Condensation state of photons in a Kerr nonlinear blackbody

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Within the framework of quantum field theory, we show that the photon system in a blackbody whose interior is filled by a Kerr nonlinear crystal is in a condensation state. In the condensation state, bare photons with opposite wave vectors and helicities are bound into pairs and unpaired bare photons are transformed into a new kind of quasiparticle, the nonpolariton. The photon-pair system is a condensate and the nonpolariton system is a boson gas. At zero temperature the condensate possesses a largest persistent energy density. The persistent energy density of the condensate is a monotonically decreasing function of temperature and Kerr nonlinear coefficient. The Q function of a Kerr nonlinear blackbody at any temperature is derived analytically. In the transition from the normal to the condensation state, the phase symmetry of the photon system is spontaneously broken.

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I. INTRODUCTION

Nowadays it is recognized that the Bose-Einstein condensation is a common quantum property of the many-particle systems in which the number of particles is conserved. These many-particle systems include the systems of atoms, electrons, and magnons in magnetic fields. The original Bose-Einstein condensation is that an ideal gas of bosonic atoms goes into a phase in which a macroscopic number of atoms occupy the zero-momentum state. The superconducting state of metals discovered by Kamerlingh Onnes in 1911 [1] is a manifestation of the Bose-Einstein condensation of Cooper pairs. A Cooper pair is an electron pair mediated by acoustic phonons. The superfluid phase of liquid ^{4}He discovered by Kapitza in 1932 [2] can be viewed as a Bose-Einstein condensation among strongly interacting ^{4}He atoms. In the year 1995, Cornell and Wieman observed Bose-Einstein condensation in a vapor of rubidium-87 atoms at temperatures of about 170 nanokelvins [3–5]. The magnetization of quantum spin systems in magnetic fields is interpreted as a Bose-Einstein condensation of repulsively interacting magnons [6,7].

The electromagnetic field is a quantum system of photons. Since the number of photons is not conserved, the photon system cannot undergo a Bose-Einstein condensation. In this paper, we shall show that the photon system can undergo a Bardeen-Cooper-Schrieffer (BCS) condensation. Now consider a blackbody whose interior is filled by a Kerr nonlinear crystal. The crystal constitutes a Kerr nonlinear medium for the electromagnetic field. Further, the crystal is in thermal equilibrium with the electromagnetic field. The crystal and the thermal radiation constitute a system. We call this system a Kerr nonlinear blackbody. Such a Kerr nonlinear blackbody can be regarded as a rectangular crystal that has perfectly conducting walls and is kept at a constant temperature T. As shown in Fig. 1, there is a small hole in a wall through which thermal radiation can pass. In a recent work [8,9], we have shown that a photon blackbody field in Kerr nonlinear crystal is a squeezed thermal radiation state. In the present paper, we shall show that the squeezed thermal radiation state is a BCS condensation state. We shall investigate the properties of the BCS condensation state in a Kerr nonlinear blackbody. Inasmuch as such a condensation state was never explored previously, features that are worthy of exploration are pointed out here.

In an earlier work [10,11], we showed that optical solitons in nonlinear polar media can be in a photonic superguiding state in which thermal scattering effects are suppressed, compared to optical fibers for which thermal scattering is the main source of soliton losses and noise. The electromagnetic field in thermal equilibrium is called blackbody radiation or thermal radiation. Within the framework of quantum field theory, we show that the photon system in a Kerr nonlinear blackbody is in a BCS condensation state. The bare photons in blackbody radiation can sense an attractive effective interaction by exchange of virtual nonpolar phonons. Such an interaction leads to a BCS condensation state, in which the bare photons with opposite wave vectors and helicities are bound into pairs and unpaired bare photons are transformed into a new kind of quasiparticle, the nonpolariton. A nonpolariton is the condensate of virtual nonpolar phonons in momentum space below a transition temperature through a nonlinear photon-phonon interaction, with a bare photon acting as the nucleus of condensation. The vacuum for nonpolaritons is a condensate consisting of photon pairs and single nonpolaritons are elementary excitations from such a condensate. The BCS condensation state of photons possesses some peculiar properties. First, the photon-pair system is a condensate and the condensate has a large persistent energy density at zero temperature, compared with the normal black

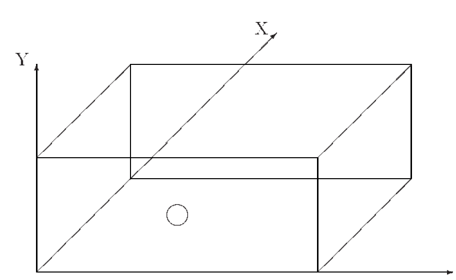

FIG. 1. A Kerr nonlinear blackbody: a rectangular Kerr nonlinear crystal enclosed by perfectly conducting walls and kept at a constant temperature; there is a very small hole in a wall.

body whose energy density is zero at zero temperature. Second, the nonpolariton system is a free boson gas and a single nonpolariton is an elementary excitation from the condensate. Third, in the transition from the normal state to the condensation state, the phase symmetry of the photon system is spontaneously broken. The predicted properties of the photonic condensation state will be verified in physics laboratories for the not too distant future.

The remainder of this paper is organized as follows. Section II describes some properties of a normal blackbody. In Sec. III, we diagonalize the Hamiltonian of the photon system in a Kerr nonlinear blackbody. Section IV describes some properties of the photonic condensation state and gives the numerical calculation of physical quantities concerned. In Sec. V, we derive the Q function of a Kerr nonlinear blackbody at any temperature. The comprehensive discussion is given in Sec. VI.

II. NORMAL BLACKBODY

A. Quantization procedure

At the beginning of the 20th century the interpretation of the blackbody radiation spectrum revealed the dual character of electromagnetic radiation and became one of the origins of quantum theory. By definition, a blackbody absorbs 100% of all thermal radiation falling upon it. A close approximation to the blackbody is a small hole in a cavity in a solid that is maintained at some steady absolute temperature T. We shall call this system a normal blackbody. The electromagnetic field is composed of mutually exciting electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$. The electromagnetic field is a transverse field, propagates in vacuum with the speed c of light, and satisfies the Maxwell equations. Since there are no free charges in the blackbody, we can set the scalar potential of the electromagnetic field to be zero. Hence, the electromagnetic field can be characterized by a single vector potential $\mathbf{A}$, which satisfies the Coulomb gauge $\nabla\cdot\mathbf{A}=0$. Consequently, the electric and magnetic fields are given by

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}. \tag{1}$$

The Hamiltonian of the electromagnetic field reads as

$$H_{em} = \int d\mathbf{r}\left(\frac{\epsilon_0}{2}\mathbf{E}^2 + \frac{1}{2\mu_0}\mathbf{B}^2\right), \tag{2}$$

where ϵ_0 and μ_0 are the permittivity and the permeability of vacuum, respectively, with $\epsilon_0\mu_0=c^{-2}$.

Now we need to quantize the electromagnetic field. Since plane-wave modes constitute a complete orthonormal set, they can be used for the expansion of the electromagnetic field in any arbitrary geometry. The blackbody occupies a volume V. In terms of the creation and annihilation operators $a^\dagger_{\mathbf{k}\sigma}$ and $a_{\mathbf{k}\sigma}$ of circularly polarized photons with wave vector $\mathbf{k}$ and helicity $\sigma=\pm 1$, the vector potential of the electromagnetic field is expanded as

$$\mathbf{A}(\mathbf{r},t) = \sum_{\mathbf{k}\sigma}\left(\frac{\hbar}{2V\epsilon_0\omega_{\mathbf{k}}}\right)^{1/2}[a_{\mathbf{k}\sigma}(t)\mathbf{e}_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}} + a^\dagger_{\mathbf{k}\sigma}(t)\mathbf{e}^*_{\mathbf{k}\sigma}e^{-i\mathbf{k}\cdot\mathbf{r}}], \tag{3}$$

where $\hbar$ is Planck's constant reduced, $\omega_{\mathbf{k}}=c|\mathbf{k}|$ is the angular frequency of a photon, and $\mathbf{e}_{\mathbf{k},\pm1}$ are two orthonormal circular polarization vectors perpendicular to $\mathbf{k}$. The photon operators obey the Bose equal-time commutation relations,

$$[a_{\mathbf{k}\sigma}(t),a^\dagger_{\mathbf{k}'\sigma'}(t)]_- = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\sigma\sigma'}, \quad [a_{\mathbf{k}\sigma}(t),a_{\mathbf{k}'\sigma'}(t)]_- = 0. \tag{4}$$

They have the time dependence: $a_{\mathbf{k}\sigma}(t)=a_{\mathbf{k}\sigma}(0)\exp(-i\omega_{\mathbf{k}}t)$ and $a^\dagger_{\mathbf{k}\sigma}(t)=a^\dagger_{\mathbf{k}\sigma}(0)\exp(i\omega_{\mathbf{k}}t)$. On substituting Eqs. (1) and (3) into Eq. (2), the Hamiltonian of the electromagnetic field is quantized as

$$H_{em} = \sum_{\mathbf{k}\sigma}\hbar\omega_{\mathbf{k}}a^\dagger_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}, \tag{5}$$

where the zero-point energy terms are dropped. Equation (5) represents the Hamiltonian of the system of noninteracting photons in a normal blackbody.

$N_{\mathbf{k}\sigma}=a^\dagger_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}$ are known as the number operators of photons. The number operators have the eigenvalues $n_{\mathbf{k}\sigma}=0,1,2,\ldots$. Since the number operators commute with H_{em}, the number of photons in each mode $\mathbf{k}\sigma$ is constant in time. The number operators form a complete commuting set and simultaneous eigenstates of this set are given by

$$|\{n_{\mathbf{k}\sigma}\}\rangle = \prod_{\mathbf{k}\sigma}\left[\frac{1}{\sqrt{n_{\mathbf{k}\sigma}!}}(a^\dagger_{\mathbf{k}\sigma})^{n_{\mathbf{k}\sigma}}\right]|0\rangle, \tag{6}$$

where $|0\rangle$ is the vacuum state of the electromagnetic field. The state vector (6) is symmetric under the interchange of any two creation operators, consistent with the Bose-Einstein statistics. Because the number of photons is variable, the chemical potential of the photon system is null. Consequently, H_{em} is a grand canonical Hamiltonian.

B. Thermal radiation state

The state vector (6) signifies a multimode number state of photons, which is a pure state and therefore far from thermal equilibrium. However, the electromagnetic field within a blackbody is in thermal equilibrium [12]. Such equilibrium is established via the continual absorption and emission of photons by matter. The electromagnetic field in thermal equilibrium is called blackbody radiation and characterized by a definite temperature T. The photons in blackbody radiation are in a thermal radiation state, which is called a normal state. In order to characterize the thermal radiation state, we need to conceive a grand canonical ensemble of photons. Some identical systems of the ensemble may be in an eigenstate of the Hamiltonian H_{em} given by Eq. (5), while the distribution of the ensemble over the eigenstates is described by the density operator of the thermal radiation state,

$$\rho = \frac{\exp(-H_{em}/k_B T)}{\mathrm{Tr}\,\exp(-H_{em}/k_B T)}, \tag{7}$$

where k_B is Boltzmann's constant. The basis states used in the trace are the eigenstates of the Hamiltonian H_{em}, which are given by Eq. (6). The main thermodynamic quantity in normal blackbody radiation is the total energy E_n or the energy density $u_n = E_n/V$, which is the ensemble average of the corresponding microscopic quantity,

$$E_n = \sum_{\mathbf{k}\sigma} \hbar\omega_{\mathbf{k}} \langle N_{\mathbf{k}\sigma} \rangle. \tag{8}$$

Here we have utilized the average notation $\langle N_{\mathbf{k}\sigma}\rangle = \mathrm{Tr}(\rho N_{\mathbf{k}\sigma})$.

It is easily found that the ensemble average of the number operator of photons in a mode $\mathbf{k}\sigma$ satisfies the well-known Bose-Einstein distribution,

$$\langle N_{\mathbf{k}\sigma}\rangle = \frac{1}{e^{\hbar\omega_{\mathbf{k}}/k_B T} - 1}. \tag{9}$$

Putting Eq. (9) into Eq. (8) and in the usual way altering the summation to an integration, we obtain

$$E_n = V\int_0^\infty \rho_n(\omega, T)\, d\omega, \tag{10}$$

$$\rho_n(\omega, T) = \frac{\hbar}{\pi^2 c^3}\frac{\omega^3}{e^{\hbar\omega/k_B T} - 1}. \tag{11}$$

Equation (11) for the spectral energy density of blackbody radiation is called Planck's formula. The spectral energy density of blackbody radiation has a maximum at a frequency ω_m defined by the equation

$$3 - \frac{xe^x}{e^x - 1} = 0, \tag{12}$$

where $x = \hbar\omega_m/k_B T$. The numerical solution of Eq. (12) gives

$$\hbar\omega_m/k_B T = 2.82144. \tag{13}$$

With the new variable of integration $x = \hbar\omega/k_B T$, the resulting integral in Eq. (10) is equal to $\pi^4/15$. Equation (10) yields

$$E_n = 4\sigma V T^4/c, \tag{14}$$

where $\sigma = \pi^2 k_B^4/60\hbar^3 c^2$ is called the Stefan-Boltzmann constant. Thus the total energy of blackbody radiation is proportional to the fourth power of the temperature. This is the Stefan-Boltzmann law. For future study, we need to write the energy density of normal blackbody radiation,

$$u_n(T) = 4\sigma T^4/c. \tag{15}$$

III. KERR NONLINEAR BLACKBODY

The model of a Kerr nonlinear blackbody was described in Sec. I. The crystal under study is a covalent one. The optical vibration modes of a covalent crystal are all the nonpolar modes that carry no electric-dipole moments, so they are infrared inactive. For convenience the crystal is taken to be of the cubic symmetry, so it is optically isotropic. A Kerr nonlinear crystal must be centrosymmetric. By "nonlinearity" we mean that the crystal is first-order Raman active. Nonpolar modes in a centrosymmetric crystal have even parity and are Raman active [13]. In the cubic system, the common covalent crystals that are both centrosymmetric and Raman active have a diamond structure. At this point, the crystal studied is determined as a specific crystal with a diamond structure, such as C. In a diamond-structure crystal a primitive cell contains two identical atoms that exhibit a triply degenerate nonpolar mode at zero wave vector, which is Raman active. For the Raman-active mode the two atoms in the primitive cell move in antiphase. Because the following treatment has no relation to acoustic modes, the vibrational modes of the crystal are limited to the Raman-active mode, whose zero-wave-vector frequency is denoted by ω_R.

In Ref. [9] we have known that the interaction between photons and phonons can lead to an attractive effective interaction among the photons themselves. The attractive effective interaction leads to bound photon pairs. The physical background for pairing is simple: A photon can emit or absorb a virtual nonpolar phonon. The emission of virtual nonpolar phonons by photons means that the photon is clothed with a cloud of virtual nonpolar phonons. If a second photon is near this cloud, it experiences a force of attraction. In the standing-wave configuration a photon pair is stable only if the two photons have opposite wave vectors and helicities. The pair Hamiltonian of the photon system is

$$H'_{em} = \frac{1}{2}\sum_{\mathbf{k}\sigma} \hbar\omega_{\mathbf{k}} (a^\dagger_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + a^\dagger_{-\mathbf{k},-\sigma} a_{-\mathbf{k},-\sigma}) + \sum_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} a^\dagger_{\mathbf{k}'\sigma'} a^\dagger_{-\mathbf{k}',-\sigma'} a_{-\mathbf{k},-\sigma} a_{\mathbf{k}\sigma}, \tag{16}$$

where the photons have the pair potential

$$V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} = \begin{cases} -V_0 \hbar\omega_{\mathbf{k}}\hbar\omega_{\mathbf{k}'}, & \text{if } \omega_{\mathbf{k}} \text{ and } \omega_{\mathbf{k}'} < \omega_R \\ 0, & \text{otherwise}, \end{cases} \tag{17}$$

where V_0 is a positive constant. We shall assume that the crystal has a dispersionfree refractive index n, so that the photonic frequency is given by $\omega_{\mathbf{k}} = c|\mathbf{k}|/n$.

Unpaired bare photons in the photon system are transformed into a new kind of quasiparticle, the nonpolariton. A nonpolariton is the condensate of virtual nonpolar phonons in momentum space, with a bare photon acting as the nucleus of condensation. The diagonalization of the pair Hamiltonian (16) can be performed by the Bogoliubov transformation,

$$c_{\mathbf{k}\sigma} = U a_{\mathbf{k}\sigma} U^\dagger = a_{\mathbf{k}\sigma}\cosh\varphi_{\mathbf{k}\sigma} - a^\dagger_{-\mathbf{k},-\sigma}\sinh\varphi_{\mathbf{k}\sigma},$$

$$c^\dagger_{\mathbf{k}\sigma} = U a^\dagger_{\mathbf{k}\sigma} U^\dagger = a^\dagger_{\mathbf{k}\sigma}\cosh\varphi_{\mathbf{k}\sigma} - a_{-\mathbf{k},-\sigma}\sinh\varphi_{\mathbf{k}\sigma}, \tag{18}$$

where the parameter $\varphi_{\mathbf{k}\sigma}$ is assumed to be real and spherically symmetric: $\varphi_{-\mathbf{k},-\sigma} = \varphi_{\mathbf{k}\sigma}$. $c^\dagger_{\mathbf{k}\sigma}$ and $c_{\mathbf{k}\sigma}$ are the creation and annihilation operators, respectively, of nonpolaritons in the photon system; they also obey the Bose equal-time commutation relations like Eq. (4). The transition from the operators of bare photons to those of nonpolaritons can be effected by a unitary transformation,

$$U=\exp\left[\frac{1}{2}\sum_{\mathbf{k}\sigma}\varphi_{\mathbf{k}\sigma}(a^{\dagger}_{\mathbf{k}\sigma}a^{\dagger}_{-\mathbf{k},-\sigma}-a_{-\mathbf{k},-\sigma}a_{\mathbf{k}\sigma})\right]. \tag{19}$$

It is well known that the unitary transformation does not change the energy spectrum of the photon system. The normalized state vector of photon pairs in the photon system may be constructed as $|G\rangle=U|0\rangle$, such that $c_{\mathbf{k}\sigma}|G\rangle=0$.

As we know, the pair Hamiltonian (16) can be solved only when the pair potential $V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'}$ is negative. Under the mean-field approximation [9], the pair Hamiltonian of the photon system is diagonalized into

$$H'_{em}=E_p+\sum_{\mathbf{k}\sigma}\hbar\tilde{\omega}_{\mathbf{k}}(T)c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}. \tag{20}$$

The idea of a mean-field approximation was introduced by the Weiss theory of ferromagnetism to deal with phase transitions [14]. Here the idea is that individual nonpolaritons move independently in a mean field caused by all other photons, which includes parts of the photon-photon interaction. The frequency of nonpolaritons is acquired as $\tilde{\omega}_{\mathbf{k}}(T)=v(T)|\mathbf{k}|$, where $v(T)$ is the velocity of nonpolaritons determined by the equation

$$v(T)=2(c/n)V_0{\sum_{\mathbf{k}}}'\hbar\omega_{\mathbf{k}}\coth\frac{\hbar v(T)|\mathbf{k}|}{2k_BT}, \tag{21}$$

where the prefactor 2 arises from the summation over helicities and the prime on the summation symbol means that $\omega_{\mathbf{k}}<\omega_R$. E_p is the energy of the system of photon pairs, as given by

$$E_p=\sum_{\mathbf{k}\sigma}\left[\hbar\omega_{\mathbf{k}}\sinh^2\varphi_{\mathbf{k}\sigma}+\frac{1}{4}\sinh 2\varphi_{\mathbf{k}\sigma}\sum_{\mathbf{k}'\sigma'}V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'}\sinh 2\varphi_{\mathbf{k}'\sigma'}\right], \tag{22}$$

where the parameter $\varphi_{\mathbf{k}\sigma}$ is determined by the relations

$$\tanh 2\varphi_{\mathbf{k}\sigma}=\Delta(T),\quad v(T)=(c/n)\sqrt{1-\Delta^2(T)}. \tag{23}$$

$\Delta(T)$ is the order parameter for pairing of photons.

The velocity $v(T)$ determined by Eq. (21) is a monotonically increasing function of temperature T, which is equal to c/n at the transition temperature T_c. In other words, the order parameter $\Delta(T)$ is a monotonically decreasing function of temperature T, which vanishes at the transition temperature T_c. In Ref. [8] we have shown that below T_c the photon system is in a squeezed thermal radiation state, in which the photons with opposite wave vectors and helicities are bound into pairs and unpaired photons are transformed into nonpolaritons. At T_c, both photon pairs and nonpolaritons become single bare photons. Above T_c, a Kerr nonlinear blackbody behaves like a normal blackbody.

IV. BCS CONDENSATION STATE OF PHOTONS

A. Formulas

For future study it will be convenient to define the number operators $N_{\mathbf{k}\sigma}=c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}$ for nonpolaritons. The number operators have the eigenvalues $n_{\mathbf{k}\sigma}=0,1,2,\cdots$. The eigenstates of number operators $N_{\mathbf{k}\sigma}$ are given by

$$|\{n_{\mathbf{k}\sigma}\}\rangle=\prod_{\mathbf{k}\sigma}\left[\frac{1}{\sqrt{n_{\mathbf{k}\sigma}!}}(c^{\dagger}_{\mathbf{k}\sigma})^{n_{\mathbf{k}\sigma}}\right]|G\rangle. \tag{24}$$

The Hilbert space of the photon system is spanned by the complete orthonormal basis vectors $|\{n_{\mathbf{k}\sigma}\}\rangle$. We first point out that the squeezed thermal radiation state of a Kerr nonlinear blackbody is just the BCS condensation state of photons. In order to characterize the photonic condensation state, we need to conceive a grand canonical ensemble of nonpolaritons. Some identical systems of the ensemble may be in an eigenstate of the Hamiltonian H'_{em} given by Eq. (20), while the distribution of the ensemble over the eigenstates is described by the density operator of the photonic condensation state,

$$\rho=\frac{\exp(-H'_{em}/k_BT)}{\mathrm{Tr}\exp(-H'_{em}/k_BT)}, \tag{25}$$

where the basis states used in the trace are the eigenstates of the Hamiltonian H'_{em}, which are given by Eq. (24).

We next point out that the system of photon pairs is the condensate in a Kerr nonlinear blackbody. The energy of the condensate is given by E_p, which is one main thermodynamic quantity in a Kerr nonlinear blackbody. The gas of free nonpolaritons constitutes the thermal radiation in a Kerr nonlinear blackbody. Another main thermodynamic quantity in a Kerr nonlinear blackbody is the energy E_r of the thermal radiation, as given by

$$E_r=\sum_{\mathbf{k}\sigma}\hbar\tilde{\omega}_{\mathbf{k}}(T)\langle N_{\mathbf{k}\sigma}\rangle. \tag{26}$$

Therefore the photon system in a Kerr nonlinear blackbody consists of the condensate and the thermal radiation.

Our first task is to calculate the energy E_p of the condensate defined by Eq. (22). From Eq. (23), the hyperbolic sine functions in Eq. (22) obtain the expressions in terms of the velocity $v(T)$ of nonpolaritons,

$$\sinh^2\varphi_{\mathbf{k}\sigma}=\frac{1}{2}\left[\frac{c}{nv(T)}-1\right],\quad \sinh 2\varphi_{\mathbf{k}\sigma}=\left\{\left[\frac{c}{nv(T)}\right]^2-1\right\}^{1/2}. \tag{27}$$

Further, the pair potential in Eq. (22) is given by Eq. (17). The insertion of Eqs. (17) and (27) into Eq. (22) leads to

$$E_p = \left[\frac{c}{nv(T)} - 1\right]\left\{1 - \left[\frac{c}{nv(T)} + 1\right]V_0 \sum_{\mathbf{k}}{}' \hbar\omega_{\mathbf{k}}\right\}\sum_{\mathbf{k}}{}' \hbar\omega_{\mathbf{k}}, \tag{28}$$

where the prime on the summation symbol means that $\omega_{\mathbf{k}} < \omega_R$. It is useful to note that the Eq. (21) at zero temperature reduces to

$$v(0) = 2(c/n)V_0 \sum_{\mathbf{k}}{}' \hbar\omega_{\mathbf{k}}. \tag{29}$$

On the other hand, the zero-temperature velocity of nonpolaritons can be written as $v(0)=\gamma c/n$ where γ is a dimensionless constant. The constant γ is meaningful only if $\gamma < 1$ and is directly proportional to the Kerr nonlinear coefficient. Thereby Eq. (29) is simplified as

$$V_0 \sum_{\mathbf{k}}{}' \hbar\omega_{\mathbf{k}} = \gamma/2. \tag{30}$$

Further, in the usual way it can be found that

$$\sum_{\mathbf{k}}{}' \hbar\omega_{\mathbf{k}} = V\pi\left(\frac{n\omega_R}{2\pi c}\right)^3 \hbar\omega_R. \tag{31}$$

The substitution of Eqs. (30) and (31) into Eq. (28) yields the final result: $E_p(T)=Vu_p(T)$ and

$$u_p(T) = \pi\left(\frac{n\omega_R}{2\pi c}\right)^3\left[\frac{c}{nv(T)} - 1\right]\left\{1 - \frac{\gamma}{2}\left[\frac{c}{nv(T)} + 1\right]\right\}\hbar\omega_R, \tag{32}$$

where $u_p(T)$ is the persistent energy density of the condensate. $u_p(T)$ is a monotonically decreasing function of temperature T. At zero temperature it attains a maximal value

$$u_p(0) = \frac{\pi}{2\gamma}\left(\frac{n\omega_R}{2\pi c}\right)^3 (1-\gamma)^2 \hbar\omega_R, \tag{33}$$

and at transition temperature T_c it is equal to zero.

Our second task is to calculate the energy E_r of the thermal radiation defined by Eq. (26). It is easily found that the ensemble average of the number operator of nonpolaritons in a mode $\mathbf{k}\sigma$ satisfies the well-known Bose-Einstein distribution,

$$\langle N_{\mathbf{k}\sigma}\rangle = \frac{1}{e^{\hbar\tilde{\omega}_{\mathbf{k}}(T)/k_BT} - 1}. \tag{34}$$

Putting Eq. (34) into Eq. (26) and in the usual way altering the summation to an integration, we obtain

$$E_r = V\int_0^\infty \rho_r(\tilde{\omega},T)d\tilde{\omega}, \tag{35}$$

$$\rho_r(\tilde{\omega},T) = \frac{\hbar}{\pi^2 v^3(T)}\frac{\tilde{\omega}^3(T)}{e^{\hbar\tilde{\omega}(T)/k_BT} - 1}, \tag{36}$$

where $\tilde{\omega}(T)=v(T)|\mathbf{k}|$. With the new variable of integration $x=\hbar\tilde{\omega}/k_BT$, the resulting integral in Eq. (35) is equal to $\pi^4/15$. Then Eq. (35) yields the result: $E_r(T)=Vu_r(T)$ and

$$u_r(T) = 4\sigma(T)T^4/v(T), \tag{37}$$

where $u_r(T)$ is the energy density of the thermal radiation and $\sigma(T)=\pi^2k_B^4/60\hbar^3v^2(T)$ is the temperature-dependent Stefan-Boltzmann constant. $u_r(T)$ is a monotonically increasing function of temperature T and at zero temperature it is equal to zero.

The photon-pair system is also a superfluid in a Kerr nonlinear blackbody. In the standing-wave configuration the propagation velocity of the superfluid is zero. Hence the superfluid has no contribution to the radiation pressure. The gas of free nonpolaritons constitutes the normal fluid in a Kerr nonlinear blackbody. In the standing-wave configuration the normal fluid has a definite propagation velocity. Therefore the normal fluid makes a contribution to the radiation pressure. The photon system in a Kerr nonlinear blackbody consists of the superfluid and normal fluid. The analogy has the following advantage: in a traveling-wave configuration, the propagation of the photon-pair system gets rid of thermal scattering effects but nonpolaritons suffer thermal scattering. The analogy has the following significance: when propagating in an optical fiber the photon-pair system carries a persistent light intensity and hence we can realize repeaterless optical communications.

B. Numerical calculation

To make a numerical calculation, we take the diamond crystal as a Kerr nonlinear crystal. The zero-wave-vector frequency of the Raman-active mode of the diamond crystal is $\omega_R=2.51\times10^{14}$ s^{-1} [13]. For convenience, we set the refractive index $n=1$, such that the Kerr nonlinear blackbody can be compared with the normal blackbody. As known, transition temperature T_c depends on dimensionless parameter γ. $T_c=464.9$ K at $\gamma=0.9$. The energy density $u_r(T)$ of the thermal radiation in a Kerr nonlinear blackbody depends on dimensionless parameter γ and thus we set $\gamma=0.9$. The variation in $u_r(T)$ with relative temperature $x=k_BT/\hbar\omega_R$ is shown in Fig. 2 using the dotted-dashed line, where temperature T varies from zero to transition temperature T_c. Figure 2 also shows the energy density $u_p(T)$ of the condensate and the energy density $u_n(T)$ of the normal blackbody, using the dashed and solid lines, respectively. There are the three features: (1) $u_r(T)$ and $u_n(T)$ are monotonically increasing functions of T while $u_p(T)$ is a monotonically decreasing functions of T; (2) at zero temperature $u_p(0)=1.093$ μJ m^{-3} but $u_r(0)=u_n(0)=0$ and at transition temperature $T_c u_p(T_c)=0$ but $u_r(T_c)=u_n(T_c)=35.273$ μJ m^{-3}; (3) as $0<T<T_c$, $u_n(T) < u_r(T)$.

At this point, we must point out that the value $\gamma=0.9$ used in the above figure is proper. In order to give reasons, we select the persistent energy density for example. As known, transition temperature T_c depends on dimensionless parameter γ, i.e., $T_c=T_c(\gamma)$. Now we need to introduce a new relative temperature $T/T_c(\gamma)$. For varying relative temperature $T/T_c(\gamma)$, Fig. 3 shows variation in the persistent energy density u_p with the parameter γ. The persistent energy density is a monotonically decreasing function of the parameter γ at a fixed relative temperature. The reason for this is as follows. The parameter γ signifies the coupling strength between a

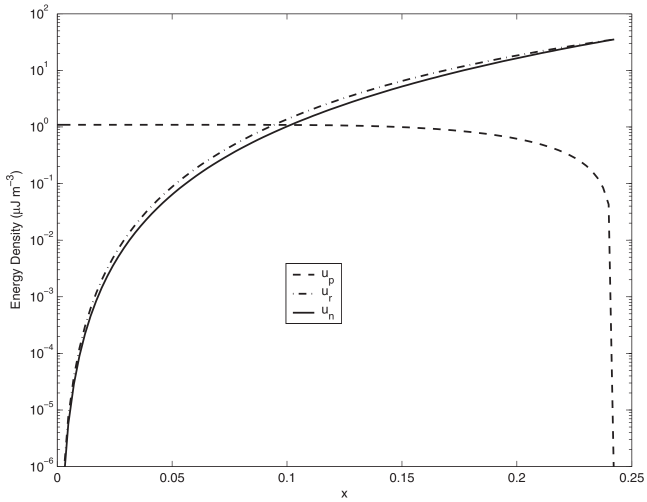

FIG. 2. Variation of energy density with relative temperature $x = k_B T/\hbar\omega_R$, where temperature T varies from zero to transition temperature T_c. u_n denotes the energy density of the normal blackbody; u_p and u_r denote the energy densities of the superfluid and normal fluid, respectively.

bare photon and virtual nonpolar phonons. The larger the parameter γ is, the smaller the coupling strength is. A photon pair consists of two bare photons and virtual nonpolar phonons. When the parameter γ is increased, the nonpolar phonon weight in a photon pair is decreased, so that the energy of the photon-pair system becomes smaller. In the strong-coupling limit $0 < \gamma \ll 1$, the energy of the photon-pair system is largest. We have made a rude approximation that the refractive index of the crystal is taken to be $n=1$. In quality this approximation does not affect our conclusions. In fact the refractive index n of the crystal is larger than one. If the refractive index n of the crystal takes its real value, in quantity the spectral energy density, energy density, and radiation pressure of a Kerr nonlinear blackbody become larger correspondingly. The numerical calculation shows that the BCS condensation state of photons exists indeed.

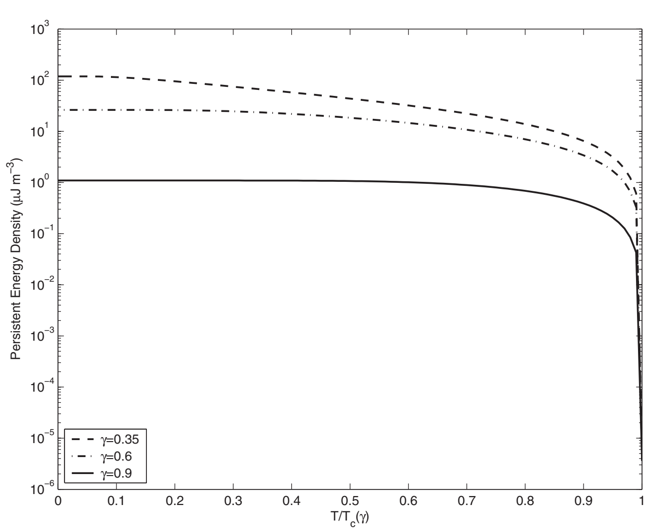

FIG. 3. For three values of γ, variation of the persistent energy density of a Kerr nonlinear blackbody with relative temperature $T/T_c(\gamma)$.

V. Q FUNCTION

The quasiprobability functions such as the Glauber-Sudarshan P, Wigner W, and Q functions are widely used to describe quantum states of the electromagnetic field [15,16]. However, when compared with other quasiprobability functions, the Q function has many advantages, such as nonnegative, well behaved, and simply expressed, which bring the Q function into a subject of increasing interest in quantum optics. The quasiprobability distribution of Q function was first introduced by Cahill and Glauber to reflect the distribution of the density operator in the phase space [16]. If we want to expound the statistical properties of a Kerr nonlinear blackbody, we need to calculate the expression of the Q function of a Kerr nonlinear blackbody at any temperature.

In order to complete the task, we first discuss the BCS condensation state of the photon system below the transition temperature T_c. Photon pairs and nonpolaritons coexist in the BCS condensation state. The subsystem of photon pairs is in a many-mode squeezed vacuum state $|G\rangle = U|0\rangle$, where the many-mode squeeze operator U is given by Eq. (19). The subsystem of nonpolaritons is in the squeezed thermal radiation state. With a single-mode index $\mu = \mathbf{k}\sigma$, we concentrate on a single-mode squeezed vacuum state $|r_\mu\rangle = S(r_\mu)|0\rangle$, where the single-mode squeeze operator $S(r_\mu)$ is given by

$$S(r_\mu) = \exp\left[\tfrac{1}{2} r_\mu (a_\mu^2 - a_\mu^{\dagger 2})\right], \tag{38}$$

with $r_\mu = -\varphi_\mu$ being a real number. The single-mode squeezed number state of nonpolaritons is defined by

$$|n_\mu, r_\mu\rangle = \frac{1}{\sqrt{n_\mu!}} (c_\mu^\dagger)^{n_\mu} |r_\mu\rangle, \tag{39}$$

where $n_\mu = 0, 1, 2, \ldots$. One can also introduce the single-mode squeezed coherent state of nonpolaritons: $|\beta_\mu, r_\mu\rangle = D(\beta_\mu)|r_\mu\rangle$, where $D(\beta_\mu)$ is the so-called displacement operator

$$D(\beta_\mu) = \exp(\beta_\mu c_\mu^\dagger - \beta_\mu^* c_\mu), \tag{40}$$

with β_μ being a complex number. Further one can introduce the density operator of the squeezed thermal radiation state of the μth mode,

$$\rho_\mu = \frac{\exp[-\hbar \tilde{\omega}_\mu(T) N_\mu / k_B T]}{\mathrm{Tr}\,\exp[-\hbar \tilde{\omega}_\mu(T) N_\mu / k_B T]}, \tag{41}$$

where $N_\mu = c_\mu^\dagger c_\mu$ is the number operator of nonpolaritons. Note that the operators $c_\mu^\dagger$ and c_μ of nonpolaritons are related to the operators $a_\mu^\dagger$ and a_μ of photons through Eq. (18).

In terms of the squeezed number states, the density operator of the squeezed thermal radiation state can be expressed as

$$\rho_\mu = \sum_{n_\mu=0}^{\infty} \frac{\langle N_\mu\rangle^{n_\mu}}{(1 + \langle N_\mu\rangle)^{n_\mu+1}} |n_\mu, r_\mu\rangle\langle n_\mu, r_\mu|, \tag{42}$$

where the mean number $\langle N_\mu\rangle$ of nonpolaritons of the μth mode is given by Eq. (9). Alternatively, in terms of the squeezed coherent states, the density operator of the squeezed thermal radiation state can be expressed as

$$\rho_\mu = \int d^2\beta_\mu[\pi\langle N_\mu\rangle]^{-1}\exp\left(-\frac{|\beta_\mu|^2}{\langle N_\mu\rangle}\right)|\beta_\mu,r_\mu\rangle\langle\beta_\mu,r_\mu| = \int d^2\beta_\mu[\pi\langle N_\mu\rangle]^{-1}\exp\left(-\frac{|\beta_\mu|^2}{\langle N_\mu\rangle}\right)D(\beta_\mu)S(r_\mu)|0\rangle \times\langle 0|S^\dagger(r_\mu)D^\dagger(\beta_\mu). \quad (43)$$

Now we need to introduce the single-mode coherent state of photons: $|\alpha_\mu\rangle = D(\alpha_\mu)|0\rangle$, where

$$D(\alpha_\mu) = \exp(\alpha_\mu a_\mu^\dagger - \alpha_\mu^* a_\mu), \quad (44)$$

In quantum optics, it is well known that the probability of finding the μth mode in the state $|\alpha_\mu\rangle$ is defined by the Q representation

$$Q(\alpha_\mu) = \frac{1}{\pi}\langle\alpha_\mu|\rho_\mu|\alpha_\mu\rangle. \quad (45)$$

From this definition, we obtain the Q representation for the squeezed thermal radiation state,

$$Q(\alpha_\mu) = \int d^2\beta_\mu[\pi\langle N_\mu\rangle]^{-1}\exp\left(-\frac{|\beta_\mu|^2}{\langle N_\mu\rangle}\right)Q_{SC}(\alpha_\mu,\beta_\mu), \quad (46)$$

where $Q_{SC}(\alpha_\mu,\beta_\mu)$ is the Q representation for the squeezed coherent state and is given by

$$Q_{SC}(\alpha_\mu,\beta_\mu) = \frac{1}{\pi}|\langle\alpha_\mu|D(\beta_\mu)S(r_\mu)|0\rangle|^2. \quad (47)$$

It is convenient to write down $\beta_\mu = \beta_x + i\beta_y$, where β_x and β_y are the real and imaginary parts of β_μ, respectively. Employing Eq. (18), we can rewrite the displacement operator $D(\beta_\mu)$ as

$$D(\beta_\mu) = D(\beta'_\mu) = \exp(\beta'_\mu a_\mu^\dagger - \beta'^*_\mu a_\mu), \quad (48)$$

where β'_μ is given by

$$\beta'_\mu = \beta_\mu \cosh r_\mu - \beta^*_\mu \sinh r_\mu = \beta_x \exp(-r_\mu) + i\beta_y \exp(r_\mu). \quad (49)$$

It is easily shown that

$$Q_{SC}(\alpha_\mu,\beta_\mu) = \frac{1}{\pi}|\langle\alpha_\mu - \beta'_\mu|S(r_\mu)|0\rangle|^2. \quad (50)$$

We now factorize the squeeze operator into a product of exponentials following Schumaker and Caves [17],

$$S(r_\mu) = \frac{1}{\sqrt{\cosh r_\mu}}\exp\left[-\frac{1}{2}(\tanh r_\mu)a_\mu^{\dagger 2}\right] \times(\cosh r_\mu)^{-a_\mu^\dagger a_\mu}\exp\left[\frac{1}{2}(\tanh r_\mu)a_\mu^2\right]. \quad (51)$$

Substituting Eq. (51) into Eq. (50) we find immediately that

$$Q_{SC}(\alpha_\mu,\beta_\mu) = \frac{1}{\pi\cosh r_\mu}\exp[-2(\alpha_y - \beta_y e^{r_\mu})^2/(1+e^{2r_\mu}) - 2(\alpha_x - \beta_x e^{-r_\mu})^2/(1+e^{-2r_\mu})]. \quad (52)$$

Substituting Eq. (52) into Eq. (46) and after straightforward algebra [18], we find the Q representation for the squeezed thermal radiation state,

$$Q(\alpha_\mu) = \frac{1}{\pi\langle N_\mu\rangle\cosh r_\mu}\frac{1}{\sqrt{(1+1/\langle N_\mu\rangle)^2 - \tanh^2 r_\mu}} \times\exp\left\{-\frac{1}{2}\tanh r_\mu(\alpha_\mu^2 + \alpha_\mu^{*2}) - |\alpha_\mu|^2 + \frac{1/\cosh^2 r_\mu}{(1+1/\langle N_\mu\rangle)^2 - \tanh^2 r_\mu}\left[(1+1/\langle N_\mu\rangle)|\alpha_\mu|^2 + \frac{1}{2}\tanh r_\mu(\alpha_\mu^2 + \alpha_\mu^{*2})\right]\right\}. \quad (53)$$

At this point, we examine the two limiting cases of $Q(\alpha_\mu)$ given by Eq. (53). In the case of $T \geq T_c$, the photon system in a Kerr nonlinear blackbody goes into a normal thermal radiation state and so $r_\mu = 0$. The Q representation of the normal thermal radiation state is given by a Gaussian distribution,

$$Q(\alpha_\mu) = \frac{1}{\pi(1+\langle N_\mu\rangle)}\exp\left(-\frac{|\alpha_\mu|^2}{1+\langle N_\mu\rangle}\right). \quad (54)$$

The real and imaginary parts of α_μ represent two quadrature phase variables, hence there is an equipartition of $Q(\alpha_\mu)$ in the phase space. Therefore, the Q representation of the normal thermal radiation state has phase symmetry. In the case of $T=0$ K, the photon system in a Kerr nonlinear blackbody goes into a squeezed vacuum state $|G\rangle = U|0\rangle$ and so $\langle N_\mu\rangle = 0$. The Q representation of the squeezed vacuum state is given by

$$Q(\alpha_\mu) = \frac{\operatorname{sech} r_\mu}{\pi}\exp[-(\alpha_x^2 + \alpha_y^2) - (\alpha_x^2 - \alpha_y^2)\tanh r_\mu]. \quad (55)$$

Hence there is an unequal partition of $Q(\alpha_\mu)$ in the phase space. Therefore, the Q representation of the squeezed vacuum state apparently lacks phase symmetry. We conclude that in the transition from the normal to the BCS condensation state, the phase symmetry is spontaneously broken.

In Eq. (53), the distribution of $Q(\alpha_\mu)$ function depends strongly on temperature T, Kerr nonlinear coefficient γ, and frequency $\omega_{\mathbf{k}}$. One will display the distribution graph of $Q(\alpha_\mu)$ function on the α_μ plane. To this end, we fix frequency $\omega_{\mathbf{k}} = \omega_R$ and parameter $\gamma = 0.6$. At $\gamma = 0.6$ the transition temperature is $T_c = 1026.8$ K. We first let temperature $T = 1200$ K. In this case the Kerr nonlinear blackbody is in a normal thermal radiation state, whose Q function is given by Eq. (54). Figure 4 shows the three-dimensional contour plots of Eq. (54) on the α_μ plane. It is apparent that the $Q(\alpha_\mu)$ distribution in the normal thermal radiation state is a circle distribution where a peak appears at the center $\alpha_\mu = (0,0)$. We then let temperature $T = 0$ K. In this case the Kerr non-

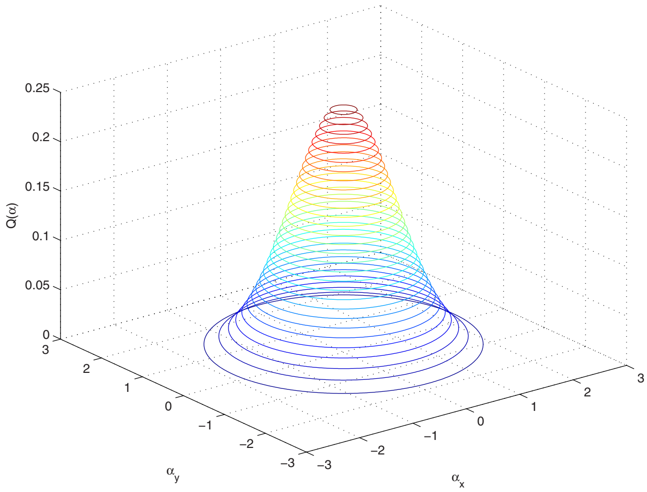

FIG. 4. (Color online) The three-dimensional contour plots of Q function of a normal blackbody at temperature $T=1200$ K.

linear blackbody is in a BCS condensation state, whose Q function is given by Eq. (55). Figure 5 shows the three-dimensional contour plots of Eq. (55) on the α_μ plane. One can observe that the $Q(\alpha_\mu)$ distribution in the BCS condensation state is an ellipse distribution where the rings are more prolate with decreasing of parameter γ. The ellipse distribution becomes the circle distribution as temperature increases from zero to T_c.

VI. DISCUSSION

The principal point of discussion in this paper is that the photon system in a Kerr nonlinear blackbody below transition temperature is in the BCS condensation state. The photon system in the BCS condensation state possesses the squeezing property. The photon system in the BCS condensation state consists of two parts: the superfluid consisting of photon pairs and the normal fluid consisting of individual nonpolaritons. The superfluid acts as a vacuum for the normal fluid. At zero temperature the superfluid possesses a largest persistent energy density. The energy density $u_p(T)$ of the superfluid is a monotonically decreasing function of temperature T, whereas the energy density of the normal fluid is a monotonically increasing function of temperature T. It is easy to understand why the energy density $u_p(T)$ decreases with increasing of temperature. The superfluid is a condensate of photon pairs. The energy density $u_p(T)$ measures the number of photon pairs in the superfluid. As the temperature is increased, some photon pairs evaporate out of the condensate into individual bare photons, so the energy density of the condensate becomes small. As the temperature approaches transition temperature T_c, all photon pairs evaporate, so the superfluid vanishes.

In a normal blackbody, the energy of the vacuum state of the electromagnetic field is the zero-point energy of the bare photon system, which is an infinite quantity. The vacuum energy is unobservable but can produce many observable effects. Among them are the spontaneous emission of an atom, the Lamb shift, and the Casimir effect. However, in a Kerr nonlinear blackbody below transition temperature, the vacuum state of the electromagnetic field is a squeezed vacuum state, which is called an effective vacuum state. The energy of the effective vacuum state is the zero-point energy of the nonpolariton system plus the energy E_p of the photon-pair system. Therefore, the zero-point fluctuation of the electromagnetic field in a Kerr nonlinear blackbody below transition temperature is larger than that in a normal blackbody. We conclude that atomic spontaneous emission in a Kerr nonlinear blackbody is enhanced. A key physical function in quantum optics is the density of states of photons. In a normal blackbody, the density of states of photons is given by

$$\rho(\omega_{\mathbf{k}}) = V\omega_{\mathbf{k}}^2/\pi^2(c/n)^3. \tag{56}$$

However, in a Kerr nonlinear blackbody, single photons are replaced by nonpolaritons. The density of states of nonpolaritons takes the following form:

$$\rho[\tilde{\omega}_{\mathbf{k}}(T)] = V\tilde{\omega}_{\mathbf{k}}^2(T)/\pi^2 v^3(T). \tag{57}$$

It is interesting to note that the velocity $v(T)$ of nonpolaritons is smaller than the velocity c/n of photons. Thus, the density of states in a Kerr nonlinear blackbody can be much larger than that in a normal blackbody. The slow velocity of nonpolaritons can also produce many observable effects.

In this paper, we have investigated the Q function distribution of the photon field in a Kerr nonlinear blackbody. As known to all, in a normal blackbody, the distribution of Q function depends only on the average photon number and is a Gaussian distribution. However, in the Kerr nonlinear blackbody below transition temperature T_c, we have found that the Q function is strongly dependent on temperature T, Kerr nonlinear coefficient γ, and frequency $\omega_{\mathbf{k}}$. We have also found that the distribution of Q function is an ellipse distribution. The ellipse distribution becomes the circle distribution as temperature increases from zero to T_c. The distribution of Q function in a Kerr nonlinear blackbody presents a richer structure than that in a normal blackbody.

Now we examine the probable candidates of a Kerr nonlinear blackbody in which the BCS condensation state of

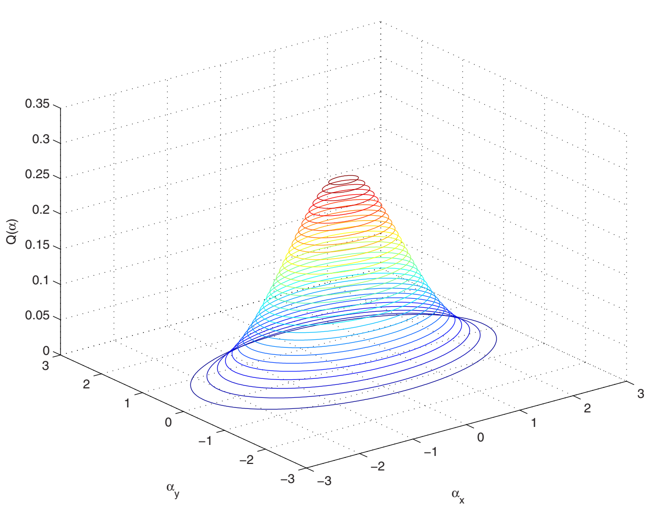

FIG. 5. (Color online) The three-dimensional contour plots of Q function of a Kerr nonlinear blackbody at zero temperature.

photons exists. In the following, we enumerate the three candidates. The first candidate is the universe. As is known to all, the universe erupted in a Big Bang about 20 billion years ago and the universe is a blackbody. In the present universe, the vacuum speed of light is 300 000 km per second. However, in the early universe, the vacuum speed of light is much larger than this value. In the early universe, there is a considerable amount of light neutral spin-zero bosons, which are called axions. Here it is accentuated that axions are speculated to exist. The interaction between photons and axions can lead to an attractive effective interaction among the photons themselves. The attractive effective interaction leads to bound photon pairs. Therefore, the universe is a big Kerr nonlinear blackbody. The system of photon pairs is a dark matter in the cosmology. The dark matter possesses the mass $M=E_p(T)/c^2$ and thus can produce gravitation to bright matters in the universe. In 1965, Penzias and Wilson discovered the cosmic microwave radiation background and here we point out that one can observe a squeezing effect in the cosmic microwave radiation background. The second candidate is a blackbody whose interior is filled by a liquid neon. In such a Kerr nonlinear blackbody, one can observe an unusual Casimir effect. The third candidate has been investigated in the present paper. From the above description, one has many means to detect a BCS condensation state of photons.

To sum up, we have proposed a BCS condensation state of photons in a blackbody whose interior is filled by a Kerr nonlinear crystal. The photon system in the BCS condensation state consists of photon pairs and individual nonpolaritons. The system of photon pairs is a superfluid and the system of individual nonpolaritons is a normal fluid. At zero temperature the superfluid possesses a largest persistent energy density. The persistent energy density of the superfluid is a monotonically decreasing function of temperature and Kerr nonlinear coefficient. The energy density of the normal fluid can be much larger than that of a normal blackbody. The Q function of a Kerr nonlinear blackbody at any temperature is derived analytically. In the transition from the normal to the condensation state, the phase symmetry of the photon system is spontaneously broken. The predicted properties of photonic condensation state are hopeful to be verified in physics laboratories for the not too distant future.

ACKNOWLEDGMENTS

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[1] H. K. Onnes, Commun. Phys. Lab. Univ. Leiden **124**, 2 (1911).
[2] P. L. Kapitza, Nature (London) **141**, 74 (1938).
[3] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
[4] K. B. Davis, M. O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[5] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature (London) **415**, 39 (2002).
[6] T. Nikuni, M. Oshikawa, A. Oosawa, and H. Tanaka, Phys. Rev. Lett. **84**, 5868 (2000).
[7] Ch. Rüegg, N. Cavadini, A. Furrer, H. U. Güdel, K. Krämer, H. Mutka, A. Wildes, K. Habicht, and P. Vorderwisch, Nature (London) **423**, 62 (2003).
[8] Z. Cheng, Phys. Lett. A **291**, 4 (2001).
[9] Z. Cheng, J. Opt. Soc. Am. B **19**, 1692 (2002).
[10] Z. Cheng, Phys. Rev. Lett. **67**, 2788 (1991).
[11] Z. Cheng, Phys. Rev. A **51**, 675 (1995).
[12] L. D. Landau and E. M. Lifshitz, *Statistical Physics, Part 1*, 3rd ed. (Reed Educational and Professional Publishing Ltd, Oxford, 1980), p. 183.
[13] W. Hayes and R. Loudon, *Scattering of Light by Crystals* (Wiley, New York, 1978).
[14] L. E. Reichl, *A Modern Course in Statistical Physics* (University of Texas Press, Austin, 1980).
[15] M. Hillery, R. F. O'Connell, M. O. Scully, and E. P. Wigner, Phys. Rep. **106**, 121 (1984).
[16] K. E. Cahill and R. J. Glauber, Phys. Rev. **177**, 1857 (1969).
[17] B. L. Schumaker and C. M. Caves, Phys. Rev. A **31**, 3093 (1985).
[18] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997), p. 86.