

Multimode-to-monomode guided-atom lasers: An entropic analysis

G. L. Gattobigio,^{1,2} A. Couvert,¹ M. Jeppesen,^{1,3} R. Mathevet,² and D. Guéry-Odelin^{1,2}

¹Laboratoire Kastler Brossel, Ecole Normale Supérieure, 24 rue Lhomond, 75005 Paris, France

²Laboratoire de Collisions Agrégats Réactivité, CNRS UMR 5589, IRSAMC, Université Paul Sabatier, 118 Route de Narbonne, 31062 Toulouse Cedex 4, France

³Australian Centre for Quantum Atom Optics, Physics Department, The Australian National University, Canberra 0200, Australia

(Received 29 May 2009; published 19 October 2009)

We have experimentally demonstrated a high level of control of the mode populations of guided-atom lasers (GALs) by showing that the entropies per particle of an optically GAL and the one of the trapped Bose-Einstein condensate (BEC) from which it has been produced are the same. The BEC is prepared in a crossed beam optical dipole trap. We have achieved isentropic outcoupling for both magnetic and optical schemes. We can prepare GAL in a nearly pure monomode regime (85% in the ground state). Furthermore, optical outcoupling enables the production of spinor guided-atom lasers and opens the possibility to tailor their polarization.

DOI: [10.1103/PhysRevA.80.041605](https://doi.org/10.1103/PhysRevA.80.041605)

PACS number(s): 03.75.Pp, 37.25.+k, 41.85.Ew, 42.60.Jf

Isentropic transformations have been used extensively to manipulate classical and degenerate quantum gases. For instance, the reversible formation of a molecular Bose-Einstein condensate (BEC) from ultracold fermionic atoms having two spin components was performed by adiabatically tuning the interspecies scattering length from positive to negative values [1–5]. Another illustration is the adiabatic change of the shape of the confining trap of cold atoms giving the possibility to change the phase-space density in a controlled manner [6,7]. A spectacular demonstration of this idea has been the multiple reversible formations of BEC by adiabatically superimposing an optical dipole trap on a magnetically trapped and precooled sample of atoms [7]. For an ideal transformation, the entropy of the initial cloud should remain constant. However, the limitations of the experimental setup always introduce an extra source of entropy. The challenge for the experimentalists is to minimize this latter contribution.

In this Rapid Communication, we demonstrate the control and the characterization of a guided-atom laser (GAL) [8,9]. The experiments performed to date on the beam quality were addressing the spatial mode of free-falling-atom lasers. The importance of the outcoupling scheme or the role played by atom-atom interactions has been extensively studied [10–12]. GALs are characterized by their population in each transverse modes of the guide. We have extended the use of isentropic analysis to propagating matter waves in order to relate quantitatively these populations to the characteristics of the BEC from which the GAL originates. This approach turns out to be possible because of both the validity of the local thermal equilibrium and the sufficiently large reduction in the extra entropy production generated by the experimental manipulation. Improvements to the production and characterization of the GAL are crucial for fundamental studies such as quantum transport [13] and applications in metrology [14], among others.

The experiment starts by loading 3×10^7 ^{87}Rb atoms in a crossed beam optical dipole trap at a wavelength of 1070 nm from an elongated magneto-optical trap (MOT). To transfer the atoms in the lower hyperfine level $F=1$, we align the horizontal arm (1) of the optical trap with the long \hat{y} axis of

the MOT [see the inset of Fig. 1(a)] and we mask the repump light in the overlapping region between the two traps. The other arm (2) of the crossed dipole trap makes an angle of $\theta=45^\circ$, in the \hat{y} - \hat{z} plane, where the \hat{z} axis is the vertical axis [9]. In practice, beam (1) [(2)] has a waist of $w_1=40 \mu\text{m}$ ($w_2=130 \mu\text{m}$) and an initial power of $P_1 \approx 20 \text{ W}$ ($P_2 \approx 100 \text{ W}$).

The BEC transition temperature is on the order of 300 nK. Typically, a BEC of 2×10^5 atoms is formed after a 4 s evaporation ramp carried out by lowering the power of both beams (see Fig. 2). Using the spin-distillation technique during the evaporation process [9], we prepare the BEC in a purely $F=1$, $m_F=0$ state. The density profile distribution of the BEC is obtained from absorption images after a 20 ms time of flight (TOF). Figure 1(a) shows a typical BEC profile with a significant thermal fraction.

We first describe experiments performed using magnetic

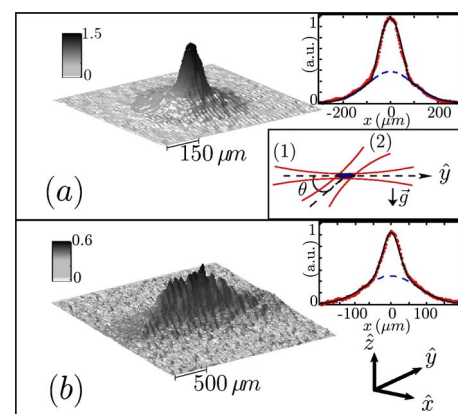


FIG. 1. (Color online) (a) Absorption image of a BEC of ^{87}Rb atoms released from a crossed dipole trap, after 20 ms time of flight. Inset, top right: atom density across a line through the cloud center. Inset, bottom right: crossed dipole trap laser geometry. (b) Absorption image of a guided-atom laser, outcoupled from the same BEC and imaged in the same conditions. The atom laser has a mean excitation number in the transverse modes of $\langle n \rangle = 4$. Inset: atom density integrated along the guide axis (1). A bimodal density structure is evident, as in (a) above.

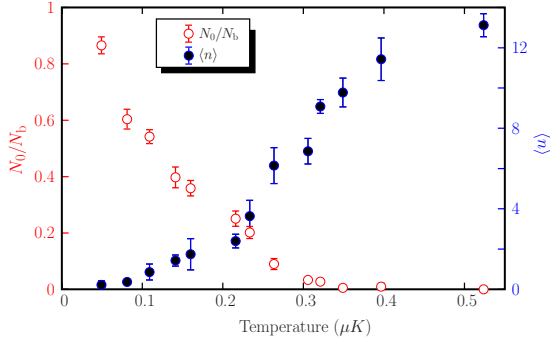


FIG. 2. (Color online) Measured condensed fraction N_0/N_b (open circles) of the source BEC and mean excitation number $\langle n \rangle$ (solid circles) of the guided-atom laser produced from the source BEC by magnetic outcoupling. Both are shown as a function of the temperature of the trapped source BEC immediately prior to the outcoupling.

outcoupling. At the desired temperature, we prepare the cloud by increasing the power of the two beams in 200 ms to limit residual evaporation. Beam (1) is used as a horizontal guide whose frequency $\omega_{\perp}/2\pi=289$ Hz is measured by exciting the transverse sloshing mode and varies by less than 4% over 1 mm. The power of beam (2) is then reduced to facilitate the outcoupling, while the power of beam (1) is kept constant [inset of Fig. 1(a)]. The outcoupling is performed by applying a magnetic gradient with increasing strength as a function of time. During this 100 ms outcoupling phase, outcoupled atoms propagate in the optical guide over 1 mm before being imaged. Figure 1(b) shows a density profile of the GAL generated from the BEC of Fig. 1(a). Such pairs of images have been acquired for different temperatures of the atomic cloud.

To relate quantitatively the characteristics of the GAL to those of the cloud it originated from, we model the GAL using a thermodynamic approach within the grand-canonical ensemble [9,15,16]. We consider an ideal Bose gas enclosed in a fictitious box along the longitudinal axis and confined transversally by a harmonic potential. For a given linear density ρ , a transverse condensation occurs below a critical temperature T_a^c , i.e., a macroscopic fraction of the atoms occupy the transverse ground state independently of their longitudinal state [17]. Indeed, there is no Bose-Einstein condensation in the true ground state (transverse and longitudinal) of this quasi-one-dimensional system in the thermodynamic limit [16].

The transverse profile (integrated along the \hat{y} axis) is readily inferred from the thermodynamic model

$$\lambda\rho(x) = \frac{g_{1/2}(z_a)}{\sqrt{\pi\sigma^2}} e^{-x^2/\sigma^2} + \frac{1}{a_0\sqrt{2\pi\xi_a^3}} g_2(z_a e^{-\xi_a x^2/2a_0^2}), \quad (1)$$

where $a_0=(\hbar/m\omega_{\perp})^{1/2}$ is the transverse harmonic-oscillator length, σ is the size of the ground state [18], $z_a=\exp(\beta\mu)$ is the fugacity, $\xi_a=\beta\hbar\omega_{\perp}$ with $\beta=1/k_B T$ [19], $\lambda=h/(2\pi mk_B T)^{1/2}$, and $g_p(z)=\sum_{n=1}^{\infty} z^n/n^p$ is the Bose function of index p .

To extract quantitative data from our profiles, we have approximated the g_2 function by a Gaussian and fit our data with the sum of two Gaussians. We have checked numerically that this method does not introduce significant errors with respect to experimental uncertainties. In this way, we can calculate the fraction of condensed atoms, the temperature, and we can thus obtain the mean excitation number through the formula [20] $\langle n \rangle = [1 - (\rho_0/\rho)] \langle n \rangle_{\text{th}}$, where $\rho_0 = g_{1/2}(z_a)/\lambda$ is the mean linear density in the transverse ground state and $\langle n \rangle_{\text{th}}$ is the average excitation number of the excited atoms, which is a function of ξ_a . The first-order expansion $\langle n \rangle_{\text{th}} \simeq \xi_a^{-1} - 1/2$ gives a satisfactory estimate over the whole range of parameters that we have used.

In practice, the two measurements of the linear density and of $\langle n \rangle$ give access to the two parameters ξ_a and z_a , and therefore characterize completely the thermodynamic equilibrium quantities. We have plotted in Fig. 2 the measured value for $\langle n \rangle$ along with the condensed fraction of the source cloud of atoms, from which the GAL has been produced, as a function of the temperature of the cloud. There is a clear correlation between the two quantities: the colder the sample, the more monomode the guided-atom laser. The mean number of populated energy levels, taking into account their degeneracy, varies from more than ~ 100 levels for the atom laser extracted from a cloud at 500 nK to just one level populated. Indeed, for the coldest point at 50 nK, 85% of atoms are in the transverse ground state of the guide, to be compared with previously reported values of 70% [9], 65% [21], and 40% [8]. This unprecedented value is due to the intrinsically better mode matching between the initial trap and the guide.

In the following, we compare quantitatively the entropy per atom in both systems. This comparison is carried out in a range of temperature for the source cloud such that $\hbar\omega \leq gn_b \leq k_B T < 0.9k_B T_b^c$, where T_b^c is the BEC transition temperature, n_b is the atomic density of the condensate, and $g=4\pi\hbar^2 a/m$ is the strength of the interaction. In this temperature regime, the Hartree-Fock and Popov theories yield very similar predictions for thermodynamic quantities, which are well reproduced by the ideal gas formula [1,22]. For our GAL, the interactions are such that $\rho_0 a < 0.5$, which validates the use of a noninteracting model described above.

We have therefore computed independently the entropy per particle $S/k_B N$ using the noninteracting formula for both systems. This quantity is intensive and, in both cases, depends only on the two dimensionless and intensive parameters: the fugacity z and $\xi=\beta\hbar\omega$. Within our thermodynamic model, the entropy per particle of the GAL reads

$$\frac{S_a(z_a, \xi_a)}{k_B N_a} = -\ln z_a + \rho\lambda^{-1} \sum_{p=1}^{\infty} \left[\frac{z_a^p}{p^{3/2}} \frac{1}{(1-e^{-p\xi_a})^2} + \frac{p+1}{2} g_{3/2}(z_a e^{-p\xi_a}) + \frac{2\xi_a z_a^p e^{-p\xi_a}}{p^{1/2}} \frac{1}{(1-e^{-p\xi_a})^3} \right]. \quad (2)$$

Figure 3 shows experimental measurements of the entropy per particle of the source BEC compared to the

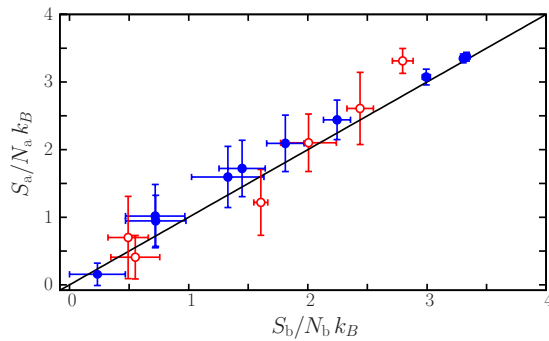


FIG. 3. (Color online) Entropy per atom of the atom laser ($S_a/N_a k_B$) against the entropy per atom of the source BEC ($S_b/N_b k_B$). Each point is the average of ten measurements. The different points correspond to different initial temperatures of the BEC. Solid circles are for magnetic outcoupling, while open circles are for optical outcoupling. The solid line is the expected result for equal entropy per particle of both systems.

corresponding GAL. The solid line represents perfect isentropic outcoupling (that is, $S_a/N_a k_B = S_b/N_b k_B$). Our data (solid circles) are therefore in good agreement with the isentropic assumption used to model the outcoupling. The error bars originate from the nonperfect pointing stability of both dipole beams yielding fluctuations of the temperature and the number of atoms of the BEC, and residual excitations of the propagating atom laser.

The robustness of the entropic analysis has been confirmed by studying the GAL generated by optical outcoupling (i.e., outcoupling by reducing the power in laser beam (2), with no magnetic field used). After the evaporation ramp and subsequent recompression of the crossed dipole trap (as with magnetic outcoupling), the outcoupling is performed by decreasing the intensity of beam (2) progressively until atoms spill into the optical guide formed by beam (1) (see Fig. 1). The outcoupled atoms experience a force due both to gravity [since beam (1) is not perfectly horizontal] and the local dipole force [since beam (2) does not necessarily cross beam (1) at its waist position]. The comparison between the entropies of the atom laser and the BEC from which it has been generated are plotted in Fig. 3 (open circles), which confirms that this optical scheme is also nearly isentropic. Conversely, one can predict the characteristics of the guided-atom laser by knowing those of the condensate from which it has been outcoupled (assuming an isentropic transformation).

The control of the mean number of excitations $\langle n \rangle$ could be used to explore the transition between the classical regime, in which a large number of energy levels are populated, and a pure quantum regime where nearly all atoms are in the transverse ground state. For instance, this system could be well suited to investigate classical versus quantum chaos where significant differences are expected between the two regimes [23].

The model used to deduce the entropy assumes that the GAL is at local thermodynamic equilibrium. The collision rate evaluated in the moving frame and using the formula of the classical limit is on the order of 25 s^{-1} for typical experimental parameters. It is strongly enhanced because of

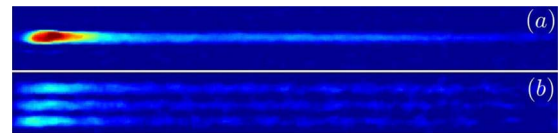


FIG. 4. (Color online) (a) Spinor guided-atom laser obtained by optically outcoupling atoms from a spinor condensate. (b) The three components $m_F = -1, 0, +1$ are separated by applying a Stern and Gerlach field during the time of flight.

bosonic amplification [24]. During the 100 ms of the outcoupling process, the local thermal equilibrium assumption is thus valid. We have also performed outcoupling over shorter times (50 ms) and obtained similar agreement for the entropy per particle.

As the GAL propagates, its density is diluted due to velocity dispersion and inhomogeneous forces on the atoms. The propagation is therefore accompanied by a modification of the transverse mode population, as soon as the collision rate is sufficient to validate the local equilibrium assumption. This is a significant difference with light propagation in fibers. The expected modification for a 1 mm propagation length is compatible with our error bars when we compare the results of long outcoupling time with respect to short outcoupling time. After a sufficiently long propagation, the mode population is expected to freeze out since the atomic density decreases and the collision rate becomes too small for restoring the thermal equilibrium. In this limit, the thermodynamic model no longer applies.

The optical outcoupling process affects atoms of all Zeeman states equally. Also, the optical waveguide confines all Zeeman states equally, compared to a magnetic guides in which only certain Zeeman states may propagate. It is therefore possible to produce atom lasers that contain mixtures of atoms in different spin states, or even linear superpositions of spin states. Starting with a spinor BEC (see [25], and references therein) in a incoherent mixture of $m_F = \pm 1, 0$, we have been able to generate a spinor guided-atom laser as shown in Fig. 4 where the three components are separated using a Stern and Gerlach field during the TOF.

As a preliminary study, we have produced a BEC in superposition of states of the $F=1$ and $F=2$ hyperfine levels, by combining the spin-distillation technique [9], and microwave pulses with well-defined polarization [26]. The internal degrees of freedom of the atoms are the analog of polarization for light. The dimension of the corresponding subspace is much larger for atoms than for light. An appealing perspective, which requires a well-controlled magnetic environment, lies in the full control of the polarization of the guided-atom laser over its propagation.

In this Rapid Communication, we have demonstrated an unprecedented control over the transverse degrees of freedom of a guided-atom laser. Two different outcoupling schemes are analyzed by a quantitative model based on an entropy analysis. In addition, optical outcoupling offers the possibility to control the internal degrees of freedom. A related prospect is the generation of a GAL with a very narrow

longitudinal velocity dispersion. This is in principle achievable by exploiting the tunnel effect for the outcoupling [27].

Another promising perspective of the techniques developed in this Rapid Communication is the production of a continuous guided-atom laser. The periodic replenishing of Bose-Einstein condensates held in an optical trap has already been demonstrated in Ref. [28]. Using the spin-distillation technique [9], this reservoir of condensates could be produced in a $m_F=0$ state. Applying a gradient of magnetic field along one arm of the dipole trap, atoms could be continuously extracted in it by applying a radio frequency

to transfer atoms into a magnetically sensitive state $m_F = \pm 1$.

We acknowledge fruitful discussions with T. Lahaye, Y. Castin, and J. Dalibard and financial support from the Délégation Générale pour l'Armement, the Institut Francilien de Recherche sur les Atomes Froids (IFRAF), the Plan-Pluri Formation (PPF), Région Midi-Pyrénées, and the Agence Nationale de la Recherche (Contract No. ANR-09-BLAN-0134-01).

-
- [1] L. D. Carr, G. V. Shlyapnikov, and Y. Castin, Phys. Rev. Lett. **92**, 150404 (2004).
- [2] J. E. Williams, N. Nygaard, and C. W. Clark, New J. Phys. **6**, 123 (2004).
- [3] T. Bourdel *et al.*, Phys. Rev. Lett. **93**, 050401 (2004).
- [4] G. B. Partridge, K. E. Strecker, R. I. Kamar, M. W. Jack, and R. G. Hulet, Phys. Rev. Lett. **95**, 020404 (2005).
- [5] M. W. Zwierlein, C. H. Schunck, C. A. Stan, S. M. F. Raupach, and W. Ketterle, Phys. Rev. Lett. **94**, 180401 (2005).
- [6] P. W. H. Pinkse *et al.*, Phys. Rev. Lett. **78**, 990 (1997).
- [7] D. M. Stamper-Kurn *et al.*, Phys. Rev. Lett. **81**, 2194 (1998).
- [8] W. Guerin *et al.*, Phys. Rev. Lett. **97**, 200402 (2006).
- [9] A. Couvert *et al.*, EPL **83**, 50001 (2008); **85**, 19901 (2009).
- [10] I. Bloch, T. W. Hänsch, and T. Esslinger, Phys. Rev. Lett. **82**, 3008 (1999).
- [11] J.-F. Riou *et al.*, Phys. Rev. Lett. **96**, 070404 (2006).
- [12] M. Jeppesen *et al.*, Phys. Rev. A **77**, 063618 (2008).
- [13] P. Leboeuf and N. Pavloff, Phys. Rev. A **64**, 033602 (2001); I. Carusotto and G. C. La Rocca, Phys. Rev. Lett. **84**, 399 (2000); T. Paul, K. Richter, and P. Schlagheck, *ibid.* **94**, 020404 (2005).
- [14] *Proceedings of the 6th Symposium on Frequency Standards and Metrology*, edited by P. Gill (World Scientific, Singapore, 2002).
- [15] T. T. Chou, C. N. Yang, and L. H. Yu, Phys. Rev. A **53**, 4257 (1996).
- [16] E. Mandonnet *et al.*, Eur. Phys. J. D **10**, 9 (2000); Y. Castin *et al.*, J. Mod. Opt. **47**, 2671 (2000).
- [17] $k_B T_a^c = \hbar \omega_{\perp} [\rho \lambda_c / \zeta(5/2)]^{1/2}$, with $\zeta(5/2) \approx 1.34$ and $\lambda_c = \hbar / (2\pi m k_B T_a^c)^{1/2}$. This semiclassical expression is valid for our parameters since, for the lowest measured temperature, we found $k_B T \approx 4 \hbar \omega_{\perp}$.
- [18] A two-dimensional Gaussian ansatz for the transverse degrees of freedom inserts in the Gross-Pitaevskii energy functional yields the size of the ground state for our model $\sigma = a_0(1 + 2\rho_0 a)^{1/4}$, where a is the scattering length and ρ_0 is the linear density of the ground state.
- [19] We use the index b for the BEC cloud and a for the guided-atom laser.
- [20] $\langle n \rangle = \sum_{\mathbf{n}} (n_x + n_y) \pi_{\mathbf{n}} / 2$, where $\pi_{\mathbf{n}}$ is the population of the state $\mathbf{n} = (n_x, n_y)$. Therefore, $\langle n \rangle = (1 - N_0/N) \sum_{\mathbf{n} \neq \mathbf{0}} (n_x + n_y) \pi_{\mathbf{n}}^{\text{th}} / 2 = (1 - N_0/N) \langle n \rangle_{\text{th}}$ with $\pi_{\mathbf{n}}^{\text{th}} = \pi_{\mathbf{n}} N_0 / (N - N_0)$ and $\pi_{\mathbf{0}}^{\text{th}} = 0$.
- [21] R. G. Dall, S. S. Hodgman, M. T. Johnsson, K. G. H. Baldwin, and A. G. Truscott (unpublished).
- [22] S. Giorgini, L. P. Pitaevskii, and S. Stringari, Phys. Rev. Lett. **78**, 3987 (1997).
- [23] Katsuhiko Nakamura, *Quantum versus Chaos: Questions Emerging from Mesoscopic Cosmos* (Kluwer Academic, New York, 2002).
- [24] D. Jaksch, C. W. Gardiner, and P. Zoller, Phys. Rev. A **56**, 575 (1997).
- [25] D. M. Stamper-Kurn and W. Ketterle, in *Les Houches Summer School Session LXXII, 1999*, edited by R. Kaiser, C. Westbrook, and F. David (Springer, New York, 2001).
- [26] M. R. Matthews *et al.*, Phys. Rev. Lett. **81**, 243 (1998).
- [27] F. Delgado, J. G. Muga, and A. Ruschhaupt, Phys. Rev. A **74**, 063618 (2006).
- [28] A. P. Chikkatur *et al.*, Science **296**, 2193 (2002).