Enhancement of parameter estimation by Kerr interaction

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We address quantum estimation of displacement and squeezing parameters by the class of probes made of Gaussian states undergoing Kerr interaction. If we fix the overall energy available to the probe, without posing any constraint on the available Gaussian squeezing, then Gaussian squeezing represents the optimal resource for parameter estimation. On the other hand, in the more realistic case where the amount of Gaussian squeezing is fixed, or even absent, then Kerr interaction turns out to be useful to improve estimation, especially for probe states with large amplitude. Our results indicate that precision achievable with current technology Gaussian squeezing may be attained and surpassed for realistic values of the Kerr coupling.

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I. INTRODUCTION

Nonclassical states of light represent a resource for high-precision measurements. They are generally produced in active optical media, which couple one or more modes of the field through the nonlinear susceptibility of the matter. In particular, parametric processes in second order $\chi^{(2)}$ media correspond to Gaussian operations and are used to generate squeezing, hereafter *Gaussian* squeezing, and entanglement. Gaussian squeezing is the basic ingredient of quantum enhanced interferometry [1–7] and found several applications in quantum metrology and communication [8–13]. In addition, Gaussian squeezing is the key resource to achieve precise estimation of unitary [14,15] and nonunitary parameters [16]. In turn, squeezed vacuum state has been addressed as a universal optimal probe [15–17] within the class of Gaussian states.

On the other hand, the Kerr effect taking place in third-order nonlinear $\chi^{(3)}$ media leads to a non-Gaussian operation, and has been suggested to realize quantum nondemolition measurements [18,19], and to generate quantum superpositions [20–22] as well as squeezing [23] and entanglement [24]. A well-known example of Kerr media are optical fibers where, however, nonlinearities are very small and accompanied by other unwanted effects. Larger Kerr nonlinearities have been observed with electromagnetically induced transparency [25] and with Bose Einstein condensates [26] and cold atoms [27]. Recently, nonlinearities on 9 orders of magnitude higher than natural Kerr interactions have been proposed by using the Purcell effect [28], Rydberg atoms [29], interaction of a cavity mode with atoms [30], and nanomechanical resonators [31]. Notice that the dynamics in a Kerr medium may be accurately described in terms of the Wigner function in the phase-space [32].

In this paper, we consider generic Gaussian states undergoing self-Kerr interaction and investigate their use in estimation of displacement and squeezing parameters. Indeed, displacement and squeezing are basic Gaussian operations in continuous variable systems and represent building blocks to manipulate Gaussian states for quantum information processing. Besides, they represent the ultimate description of interferometric interaction. As a consequence, their characterization, i.e., the optimal estimation of displacement and squeezing parameters has been widely investigated [15,33–37] by using different tools from quantum estimation theory (QET) [38–45].

Our main goal is to assess Kerr interaction and the resulting non-Gaussianity (nonG) as a resource for parameter estimation, and to this aim we consider two different situations with different physical constraints. On the one hand, we study schemes where we fix the overall energy available to the probe, without posing any constraint on the available Gaussian squeezing; this will be referred to as the fixed energy case. On the other hand, we will analyze the more realistic case where the amount of Gaussian squeezing is fixed, or even absent, and refer to this case as the fixed squeezing case. As we will see, at fixed energy Gaussian squeezing still represents the optimal resource for parameter estimation. On the other hand, when the amount of Gaussian squeezing is fixed then Kerr interaction turns out to be useful to improve estimation, especially when the probe states have a large number of *nonsqueezing* photons, i.e., large amplitude. In this case, precision obtained by Gaussian states is achieved or enhanced.

The paper is structured as follows: in Sec. II, we review few basic ingredients of local quantum estimation theory and illustrate the content of the quantum Cramer-Rao bound. In Sec. III, we analyze the use of Kerr interaction to improve estimation of the displacement amplitude, whereas in Sec. IV we focus on squeezing estimation. Section VI closes the paper with some concluding remarks.

II. QUANTUM ESTIMATION THEORY

Let us start by reviewing some basic concepts of local quantum estimation theory: when a physical parameter is not directly accessible one has to resort to indirect measure-

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ments, i.e., measuring an observable somehow related to the quantity of interest and estimate its value from the experimental sample. Let us denote by λ the quantity of interest, X the measured observable, and $\chi=(x_1,\ldots,x_M)$ the observed sample. The *estimation problem* amounts to find an estimator, that is, a map $\hat{\lambda}=\hat{\lambda}(\chi)$ from the set of the outcomes to the space of parameters. Classically, optimal estimators are those saturating the Cramer-Rao inequality $\mathrm{Var}(\lambda)\geq[MF(\lambda)]^{-1}$, which bounds from below the variance $\mathrm{Var}(\lambda)=E[\hat{\lambda}^2]-E[\hat{\lambda}]^2$ of any unbiased estimator of the parameter λ. In the Cramer-Rao inequality, M is the number of measurements and $F(\lambda)$ is the so-called Fisher Information (FI) $F(\lambda)=\int dx p(x|\lambda)[\partial_\lambda \ln p(x|\lambda)]^2$ where $p(x|\lambda)$ is the conditional probability of obtaining the value x when the parameter has the value λ. The quantum analog of the Cramer-Rao bound is obtained starting from the Born rule $p(x|\lambda)=\mathrm{Tr}[\Pi_x\varrho_\lambda]$ where $\{\Pi_x\}$ is the probability operator-valued measure (POVM) describing the measurement and ϱ_λ the density operator, labeled by the parameter of interest. In order to evaluate the ultimate bounds to precision, one introduces the Symmetric Logarithmic Derivative (SLD) L_λ as the operator satisfying $2\partial_\lambda\varrho_\lambda=L_\lambda\varrho_\lambda+\varrho_\lambda L_\lambda$, and prove that the FI is upper-bounded by the Quantum Fisher Information (QFI) [44] $F(\lambda)\leq H(\lambda)\equiv\mathrm{Tr}[\varrho_\lambda L_\lambda^2]$. In turn, the ultimate limit to precision is given by the quantum Cramer-Rao bound $\mathrm{Var}(\lambda)\geq[MH(\lambda)]^{-1}$. Let us consider the case where the parameter of interest is the shift imposed by a unitary evolution $U_\lambda=\exp(-i\lambda G)$ to a given initial pure state $|\psi_0\rangle$, G being the corresponding Hermitian generator. The family of states we are dealing with is given by $|\psi_\lambda\rangle=U_\lambda|\psi_0\rangle$, and since for pure states $\varrho_\lambda^2=\varrho_\lambda$, one has $\partial_\lambda\varrho_\lambda=\partial_\lambda\varrho_\lambda\varrho_\lambda+\varrho_\lambda\partial_\lambda\varrho_\lambda$, i.e.,

$$L_\lambda = 2[|\psi_\lambda\rangle\langle\partial_\lambda\psi_\lambda| + |\partial_\lambda\psi_\lambda\rangle\langle\psi_\lambda|]$$

$$H(\lambda) = 4[\langle\partial_\lambda\psi_\lambda|\partial_\lambda\psi_\lambda\rangle + (\langle\partial_\lambda\psi_\lambda|\psi_\lambda\rangle)^2].$$

After some algebra one sees that the QFI turns out to be proportional to the fluctuations of the generator on the probe state, $H(\lambda)=4\langle\psi_0|\Delta G^2|\psi_0\rangle$, and thus it is independent on the value of λ. The above equation, together with the Cramer-Rao bound, expresses the ultimate quantum lower bound on the precision achievable by using a given probe $|\psi_0\rangle$ and any estimation procedure, i.e., without making reference to any specific detection scheme. In the following, we will exploit the above tools to assess and compare the use of Gaussian states and Kerr modified Gaussian states in the estimation of displacement and squeezing parameters. More specifically, we evaluate the QFI as a function of the involved parameters and analyze its behavior in different relevant regimes.

III. ESTIMATION OF DISPLACEMENT

Let us first consider the estimation of displacement, i.e., of the real parameter $\lambda\in\mathbb{R}$ imposed by the unitary $U_\lambda=\exp\{-i\lambda G_d\}$, $G_d=a^\dagger+a$ being the corresponding generator. For a generic pure Gaussian probe, i.e., a displaced squeezed state of the form $|\alpha,r\rangle=D(\alpha)S(r)|0\rangle$ (with $\alpha=|\alpha|e^{i\phi}$ and $r>0$) where $D(\alpha)=\exp\{\alpha a^\dagger-\bar{\alpha}a\}$ and $S(r)=\exp\{\frac{r}{2}(a^{\dagger 2}-a^2)\}$, the QFI, i.e., the fluctuations of the generator,

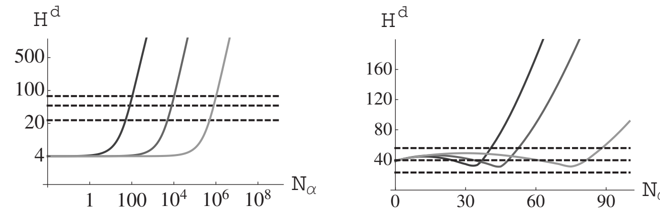

FIG. 1. Top: QFI $H_\gamma^{(d)}$ for displacement estimation by Kerr modified coherent states (solid lines) as a function of the number of photons N_α and for different values of γ. From darker to lighter gray: $\gamma=\{10^{-2},10^{-4},10^{-6}\}$. Dashed lines refer to QFI $H_G^{(d)}$ of squeezed vacuum states for different values of squeezing photons. From bottom to top: $N_{sq}=1$, 2, 3. Bottom: QFI $H_\gamma^{(d)}$ for Kerr modified displaced squeezed states, $N_{sq}=2$, for different values of γ. From darker to lighter gray: $\gamma=\{0.01,0.008,0.005\}$. Dashed lines denote QFI $H_G^{(d)}$ of squeezed vacuum states for different values of squeezing photons. From bottom to top: $N_{sq}=1$, 2, 3.

may be evaluated by normal ordering for creation and annihilation operators [46]. One obtains $H^{(d)}=4+8N\beta+8\sqrt{N\beta(1+N\beta)}$, where $N=\sinh^2 r+|\alpha|^2$ is the number of photons of the probe state and where $\beta=\sinh^2 r/N$ is the corresponding squeezing fraction $(0\leq\beta\leq1)$. As expected for a unitary family the QFI does not depend on the value of the parameter. Besides, the QFI depends only on the squeezing energy $N_{sq}=\beta N$, and thus increasing the amplitude energy $N_\alpha=|\alpha|^2$, does not lead to any enhancement of precision. Therefore, at fixed energy, the maximum QFI $H_S^{(d)}=4+8N+8\sqrt{N(1+N)}$ is achieved for $\beta=1$, i.e., for squeezed vacuum. In the opposite limit $(\beta=0)$, i.e., for coherent states, the QFI is constant: $H_C^{(d)}=4$. Let us consider now a generic Gaussian state that undergoes Kerr interaction $|\alpha,r,\gamma\rangle=U_\gamma D(\alpha)S(r)|0\rangle$ where $U_\gamma=\exp[-i\gamma(a^\dagger a)^2]$. The QFI for this class of states can be evaluated numerically upon varying the parameters γ, $|\alpha|$, ϕ, and r. We found that at fixed energy, the optimal probe state is still the squeezed vacuum state. The optimal QFI is a monotonous decreasing function of γ and the Kerr dynamics does not improve estimation precision. In other words, at fixed energy, squeezed vacuum state is the best probe not only among the class of Gaussian states, but also maximizing the QFI over the wider class of states Kerr perturbed Gaussian states.

Let us now address estimation of displacement in the more realistic configuration, where the amount of Gaussian squeezing is fixed or absent. For Kerr modified coherent states $|\alpha,\gamma\rangle$, QFI can be evaluated analytically at fixed energy $N=|\alpha|^2$ and γ, arriving at

$$\begin{aligned} H^{(d)} = 4 + 8Ne^{-4N\sin^2\gamma}\{&e^{4N\sin^2\gamma}-1 \\ &+\cos[2(\gamma-\phi+N\sin 2\gamma)]-e^{-4N\cos 2\gamma\sin^2\gamma} \\ &\times\cos[4\gamma-2\phi+N\sin 4\gamma]\}, \end{aligned} \tag{1}$$

and then optimized numerically over the coherent phase ϕ. The results are reported in Fig. 1 (top plot) as a function of the number of photons $|\alpha|^2$ and for different values of γ. The QFI increases with $|\alpha|^2$ and γ and the precision achievable with current technology squeezing, say $N_{sq}\lesssim 2$, may be attained and surpassed for realistic values of the Kerr coupling

γ and large enough signal amplitude, say $\gamma|\alpha|^2 \lesssim 1$. Better performances may be obtained by considering Kerr modified squeezed states $|\alpha, r, \gamma\rangle$ with fixed squeezing r and large amplitude $|\alpha| \gg 1$. The QFI for this case, as evaluated numerically and optimized over the amplitude phase ϕ is reported in Fig. 1 (bottom plot). We observe that, after a regime where QFI oscillates around the value obtained for vanishing γ, then it increases monotonically with $|\alpha|^2$ and exceed the corresponding Gaussian QFI for large enough values of $|\alpha|^2$ and/or γ. Due to numerical limitations, we have considered $|\alpha|^2 \leq 100$, and thus we have seen enhancement of precision only for the largest values of γ. We expect analog performances by considering smaller values of γ and larger numbers of photons.

IV. ESTIMATION OF SQUEEZING

Let us now consider estimation of squeezing that is the estimation of the real parameter $z \in \mathbb{R}$ imposed by the unitary evolution $U_z = \exp\{-izG_s\}$ with generator $G_s = \frac{1}{2}(a^{\dagger 2} + a^2)$. Given a generic single-mode Gaussian state $|\alpha, r\rangle$, the QFI for squeezing estimation has been evaluated by using the normal ordering for creation and annihilation operators [46]. The maximum is $H_G^{(s)} = 8N^2 + 8N + 2$ and is again achieved using squeezed vacuum probe [15]. In order to investigate the effect of Kerr interaction we consider Kerr modified Gaussian states $|\alpha, r, \gamma\rangle$. At fixed energy QFI has been evaluated and optimized numerically against the squeezing fraction β and phase ϕ. In this case, the optimal squeezing fraction decreases monotonically with both γ and the total number of photons N and the maximized QFI is a decreasing function of γ, that is Kerr interaction does not improve, actually degrades, the estimation precision achievable with squeezed vacuum probe.

Let us now consider situations where squeezing is not available, or its amount is fixed, and where the field amplitude may be increased at will. The QFI for probe states of the form $|\alpha, \gamma\rangle = U_\gamma D(\alpha)|0\rangle$ can be evaluated analytically as

$$\begin{aligned} H^{(s)} = 2 + 2N\{ & 2 + N - Ne^{-4N \sin^2 \gamma} \\ & \times (1 + \cos[2(4\gamma - 2\phi + N \sin 42\gamma)]) + Ne^{-N(1-\cos 8\gamma)} \\ & \times \cos[16\gamma - 4\phi + N \sin 8\gamma]\}, \end{aligned} \tag{2}$$

and then maximized numerically over the amplitude phase ϕ. In Fig. 2 we report the optimized QFI together with the QFI of displaced squeezed vacuum states with $N_{sq} \leq 3$ and the same value of $|\alpha|^2$. Results indicate that upon using coherent states with large amplitude we may achieve and improve the precision of squeezed vacuum states already for small, realistic, values of the Kerr coupling γ. When the amount of Gaussian squeezing is nonzero but fixed we can combine the effects of squeezing and Kerr interaction by considering Kerr modified displaced squeezed states with a large number of amplitude photons ($|\alpha|^2 \gg 1$). As it is apparent from Fig. 2 the QFI increases with $|\alpha|^2$ and overtake quite rapidly the values of QFI of the corresponding Gaussian state.

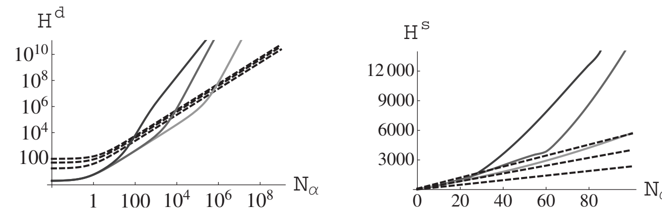

FIG. 2. Top: QFI $H_\gamma^{(s)}$ for squeezing estimation by Kerr modified coherent probes (solid lines) as a function of the number of photons N_α and for different values of γ. From darker to lighter gray: $\gamma = \{10^{-2}, 10^{-4}, 10^{-6}\}$. Dashed lines refer to the QFI $H_G^{(s)}$ for displaced squeezed probes and different values of squeezing photons. From bottom to top: $N_{sq} = 1, 2, 3$. Bottom: QFI $H_\gamma^{(s)}$ for Kerr modified displaced squeezed states (solid lines) with $N_{sq} = 2$ squeezing photons, as a function of field amplitude photons $N_\alpha = |\alpha|^2$ and for different values of γ. From darker to lighter gray: $\gamma = \{0.01, 0.005, 0.001\}$. Dashed lines refer to the QFI $H_G^{(s)}$ for displaced squeezed vacuum states and different values of squeezing photons. From bottom to top: $N_{sq} = 1, 2, 3$.

V. NON-GAUSSIANITY AS AN OVERALL INDICATOR OF PRECISION ENHANCEMENT

As pointed out in the introduction, Kerr interaction induces a non-Gaussian operation. A question thus arises on whether there is a connection between the amount of non-Gaussianity of the probe and the precision of estimation. In other words, whether or not nonG may be used as an overall indicator of precision enhancement due to Kerr interaction. The answer to this question amounts to investigate the behavior of the QFI as a function of a nonG measure. Different measures of nonG for a quantum state have been recently introduced [47–49], and here we consider the entropic measure [48] $\delta[\varrho] = S(\tau) - S(\varrho)$ where $S(\varrho)$ is the Von Neumann entropy of the state ϱ, and τ denotes the Gaussian states with the same covariance matrix of the state ϱ under investigation. Since both nonG for Kerr modified coherent states and the corresponding QFI are increasing functions of the number of photons, we consider a *normalized* nonG measure $\delta_R[\varrho] = \delta[\varrho] / \delta_m(N_\alpha)$, obtained as the ratio between $\delta[\varrho]$ and the maximum nonG $\delta_m(N_\alpha)$ achievable with the same number of photons. This is in order to discern the real contribution of nonG to the improvement of estimation from that coming from energy scaling.

In Fig. 3, we report the QFI for both displacement and squeezing estimation by Kerr modified coherent states, as a function of the normalized nonG for fixed Kerr constant γ (varying the number of photons N_α) and for fixed number of photons N_α (varying γ). As it is apparent from the plots, QFI is not a fully monotone function of nonG: the allowed region for the values of parameters we have considered is the gray area and one may find two states such that $\delta_R[\varrho_1] > \delta_R[\varrho_2]$ and $H[\varrho_1] < H[\varrho_2]$. On the other hand, if we fix one of the two parameters (γ or N_α) and vary the other one, we observe a monotonous behavior. In other words, nonG is quantitatively related to the increase in QFI and thus represents a good indicator to assess Kerr interaction in quantum estimation.

One may also ask whether the enhancement in precision obtained with Kerr interaction may be ascribed to the

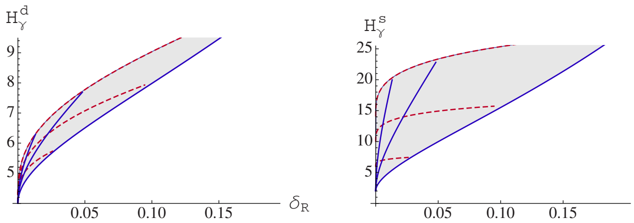

FIG. 3. (Color online) Top: QFI $H_\gamma^{(d)}$ for displacement estimation by Kerr modified coherent states as a function of the normalized non-Gaussianity δ_R. The solid blue lines refer to the case of fixed Kerr coupling γ and varying number of photons $0<N_\alpha<3$; from top to bottom we have γ=0.04, 0.06, 0.10. The dashed red lines are for fixed number of photons and varying Kerr coupling $0<\gamma<0.1$; from top to bottom we have N_α=3, 2, 1. The gray area denotes the allowed values of both QFI and nonG for the considered values of the amplitude and the coupling. Bottom: The same as in the left panel for the QFI $H_\gamma^{(s)}$ for squeezing estimation.

squeezing effect occurring in Kerr evolution at small time and nonlinearity and/or small number of photons. For coherent input, this is definitely not the case, as it can easily checked by noting that improvement in precision occurs for $\gamma|\alpha|^2 \lesssim 1$, i.e., when the state is no longer squeezed (see [32] for a phase-space picture of Kerr evolution for input coherent states). Moreover, when we consider a Kerr perturbed squeezed state as input, the QFI for displacement estimation is not monotone in the region where one may expect a further squeezing effect or at least that the initial squeezing is conserved. Also in this case, enhancement in precision is observed for increasing amplitude photons or Kerr nonlinearity, when the quantum state is no longer squeezed. At the same time, improvement is not due to the evolution toward cat states, since they are achieved by Kerr interaction only for very high nonlinearities and they present a different scaling in precision [50]. The most intuitive picture one may draw is that the involved structure of the Wigner function leads to its spread over the phase-space and consequently to a smaller overlap when displaced (squeezed). One should also notice that, since the phase of the coherent input signal is optimized for each pair of values of the coupling and the amplitude, a simple picture in terms of Wigner evolution may be even confusing rather than help intuition. For these reasons we consider nonG as a suitable quantity to summarize the improvement in the estimation precision for Kerr perturbed Gaussian states.

VI. CONCLUSIONS

In conclusion, we have addressed the use of Kerr interaction to improve estimation of displacement and squeezing parameters and analyzed in details the behavior of the quantum Fisher information as a function of probe and interaction parameters. We found that at fixed energy, with no constraint on the available Gaussian squeezing, Kerr dynamics is not useful and performances of Gaussian states are superior. On the other hand, in the more realistic case where the amount of Gaussian squeezing is fixed, or absent, then Kerr interaction improves estimation, especially for probe states with large amplitude.

It should be noticed that Gaussian squeezing in $\chi^{(2)}$ media is obtained by parametric processes and the amount of squeezing linearly increases with the pump intensity. On the other hand, in $\chi^{(3)}$ media, the energy needed to obtain significant nonlinear effects is provided by the signal itself. Overall, our results indicate that precision achievable with current technology Gaussian squeezing may be attained and surpassed for realistic values of the Kerr coupling and large enough signal amplitude. We also found that precision improvement is quantitatively related with the amount of non-Gaussianity induced by Kerr interaction, and thus conclude that Kerr non-Gaussianity is a resource, achievable with current technology, for high-precision measurements. We foresee a possible widespread use as a characterization tools in emerging quantum technologies like quantum communication and metrology.

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