider a two-qubit model where the qubits are encoded in two atoms or two quantum dots in distant cavities connected into a closed loop by optical fibers, as illustrated in Fig. 1. It was shown in [13] that eliminating the interacting light field between the two atoms in the dispersive regime leads to an effective Hamiltonian for the two-atom system of form $H_{\text{tot}}=H_{\text{local}}+H_{\text{eff}}$, where the local Hamiltonian induced by interaction with resonant light and the effective interaction Hamiltonian are:

$$H_{\text{local}} = B(X \otimes I + I \otimes X) \tag{1a}$$

$$H_{\text{eff}} = 2JZ \otimes Z \tag{1b}$$

where X, Y, and Z are Pauli operators and I is the identity operator, and the coupling constant $B=\eta J$ where η should be sufficiently smaller than 1 to ensure the derivation of H_{eff} remains valid.

This Hamiltonian can be used to generate a maximally entangled state from the initial ground state by turning on H_{tot} for a critical time t_0 before switching the field off [13].

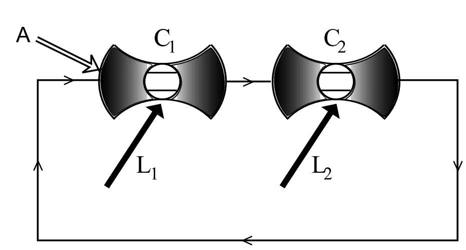

FIG. 1. Two cavities C_1 and C_2, each of which contains a two-level atom, are connected into a closed loop through optical fibers. The off-resonant driving field A generates an effective nonlocal Hamiltonian H_{eff} while the two local resonant lasers generate the local Hamiltonian H_{local}.

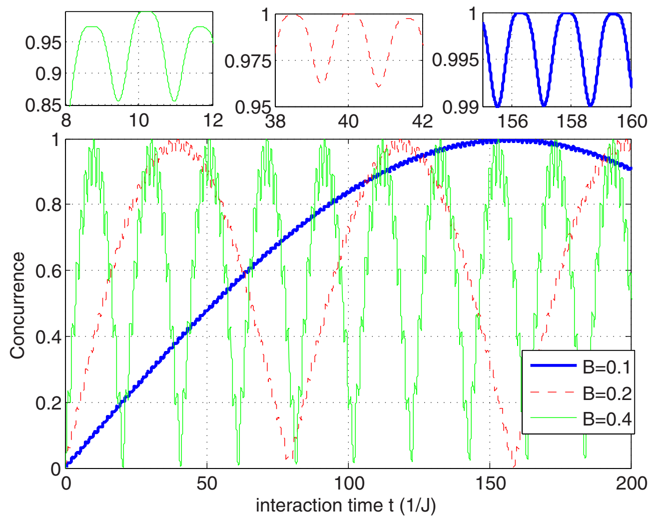

FIG. 2. (Color online) Concurrence as a function of the interaction time for the geometric control scheme for different values of coupling B. Due to fluctuations in the concurrence, achieving unit concurrence requires precise switching of the Hamiltonian. If the control Hamiltonian is switched on or off too early or too late, even by a small amount, the concurrence of the final state may be reduced significantly. The three subfigures on top of the main figure shows the zoom in of the plots.

Broadly speaking, by applying a constant Hamiltonian we effectively perform a rotation about a fixed axis in the two-qubit space, and with the correct timing we can choose the rotation angle such as to ensure that the system state is transferred to a maximally entangled state $\rho(t_f)$ at the final time t_f, where the degree of entanglement for the two-qubit system is measured by the concurrence [26]

$$\mathcal{C}(\rho) \equiv \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\} \tag{2}$$

where $\lambda_1, \ldots, \lambda_4$ are the eigenvalues of the matrix

$$\rho(Y \otimes Y)\rho^*(Y \otimes Y) \tag{3}$$

in decreasing order and ρ^* is the complex (not Hermitian) conjugate of ρ. However, plotting the entanglement of the final state versus the interaction time (Fig. 2) shows that achieving very high fidelity with respect to the maximally entangled state requires very precise switching as the concurrence is subject to small fluctuations. In the model we have assumed a fixed coupling strength J and controllable local field B. We see that increasing B significantly reduces the time required to prepare a maximally entangled state but also increases the magnitude of the fluctuations. E.g., for $B=0.1$ the fluctuations around the peak are only about 1% but it takes 157 time units to reach a maximally entangled state. For $B=0.4$ on the other hand, we can prepare a maximally entangled state in about 1/8 of the time but the concurrence fluctuations increase by a factor of approximately 15. This shows that this simple scheme for entanglement generation is quite sensitive to small variations in the switching times.

III. LYAPUNOV CONTROL DESIGN

In the previous section we have seen that the method of entanglement generation by switching a constant field on for a fixed amount of time is highly sensitive to small switching time errors. Ideally, we would like a control scheme where the concurrence of the two qubits converges to 1 asymptotically, without any fluctuations. In that way, the control is robust against switching time errors. A simple method that seems well suited to this task is Lyapunov-based design. Roughly speaking, the idea of Lyapunov control is to choose a suitable so-called Lyapunov function V and then try to find a control that ensures that V is monotonically decreasing along any dynamical evolution.

On timescales where the Hamiltonian evolution is still a good approximation, many physical systems satisfy the quantum Liouville equation (with $\hbar=1$)

$$\dot{\rho} = -i[H_0 + f(t)H_1, \rho].$$

Here we have assumed that the Hamiltonian consists of two parts, a fixed system Hamiltonian H_0 and an interaction part $f(t)H_1$ with a coupling strength $f(t)$ that can be varied in time. For example, for a two-level atom with energy level splitting Ω, interacting with a variable laser field of $f(t)$ via a dipole interaction, we might set $H_0 = \frac{\Omega}{2}\sigma_z$ and $H_1 = \sigma_x$. The fact that $f(t)$ can be varied is very crucial from control point of view, since this degree of freedom allows us to design the dynamics to derive the desired evolution.

We can define a general control task thus: for a given target state ρ_d, for example, a maximally entangled state, we wish to find a control function $f(t)$, such that the system state $\rho(t)$ will converge to ρ_d, as $t \to \infty$. In many applications, we allow $\rho_d(t)$ to evolve under H_0, and the control requirement becomes $\rho(t) \to \rho_d(t)$ as $t \to \infty$, which is generally known as tracking control [27]. In the following we assume:

$$\dot{\rho}_d = -i[H_0, \rho_d].$$

Motivated from the theory of Lyapunov function and the Hilbert Schmidt distance $\|\rho(t) - \rho_d(t)\|_2$, we define

$$V(\rho, \rho_d) = \frac{1}{2}\|\rho - \rho_d\|^2 = \frac{1}{2}\mathrm{Tr}[(\rho - \rho_d)^2]. \tag{4}$$

Assuming $\kappa > 0$, if we choose

$$f(t) = f(\rho(t), \rho_d(t)) = \kappa\, \mathrm{Tr}(\rho_d(t)[-iH_1, \rho(t)]), \tag{5}$$

we find that for $V(t) = V(\rho(t), \rho_d(t))$,

$$\dot{V}(t) = -f(t)\mathrm{Tr}\{\rho_d(t)[-iH_1, \rho(t)]\} = -\kappa f(t)^2 \leq 0. \tag{6}$$

Hence V is a Lypunov function and the value of V monotonically decreases along any solution $(\rho(t), \rho_d(t))$. Moreover, every solution $(\rho(t), \rho_d(t))$ converges to an invariant set E, called the LaSalle invariant set, on which $\dot{V}$ vanishes.

Recent theoretical work on Lyapunov-based control design [21–23] shows that most target states are almost globally asymptotically stable if the Hamiltonian satisfies certain conditions: (i) H_0 be strongly regular and (ii) H_1 be fully connected [23]. The former condition translates into the requirement that H_0 have distinct transition frequencies between any pair of energy levels. This rules out systems with degenerate or equally spaced energy levels. The latter condition is even more demanding. In the basis where H_0 is diagonal, all the off-diagonal elements of H_1 must be nonzero, i.e., transitions between any two energy level of H_0 must be possible. When the strict conditions on the Hamiltonian do not hold, the target state is generally no longer asymptotically stable, and we no longer have $\rho(t)\to\rho_d(t)$, implying that the control design becomes ineffective. This restricts the applicability of the method especially for higher-dimensional systems, including two-qubit models and spin chains.

However, for high-dimensional systems with Hamiltonians not satisfying the above conditions, it is still possible to make the target state asymptotically stable on a subspace, where the Lyapunov control can be applied effectively. In the following, for the two-distant-atom model (Fig. 1), we illustrate how the Lyapunov control design can be utilized to drive the system state from a product state to a maximally entangled state, despite the fact that the full Hamiltonian of the system clearly does not satisfy the strict conditions set out above.

IV. LYAPUNOV CONTROL DESIGN FOR ENTANGLEMENT CREATION

For the two-distant-atom model with Hamiltonian Eq. (1), we can either choose the control Hamiltonian H_1 to be the local Hamiltonian $H_1=H_{\text{local}}$ or the effective coupling Hamiltonian $H_1=H_{\text{eff}}$, depending on which scenario is easier to implement for a particular physical system.

A. Local control

First, let us consider controlling the local Hamiltonian. In this case we choose $H_0=H_{\text{eff}}=2J(Z\otimes Z)$ and $H_1=H_{\text{local}}=\eta J(X\otimes I+I\otimes X)$. To make the Hamiltonian easier to analyze, we transform from the Z-eigenbasis $\{|0\rangle,|1\rangle\}$ to the X-eigenbasis $\{|+\rangle,|-\rangle\}$. In this basis, the matrices for the Hamiltonian are rewritten as

$$H_0=2J\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix},\quad H_1=2\eta J\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&-1\end{pmatrix}.$$

and it is easy to see that the eigenvectors of H_0 are the Bell states

$$|\Psi^+\rangle=\frac{1}{\sqrt{2}}(|+-\rangle+|-+\rangle) \tag{7a}$$

$$|\Phi^+\rangle=\frac{1}{\sqrt{2}}(|++\rangle+|--\rangle) \tag{7b}$$

$$|\Phi^-\rangle=\frac{1}{\sqrt{2}}(|++\rangle-|--\rangle) \tag{7c}$$

$$|\Psi^-\rangle=\frac{1}{\sqrt{2}}(|+-\rangle-|-+\rangle). \tag{7d}$$

To generate maximally entangled state, we can choose $\rho_d=|\Phi^+\rangle\langle\Phi^+|$, for instance, and the control $f(t)=\kappa\,\text{Tr}(\rho_d[-iH_1,\rho(t)])$, according to Eq. (5). Notice that H_0 and H_1 do not satisfy the strict condition in Sec. III. Thus this design cannot drive every state to the target state, but we can see that if the initial state of the system is $\rho(0)=|++\rangle\langle++|$ or $|--\rangle\langle--|$ then the state will converge to the target state. In fact, in the Bell-state basis, the Hamiltonian can be written as

$$\tilde{H}_0=2J\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix},\quad \tilde{H}_1=2\eta J\begin{pmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{pmatrix},$$

where

$$\rho_d=\begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\quad \rho(0)=\frac{1}{2}\begin{pmatrix}0&0&0&0\\0&1&\pm1&0\\0&\pm1&1&0\\0&0&0&0\end{pmatrix}.$$

For states initially prepared in the subspace $\mathcal{S}$ spanned by $|++\rangle$ and $|--\rangle$, we clearly see that the dynamics under the Hamiltonian $H_0+f(t)H_1$ will be confined in that subspace, and thus we can consider the dynamics on this two-dimensional subspace $\mathcal{S}$ where the Hamiltonian and the state take the form:

$$H_0=2J\begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad H_1=2\eta J\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \rho_d=\begin{pmatrix}1&0\\0&0\end{pmatrix}.$$

The results in [23] now guarantee that all solutions in $\mathcal{S}$ except for $|\Phi^-\rangle$ will converge to the target state. The control field varies smoothly and steers the system gently to the target state as shown in Fig. 3. Moreover convergence is exponential and

$$\begin{aligned}|f(t)|&=\kappa|\text{Tr}(i[\rho(t),\rho_d(t)]H_1)|\\&=\kappa\|i[\rho(t),\rho_d(t)]H_1\|\\&\le\kappa\|i[\rho(t),\rho_d(t)]\|\cdot\|H_1\|\end{aligned} \tag{8}$$

shows that $f(t)$ is bounded and we can choose κ to ensure that $|f(t)|$ is sufficiently small and the approximations inherent in the model remain valid.

The method can also be utilized to increase the entanglement in the initial state, i.e, to prepare a maximally entangled state starting with a partially entangled one. More specifically, if the system initially starts in the state $|\psi_0\rangle=\lambda_1|++\rangle+\lambda_2|--\rangle$ then the control design produces a control field that steers the system from this state to the desired maximally entangled state $|\Phi^+\rangle$. Choosing $\rho_d=|\Phi^-\rangle\langle\Phi^-|$ instead, we can similarly prepare $|\Phi^-\rangle$, and it can be verified that steering the state to $|\Psi^-\rangle$ simply requires inverting the sign of the control field. Thus, not only can we prepare a maximally entangled state, but we can select which state we prepare.

If the coupling constants of the local Hamiltonian for the two atoms are not exactly identical, e.g., if

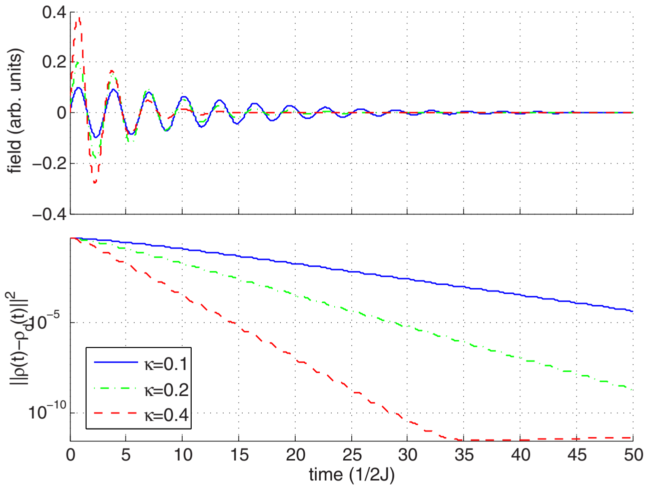

FIG. 3. (Color online) Local control: control fields obtained from Lyapunov design for different values of κ and distance between the system state and the Bell state $|\Psi^{+}\rangle$. The control design is robust in that the field amplitude gently decreases to zero, and the semilog distance plot shows that the convergence to the target state is not only monotonic but also exponential with the convergence rate determined by κ.

$H_{\text{local}}=\eta J(X\otimes I+kI\otimes X)$ then changing to the X basis gives $H_1=\eta J\ \text{diag}(1+k,1-k,-1+k,-1-k)$, which transforms to

$$\tilde{H}_1=\eta J\begin{pmatrix} 0 & 0 & 0 & 1-k\\ 0 & 0 & 1+k & 0\\ 0 & 1+k & 0 & 0\\ 1-k & 0 & 0 & 0\end{pmatrix}. \quad (9)$$

Thus for $k\neq 1$ we can also steer the system from the product states $|+-\rangle$ or $|-+\rangle$ to the Bell state $|\Phi^{\pm}\rangle$, i.e., for this two-atom model Lyapunov control can be used to prepare any of the four Bell states.

One limitation of the scheme is that the initial state must be in the subspace $\mathcal{S}$, for example, $\mathcal{S}=\text{span}\{|++\rangle,|--\rangle\}$, for the control to be effective. This is not a shortcoming of the proposed control scheme, however, because we can see from the structure of $\tilde{H}_0$ and $\tilde{H}_1$ that the control system is decomposable, hence not controllable on the whole space [28]. More specifically, the dynamics on the orthogonal subspaces $\mathcal{S}$ and $\mathcal{S}^{\perp}$ are independent, and subspace populations are conserved quantities. Thus, for the above Hamiltonian, *no* control exists that steers population from subspace $\mathcal{S}$ to $\mathcal{S}^{\perp}$ and vice versa.

B. Interaction control

Instead of controlling the atoms locally, we can alternatively control the nonlocal Hamiltonian H_{eff}, if the underlying physical system allows. In this case we choose $H_0=\eta J(X\otimes I+I\otimes X)$ and $H_1=2J(Z\otimes Z)$, or in the X eigenbasis

$$H_0=2\eta J\begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -1\end{pmatrix},\quad H_1=2J\begin{pmatrix} 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\end{pmatrix}.$$

The Bell states are no longer the eigenstates of H_0. Hence, for $\rho_d(0)=|\Phi^{+}\rangle\langle\Phi^{+}|$, the target state is also evolving with time, but for $\rho(0)=|++\rangle$ the dynamics is still confined to the subspace $\mathcal{S}$ spanned by $|++\rangle$ and $|--\rangle$. Therefore, the dynamics can again be reduced to a two-dimensional subspace on which we have

$$H_0=2\eta J\begin{pmatrix} 1 & 0\\ 0 & -1\end{pmatrix},\quad H_1=2J\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix},$$

as well as

$$\rho(0)=\begin{pmatrix} 1 & 0\\ 0 & 0\end{pmatrix},\quad \rho_d(0)=\frac{1}{2}\begin{pmatrix} 1 & 1\\ 1 & 1\end{pmatrix},$$

where the orbit of $\rho_d(t)$ is the equator of the Bloch sphere.

From the analysis in [23] we can conclude that all solutions in $\mathcal{S}$ will converge to the equator of the Bloch sphere, i.e., states of the form

$$\rho=\frac{1}{2}\begin{pmatrix} 1 & e^{-i\alpha}\\ e^{i\alpha} & 1,\end{pmatrix}$$

which corresponds to the LaSalle invariant set E of the original problem satisfying

$$\rho=\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & e^{-i\alpha}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ e^{i\alpha} & 0 & 0 & 1.\end{pmatrix}.$$

Thus, we can no longer guarantee $\rho(t)\to\rho_d(t)$ as $t\to+\infty$, i.e., that the state converges to a particular Bell state. This is illustrated in Fig. 4, which shows that the distance from the target state still decreases monotonically and exponentially but the asymptotic value of $V(\rho(t),\rho_d(t))$ for $t\to\infty$ now depends on κ and is generally larger than zero. However, since all the states in the set to which $\rho(t)$ converge are maximally entangled, we can still steer the system to a maximally entangled state, and the concurrence still increases monotonically to one (see Fig. 5) but the relative phase α of the state we converge to now depends on the exact initial state and the feedback strength κ.

Strictly speaking, as the control $f(t)$ reduces to zero, the norm of H_{local} will cease to be significantly smaller than that of H_{eff}, rendering the approximations made in the derivation of H_{eff} invalid, unless we reduce the strength of H_{local} accordingly. However, in practice, the system should already have reached a state with significant entanglement before this happens.

V. CONCLUSION

We have shown how to apply Lyapunov control to the problem of generating entanglement between two-distant

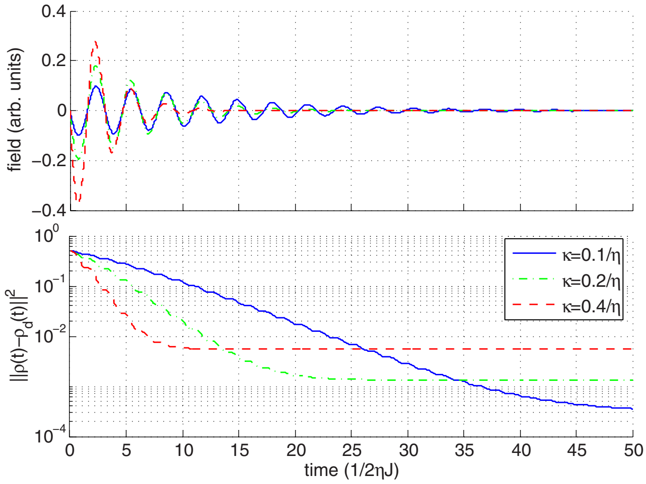

FIG. 4. (Color online) Interaction control: control fields obtained from Lyapunov design for different values of κ and distance between the system state and the target state with $\rho_d(0)=|\Psi^+\rangle\langle\Psi^+|$. The control design is robust in that the field amplitude gently decreases to zero, and the semilog distance plot shows that the convergence to the target state is not only monotonic but actually exponential, although unlike in the local control case, $\|\rho(t)-\rho_d(t)\|$ does not converge to 0. The final $\rho(t)$ is still maximally entangled with unit concurrence.

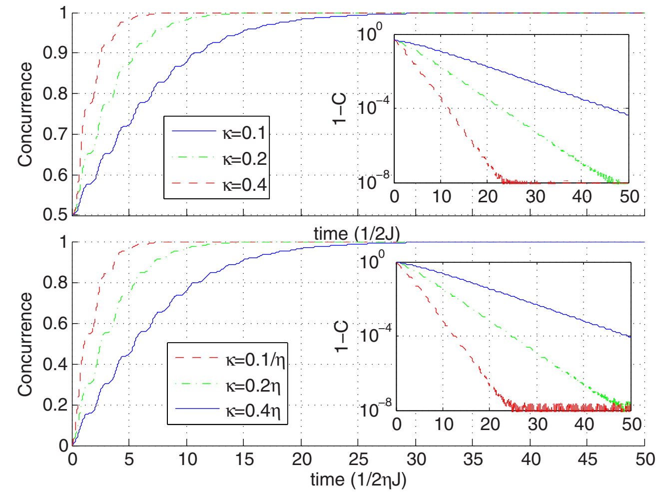

FIG. 5. (Color online) The evolution of concurrence $\mathcal{C}$ under Lyapunov control for different values of κ for local control (top) and interaction control (bottom) shows monotonic convergence to 1. The inset shows that the error, i.e., $1-\mathcal{C}$ decreases effectively exponentially.

two-levels atoms in cavities connected by optical fibers. Despite the fact that the sufficient condition for asymptotic stability of a target state is not satisfied for the entire state space, we can still ensure almost globall asymptotic stability of a subspace. Within that subspace we can drive the system from a product state to a maximally entangled state. The Lyapunov control design has the advantage of much greater robustness compared to simple geometric schemes, and optimality in the sense that the distance from the maximally entangled target state is monotonically decreasing, and the convergence speed is exponential. We have discussed two control paradigms: control of the local Hamiltonian, as well as control of the effective interaction Hamiltonian between the two atoms. In both cases we can generate a maximally entangled state from an initial product state: for the former case the system state will converge to a stationary Bell state, while for the latter case the relative phase of the final state will keep varying under the Hamiltonian, since the target state is nonstationary. Moreover, in the latter case, the model becomes invalid when the control amplitude becomes sufficiently small. Therefore, the former control paradigm is preferable. The Lyapunov control design can be also used to steer partially entangled states to a maximally entangled state, although the control is only effective for initial states in the subspace where the target state is asymptotically stable. This is not a limitation of the control design, however, but a consequence of the fact that the controlled system is decomposable into two orthogonal subspaces on each of which the dynamics is invariant. In this sense, the Lyapunov control design is as effective as is possible within the constraints of the model.

ACKNOWLEDGMENTS

We thank Prof. Sougato Bose for stimulating discussions. S.G.S. acknowledges funding from EPSRC ARF Grant No. EP/D07195X/1, Hitachi, and NSF Grant No. PHY05-51164.

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