Single-photon scattering on Λ-type three-level atoms in a one-dimensional waveguide

T. S. Tsoi and C. K. Law
Department of Physics and Institute of Theoretical Physics, The Chinese University of Hong Kong, Shatin, Hong Kong, SAR, China
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We investigate the quantum scattering involving a single photon and a chain of N polarized Λ-type three-level atoms inside a one-dimensional waveguide. By using a transfer matrix method, we obtain analytic expressions of eigenvectors of the Hamiltonian, which determine the polarization dependent transmission and reflection spectra for a single photon. We show that the scattering is most significant when the interatomic distance equals an integer multiple plus a quarter of the resonance wavelength. In such a configuration, an incident photon with an unknown polarization can be converted into a specified polarization with a transmission probability higher than that in polarizers obeying Malus's law.

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I. INTRODUCTION

The realization of deterministic sources of single photons has opened a door for some key applications in quantum information [1] and for fundamental investigations of photon-atom interactions at the single-photon level. Recently, studies of single-photon scattering on a two-level atom inside a one-dimensional waveguide have revealed some of the intriguing features of photon transport in low dimensional environments [2,3]. Such a one-dimensional waveguide, which can be realized by a line defect in photonic crystals [4] and superconducting transmission line [5], can have small transverse cross sections enabling a strong coupling between atom and the waveguide modes. This makes a single atom efficiently serve as a mirror controlling the transmission and reflection properties at various frequencies [2,6]. In addition, further enhancement of coupling can be achieved by exploiting microcavities along a waveguide, which is potentially useful for building a quantum switch for routing single-photons in quantum network [3,7,8]. More recently, quantum scattering with two photons [9,10] and multiple atoms [11] have also been explored. In most of these previous studies, the scatterer (atom) is considered as a two-level system, and therefore only one polarization of the photon participates in the dipole interaction. In order to manipulate the polarization degree of freedom, scatterers with more than two levels would be needed.

Polarization control of light has been one of the main topics in optics. In this paper we address a basic question how a single photon of an *unknown* polarization can be converted into a specified polarization inside a waveguide. Such a transformation should be distinguished from the task performed by polarizers obeying Malus's law. This is because for ordinary adsorptive polarizers, photons in the unwanted orthogonal polarization are simply blocked by absorption, and hence there would be a 50% loss in an ensemble of randomly polarized photons. A possible solution to reduce the loss is to employ Raman coupling with a polarized Λ-type three-level atoms so that whenever a pump photon is scattered into a Stokes photon, the polarization can be transformed accordingly. However, owing to the lack of exact analytic solutions describing the process [12], it remains unclear about the efficiency of the polarization transfer. In this paper we will examine this issue in a one-dimensional waveguide, and show that the polarization transformation can be achieved by a few numbers of atoms with the performance limited by reflection and relaxation loss. We note that the interaction between a single photon and Λ-type atoms have been studied extensively in the context of cavity QED [13]. Here we will address a noncavity situation and present analytically a set of eigenvectors of the Hamiltonian relevant to the scattering process. With these eigenvectors, the transmission and reflection amplitudes as a function of incident photon frequency, number of atoms, and atomic spacings can be determined.

II. MODEL HAMILTONIAN

To begin with, we show in Fig. 1 the atomic structure and interaction scheme. The Λ-atom consists of two degenerate ground states $|H\rangle$ and $|V\rangle$, and an excited state $|e\rangle$ which has an energy $\hbar\omega_A$ above the ground states. The two atomic transitions correspond to dipole couplings with two orthogonal polarizations of the field, H and V. Note that the labels H and V are used for convenience only, and they do not necessarily imply that the polarizations are horizontal and vertical. In fact, H and V can refer to left and right circularly polarized modes allowed by the dipole selection rule. Our model consists of N Λ atoms, each separated by distance L, fixed inside a waveguide. The waveguide is assumed to be infinitely long with negligible lateral loss, and the positions of atoms are located at $x=0,L,\ldots,(N-1)L$.

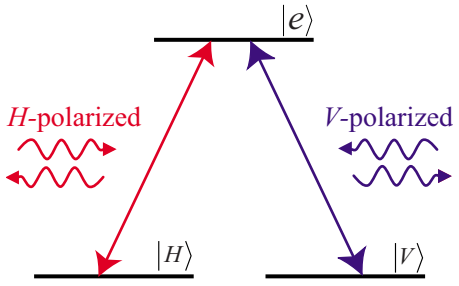

FIG. 1. (Color online) A drawing of the interaction scheme in which a single photon of two orthogonal polarizations, H and V, couples with a Λ system.

The Hamiltonian of the system under rotating wave approximation is given by ($\hbar=1$):

$$\mathcal{H}=\sum_{j=1}^{N}\omega_A|e\rangle_j\langle e|+\sum_{s=H,V}\int_{-\infty}^{\infty}dk\omega_k a_{k,s}^{\dagger}a_{k,s}+\sum_{j=1}^{N}\sum_{s=H,V}\int_{-\infty}^{\infty}dk[g_{k,s}a_{k,s}^{\dagger}e^{-ikx_j}|s\rangle_j\langle e|+g_{k,s}^{*}a_{k,s}e^{ikx_j}|e\rangle_j\langle s|] \tag{1}$$

where $a_{k,s}$ and $a_{k,s}^{\dagger}$ are the usual annihilation and creation operators associated with the traveling wave mode of wave vector k and polarization s, and x_j is the longitudinal position of the jth atom. For simplicity, the transverse positions of atoms are assumed the same. The dipole coupling constant is $g_{k,s}=\sqrt{\frac{\omega_k}{4\pi\hbar\varepsilon_0 A}}\langle e|d|s\rangle$, where d is the dipole operator, and A is an effective transverse cross section area containing the dependence on the transverse atomic position [6]. Note that dipole matrix element is a complex number in general, but since $\langle e|d|s\rangle$ does not depend on k and the atomic position, the phase factor of the matrix element can always be absorbed into the definition of $a_{k,s}$ and $a_{k,s}^{\dagger}$ in the Hamiltonian without affecting the dynamics. In this way $g_{k,s}$ will be treated as a real number for convenience.

Next we adopt the real space approach [2] and define the right and left propagating field operators,

$$\psi_{R,s}(x)\equiv\frac{1}{\sqrt{2\pi}}\int_0^{\infty}dke^{ikx}a_{k,s}, \tag{2}$$

$$\psi_{L,s}(x)\equiv\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0}dke^{ikx}a_{k,s}. \tag{3}$$

By the commutation relations for $a_{k,s}$ and $a_{k,s}^{\dagger}$, we have $[\psi_{L,s}(x),\psi_{R,s'}^{\dagger}(x')]=[\psi_{R,s}(x),\psi_{L,s'}^{\dagger}(x')]=0$, and

$$[\psi_{R,s}(x),\psi_{R,s}^{\dagger}(x')]=\delta(x-x')-\frac{1}{2\pi}\int_{-\infty}^{0}dke^{ik(x-x')}, \tag{4}$$

$$[\psi_{L,s}(x),\psi_{L,s}^{\dagger}(x')]=\delta(x-x')-\frac{1}{2\pi}\int_{0}^{\infty}dke^{ik(x-x')}. \tag{5}$$

because of the separation of positive and negative k's in the definitions (2) and (3). However, the integral terms in Eqs. (4) and (5) can be neglegted in our later calculations, since the mode functions defined in Sec. III involve narrow bands of optical wave vectors.

With the field operators (2) and (3) the Hamiltonian can be rewritten as

$$\mathcal{H}=\sum_{j=1}^{N}\omega_A|e\rangle_j\langle e|+iv_g\sum_{s=H,V}\int dx[\psi_{L,s}^{\dagger}(x)\partial_x\psi_{L,s}(x)-\psi_{R,s}^{\dagger}(x)\partial_x\psi_{R,s}(x)]+J\sum_{j=1}^{N}\sum_{s=H,V}\{[\psi_{R,s}^{\dagger}(x_j)+\psi_{L,s}^{\dagger}(x_j)]|s\rangle_j\langle e|+|e\rangle_j\langle s|[\psi_{R,s}(x_j)+\psi_{L,s}(x_j)]\}, \tag{6}$$

where $v_g=\omega_k/|k|$ is the speed of light inside the waveguide in the absence of atoms. We have assumed that the field frequencies are far away from the cutoff frequency of the waveguide, so that the linear dispersion relation holds. In addition, we have made use of the fact that $g_{k,s}$ is approximately a constant within the bandwidth of incident photon frequencies concerned. Since we are interested in scattering processes with photon frequency close to resonance, we introduce the coupling strength $J=\sqrt{2\pi}|g_{k=k_A,s}|$ (where $k_A=\omega_A/v_g$) evaluated at the resonance frequency, assuming $g_{k,s}$ are the same for the two polarizations. Note that $g_{k,s}$ is already treated as real for the reason given above.

In writing the Hamiltonians (1) and (6), we have followed the previous treatment to omit the interaction with nonwaveguide modes [2,3], and this is justified as long as the waveguide modes can capture most of the radiation from atoms [14]. Note that in the strict one-dimensional environment, the dipole-dipole interaction mediated by waveguide modes can be derived from our Hamiltonian and there is no need to add an extra dipole-dipole interaction term. This point about the fundamental derivation of dipole-dipole interaction has been emphasized by Goldstein and Meystre [15]. However, in the less ideal case where atoms and nonwaveguide modes in other dimensions are coupled, the interaction would lead to spontaneous atomic decay and a correction of atom-atom interactions. By standard calculations of two-level atoms in three-dimensional free space [16], the atom-atom interaction decreases with the interatomic distance. We expect the modification of atom-atom interaction inside waveguide can be substantially weakened by increasing the interatomic distance, since it becomes more difficult to exchange photons through nonwaveguide modes. Therefore we will neglect such an interaction. Nevertheless, we will still examine the photon loss by spontaneously atomic decay in nonwaveguide modes, which shall be discussed in Sec. IV.

III. ANALYTICAL SOLUTION OF EIGENVECTORS

We consider the scattering problem in which a photon of frequency ω_k is incident from the left with the polarization state H, and all atoms are initially polarized in the state $|H\rangle$. The solution of such a scattering problem is determined by the eigenvectors of H denoted by $|E_k\rangle$ with $E_k=\omega_k$ being the energy.

A. $N=1$ case

Let us first discuss the solution of the $N=1$ problem, with the atom located at $x=0$. The form of $|E_k\rangle$ is given by

$$|E_k\rangle=\sum_{s=H,V}\int dx[u_{k,R,s}^{*}(x)\psi_{R,s}^{\dagger}(x)+u_{k,L,s}^{*}(x)\psi_{L,s}^{\dagger}(x)]|0,s\rangle+e_k|0,e\rangle. \tag{7}$$

where $|0,\mu\rangle$ ($\mu=H,V,e$) denotes the joint state in which the field is in the vacuum state and the atom is in the state $|\mu\rangle$. In writing Eq. (7), we have introduced the mode functions

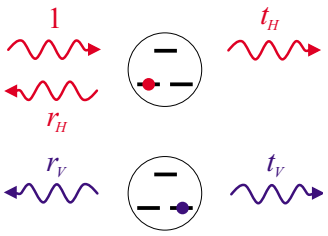

FIG. 2. (Color online) An illustration of an eigenvector of a one-atom system. The curvy lines denote the photon with the polarization specified by the corresponding subscripts, and the central circles describe the atomic states.

$$u^*_{k,R,H}(x) = e^{ikx}[\theta(-x) + t_H\theta(x)], \tag{8}$$

$$u^*_{k,L,H}(x) = e^{-ikx}r_H\theta(-x), \tag{9}$$

$$u^*_{k,R,V}(x) = e^{ikx}t_V\theta(x), \tag{10}$$

$$u^*_{k,L,V}(x) = e^{-ikx}r_V\theta(-x), \tag{11}$$

with $\theta(x)$ being the Heaviside step function and $\theta(0)\equiv 1/2$. The t_s and r_s can be interpreted as transmission and reflection coefficients of the polarization s respectively (Fig. 2).

Putting $|E_k\rangle$ into the Schrödinger's equation $\mathcal{H}|E_k\rangle = \omega_k|E_k\rangle$, we have

$$-iv_g\partial_x u^*_{k,R,s}(x) + e_k J\delta(x) = \omega_k u^*_{k,R,s}(x), \tag{12}$$

$$iv_g\partial_x u^*_{k,L,s}(x) + e_k J\delta(x) = \omega_k u^*_{k,L,s}(x), \tag{13}$$

$$\omega_A e_k + J\sum_{s=H,V}(u^*_{k,R,s}(0) + u^*_{k,L,s}(0)) = \omega_k e_k. \tag{14}$$

These eigenvectors are orthogonal and subject to normalization condition: $\langle E_{k'}|E_k\rangle = 2\pi\delta(k-k')$. Together with Eqs. (8)–(11), the transmission and reflection coefficients are obtained,

$$t_H = \frac{\Delta_k + i\Omega}{\Delta_k + 2i\Omega}, \tag{15}$$

$$r_H = t_V = r_V = -\frac{i\Omega}{\Delta_k + 2i\Omega}, \tag{16}$$

$$e_k = \frac{J}{(\Delta_k + 2i\Omega)}, \tag{17}$$

where $\Delta_k \equiv (\omega_k - \omega_A)$ is the detuning and $\Omega \equiv J^2/v_g$ is a key interaction parameter. As an estimation, suppose $v_g \approx c$ is the speed of light in vacuum and the cross section area is of the order of wavelength square, then Ω can be comparable with the natural line width of the atomic excited state in free space.

It is easy to check that $|t_H|^2 + |r_H|^2 + |t_V|^2 + |r_V|^2 = 1$, which corresponds to the conservation of probability in the scattering problem. Note that the coefficients r_H, t_V, r_V are identical, which is due to the fact that once the atom is excited by the incident photon, the field becomes a vacuum, and the atom can emit a photon into all of the four modes equally. However, the interference with the incident mode makes t_H different from others, which can be seen by rewriting Eq. (15) as $t_H = 1 - i\Omega/(\Delta_k + 2i\Omega)$.

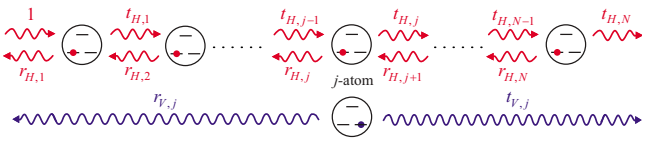

FIG. 3. (Color online) An illustration of an eigenvector for N-atom system. The upper row shows the various transmission and reflection coefficients of H-polarized photon interacting with atomic state $|H^{(1)}\ldots H^{(N)}\rangle$. The lower row shows the coupling between a V-polarized photon with an atomic state $|H^{(1)}\ldots V^{(j)}\ldots H^{(N)}\rangle$, where $1 \le j \le N$.

B. $N > 1$ case

The eigenvectors of a multiatom system relevant to our scattering problem require a more complicated set of mode functions. Explicitly, $|E_k\rangle$ takes the form:

$$\begin{aligned}|E_k\rangle = &\int dx[u^*_{k,R}(x)\psi^\dagger_{R,H}(x) + u^*_{k,L}(x)\psi^\dagger_{L,H}(x)]|0,H^{(1)}\cdots H^{(N)}\rangle \\ &+ \sum_{j=1}^{N}\int dx[v^{(j)*}_{k,R}(x)\psi^\dagger_{R,V}(x) \\ &+ v^{(j)*}_{k,L}(x)\psi^\dagger_{L,V}(x)]|0,H^{(1)}\cdots V^{(j)}\cdots H^{(N)}\rangle \\ &+ \sum_{j=1}^{N} e^{(j)}_k|0,H^{(1)}\cdots e^{(j)}\cdots H^{(N)}\rangle,\end{aligned} \tag{18}$$

with the mode functions,

$$u^*_{k,R}(x) = \begin{cases} e^{ikx} & x < 0, \\ t_{H,j}e^{ik(x-jL)} & (j-1)L < x < jL, \\ t_{H,N}e^{ik(x-NL)} & x > (N-1)L, \end{cases} \tag{19}$$

$$u^*_{k,L}(x) = \begin{cases} r_{H,1}e^{-ikx} & x < 0, \\ r_{H,j+1}e^{-ik(x-jL)} & (j-1)L < x < jL, \\ 0 & x > (N-1)L, \end{cases} \tag{20}$$

for $1 \le j \le N$, and

$$v^{(j)*}_{k,R}(x) = t_{V,j}e^{ik[x-(j-1)L]}\theta[x-(j-1)L], \tag{21}$$

$$v^{(j)*}_{k,L}(x) = r_{V,j}e^{-ik[x-(j-1)L]}\theta[x+(j-1)L]. \tag{22}$$

Here $v^{(j)*}_{k,R}(x)$ and $v^{(j)*}_{k,L}(x)$ describe the scattered photon associated with the j-th atom being transferred to the $|V\rangle$ state, and it propagates out of the chain without further interaction with other atoms (Fig. 3). Note that in the above definition of $u^*_{k,R}(x)$ and $u^*_{k,L}(x)$, $t_{H,j-1}$, and $r_{H,j}$ correspond to the respective photon amplitudes at the immediate left of the jth atom. This should not be confused with that the previous single-atom section, in which t_H and r_H were defined as the photon amplitudes at $x = 0^+$ and $x = 0^-$, respectively. Therefore in the

case for $N=1$, $t_{H,1}$ and t_H are related by $t_{H,1}=t_H e^{ikL}$.

Solving $\mathcal{H}|E_k\rangle=\omega_k|E_k\rangle$, we have, for $1\leq j\leq N$,

$$t_{H,j}e^{-ikL}-t_{H,j-1}+\frac{iJe_k^{(j)}}{v_g}=0, \tag{23}$$

$$r_{H,j+1}e^{ikL}-r_{H,j}-\frac{iJe_k^{(j)}}{v_g}=0, \tag{24}$$

$$t_{V,j}+\frac{iJe_k^{(j)}}{v_g}=0, \tag{25}$$

$$r_{V,j}+\frac{iJe_k^{(j)}}{v_g}=0, \tag{26}$$

and

$$\frac{1}{2}(t_{H,j-1}+t_{H,j}e^{-ikL}+r_{H,j}+r_{H,j+1}e^{ikL}+t_{V,j}+r_{V,j})-\frac{\Delta_k e_k^{(j)}}{J}=0, \tag{27}$$

where $t_{H,0}\equiv 1$ and $r_{H,N+1}\equiv 0$ are defined. In deriving Eq. (27), $u_{k,s}(x_j)\equiv[u_{k,s}(x_j^+)+u_{k,s}(x_j^-)]/2$ and $v_{k,s}(x_j)\equiv[v_{k,s}(x_j^+)+v_{k,s}(x_j^-)]/2 (s=R,L)$ have been used. This definition is consistent with the usual definition of $\theta(0)\equiv 1/2$.

It is more convenient to proceed by defining a transfer matrix $\mathbf{M}$ for the H-polarized photon such that

$$\begin{bmatrix} t_{H,j} \\ r_{H,j+1} \end{bmatrix}=\mathbf{M}^j\begin{bmatrix} 1 \\ r_{H,1} \end{bmatrix}, \tag{28}$$

where $\mathbf{M}=\mathbf{G}\mathbf{T}_H$ with

$$\mathbf{G}=\begin{bmatrix} e^{ikL} & 0 \\ 0 & e^{-ikL} \end{bmatrix}, \tag{29}$$

$$\mathbf{T}_H=\begin{bmatrix} 1-i\dfrac{\Omega}{\Delta_k+i\Omega} & -i\dfrac{\Omega}{\Delta_k+i\Omega} \\ i\dfrac{\Omega}{\Delta_k+i\Omega} & 1+i\dfrac{\Omega}{\Delta_k+i\Omega} \end{bmatrix}. \tag{30}$$

In this way, we have $\mathbf{M}^2=2\mathbf{M}\cos\beta-\mathbf{I}$, with β being a complex angle defined by

$$\cos\beta=\cos kL+\frac{\Omega}{\Delta_k+i\Omega}\sin kL. \tag{31}$$

Then by induction it can be shown that for $1\leq j\leq N$ [17],

$$\mathbf{M}^j=\frac{1}{\sin\beta}[\mathbf{M}\sin j\beta-\mathbf{I}\sin(j-1)\beta]. \tag{32}$$

With the explicit expression of $\mathbf{M}^N$, we can solve $r_{H,1}$ in Eq. (28) under the condition $r_{H,N+1}=0$, and then $t_{H,j}$ and $r_{H,j}$ for a general j are obtained in closed forms,

$$t_{H,j}=\csc\beta\left[\frac{e^{ikL}\sin j\beta[\Delta_k+i\Omega-e^{ikL}\Delta_k\csc N\beta\sin(N-1)\beta]}{\Delta_k+2i\Omega-e^{ikL}(\Delta_k+i\Omega)\csc N\beta\sin(N-1)\beta}-\sin(j-1)\beta\right], \tag{33}$$

$$r_{H,j}=-\frac{i\Omega\csc(N\beta)\sin(N-j+1)\beta}{\Delta_k+2i\Omega-e^{ikL}(\Delta_k+i\Omega)\csc N\beta\sin(N-1)\beta}. \tag{34}$$

Next by using Eqs. (23)–(27), the rest of the coefficients, $t_{V,j}$, $r_{V,j}$, and $e_k^{(j)}$, can be obtained in terms of $t_{H,j}$,

$$t_{V,j}=r_{V,j}=t_{H,j}e^{-ikL}-t_{H,j-1}, \tag{35}$$

$$e_k^{(j)}=\frac{iv_g}{J}(t_{H,j}e^{-ikL}-t_{H,j-1}), \tag{36}$$

and the conservation of probability

$$|t_{H,N}|^2+|r_{H,1}|^2+\sum_{j=1}^{N}(|t_{V,j}|^2+|r_{V,j}|^2)=1 \tag{37}$$

is satisfied. These eigenvectors are orthogonal and subject to the same normalization condition given in the previous subsection. For later purpose, we introduce

$$\phi\equiv\omega_A L/v_g\approx kL, \tag{38}$$

where the approximation is valid as long as $\Delta_k L/v_g\ll 1$, and this is true for L up to several thousand resonance wavelengths, given that the bandwidth Δ_k of interest here is of the order of the natural line width of the atoms.

In the case of the interatomic separation $L=(m+1/4)\lambda$ (where m is a positive integer and λ is the resonance wavelength) or $\phi=(2m+1/2)\pi$, the coefficients can be much simplified,

$$t_{H,N}=\frac{\sin\beta}{\sin\beta\cos(N\beta)-i\sin(N\beta)}, \tag{39}$$

$$r_{H,1}=-\frac{\Omega}{(\Delta_k+i\Omega)[\cot(N\beta)\sin(\beta)-i]}, \tag{40}$$

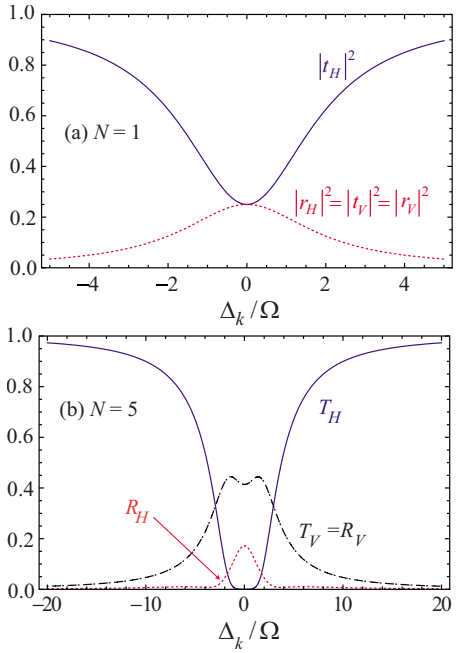

FIG. 4. (Color online) Transmission and reflection spectra in H and V polarizations for (a) $N=1$ (b) $N=5$ at $\phi=\pi/2$. (a) Solid blue line represents $|t_H|^2$ and the red dashed line represents $|t_V|^2=|r_H|^2=|r_V|^2$. (b) Solid blue line: transmission probability $T_H=|t_{H,N}|^2$ in the original polarization. Dashed red line: reflection probability $R_H=|r_{H,1}|^2$ in the original polarization. Dashed-dotted black line: probability that the photon is turned into polarization V while being transmitted or reflected $T_V\equiv\Sigma_j|t_{V,j}|^2=\Sigma_j|r_{V,j}|^2=R_V$.

$$r_{V,j}=-\frac{\Omega[i\sin[(N-j+1)\beta]+\sin[(N-j)\beta]]}{(\Delta_k+2i\Omega)\sin(N\beta)-i(\Delta_k+i\Omega)\sin[(N-1)\beta]}, \tag{41}$$

where $\beta=\cos^{-1}(\frac{\Omega}{\Delta_k+i\Omega})$ has been used.

IV. POLARIZATION DEPENDENT TRANSMISSION AND REFLECTION PROPERTIES

In this section, we study the transmission and reflection spectrum by using the eigenvectors obtained. We will focus on the configuration with the interatomic separation $L=(m+1/4)\lambda$ or $\phi=(2m+1/2)\pi$ because it gives a high probability of polarization transformation as we shall see. In Fig. 4, we illustrate different features for single atom and multiple atoms cases. In the case of single-atom system [Fig. 4(a)], an initially H-polarized photon can be transmitted in the same polarization with a 25% probability at resonance according to Eq. (15). However, such a transmission can be strongly suppressed in the $N=5$ system [Fig. 4(b)]. We see that $t_{H,N}$ is almost zero in a range of frequencies near the resonance. The emergence of such a band-gap-like structure is a consequence of the interference from multiple atoms. A closer inspection of Eq. (39) at $\Delta_k=0$ gives, for any nonzero coupling Ω,

$$|t_{H,N}|^2\approx e^{-2\eta(N-1)}/4, \tag{42}$$

where $\eta=\mathrm{Im}[\cos^{-1}(-i)]\approx-0.88$ is the numerical value of the imaginary part of β. Since η is of order 1, a few atoms can significantly block the H-polarized photon. We emphasize that $t_{H,N}\approx0$ does not have to be at the exact resonance. For the parameters used in Fig. 4(b), we observe that there is a window $|\Delta_k|\leq\Omega$ in which $|t_{H,N}|^2<5.3\times10^{-3}$. The width of such a window increases with the number of atoms. The blocked photon can either be reflected with the polarization unchanged, or converted into the polarization V. The probability of the former case is $|r_{H,1}|^2$, which is approximately a Lorentzian shape with a width of the order of Ω, and it is less sensitive to the number of atoms. The transmission probability in the V polarization is $T_V\equiv\Sigma_j|t_{V,j}|^2$, which displays a double-peak feature not observed in $N=1$ system. Such a double-peak shape emerges when $N\geq3$.

Now we examine the performance of the system as a basic model of transforming an arbitrary polarized photon into a V–polarized photon. It is sufficient to investigate the input photon with a H polarization as above, since a V–polarized photon passes through the atoms freely. For realistic calculations, we include the photon loss due to spontaneous emission into nonwaveguide modes. This is done by adding an imaginary part in the atomic frequency: $\omega_A\rightarrow\omega_A-i\gamma_T$, where γ_T is the corresponding decay rate. Such a treatment can be derived by eliminating field variables of nonwaveguide modes, assuming that these modes form a Markovian bath and the interatomic distance is sufficiently large that there is no exchange of photons among atoms through the bath. The value of γ_T depends on the geometry of the system, but it is expected to be smaller than the spontaneous decay rate in free space since the contributions from waveguides modes are excluded. For this reason we will examine the parameter range for γ_T from zero up to the order of Ω.

To quantify the performance of polarization transformation in the transmission, we introduce the fidelity defined by

$$\mathcal{F}\equiv\frac{T_V}{T_H+T_V}, \tag{43}$$

where $T_H=|t_{H,N}|^2$. Since the denominator T_H+T_V is the overall transmission probability, $\mathcal{F}$ is understood as a conditional probability. In other words $\mathcal{F}=1$ means that if a photon is transmitted then its polarization must be V with full certainty.

In Figs. 5(a)–5(c) we present typical results of the fidelity as a function of various parameters. The key feature is that there is a window of frequency where $\mathcal{F}\approx1$ [Fig. 5(a)], even though γ_T is comparable with Ω. In fact, the loss due to spontaneous decay does not seem to have significant effects on the $\mathcal{F}$'s frequency profile [18]. For the parameter considered in Fig. 5(a) (dashed line), we have $\mathcal{F}=0.9997$ at resonance. The dependence of $\mathcal{F}$ at $\Delta_k=0$ on the interatomic distance is shown in Fig. 5(b), which indicates that the fidelity is maximized at $L=(m+1/4)\lambda$ and it is quite insensitive to the atomic separation except at $L=m\lambda/2$, where the polarization transformation is much less effective. In Fig. 5(c), we see that a nonzero γ_T would lower the fidelity for a given number of atoms, but high values of $\mathcal{F}$ can be restored by increasing the atom number.

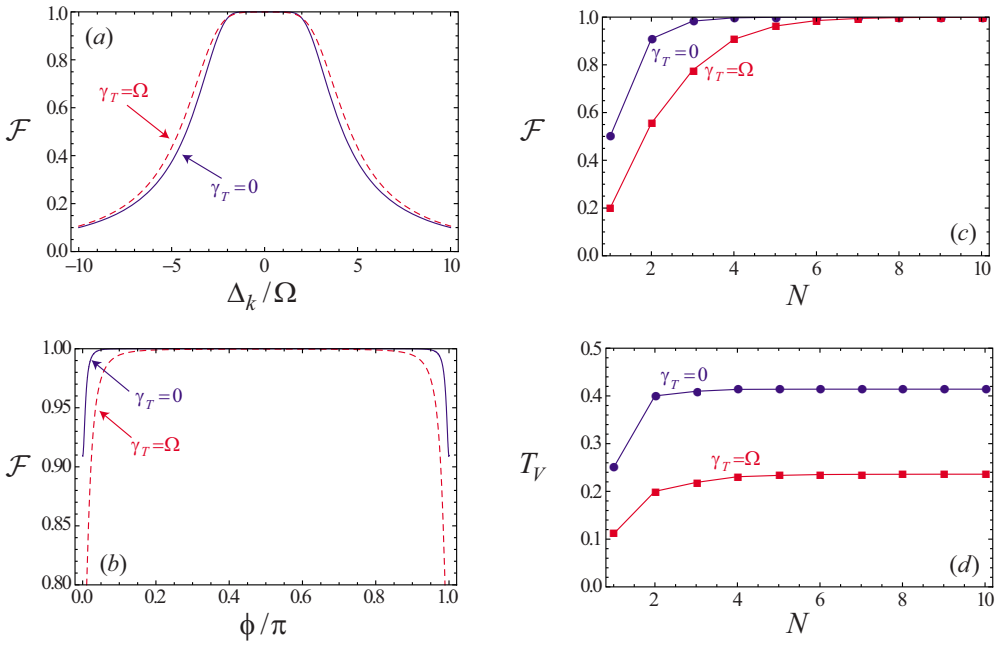

FIG. 5. (Color online) (a) $\mathcal{F}$ as a function of detuning Δ_k at $\phi=\pi/2$ and $N=10$. (b) $\mathcal{F}$ as a function of interatomic distance in terms of ϕ at $\Delta_k=0$ and $N=10$. (c) $\mathcal{F}$ as a function of N at $\Delta_k=0$. (d) Transmission probability in the polarization V, $T_V \equiv \Sigma_j |t_{V,j}|^2$, as a function of the number of atoms N at $\Delta_k=0$. In (a) and (b) The blue solid line is for $\gamma_T=0$ and the red dashed line is for $\gamma_T=\Omega$. In (c) and (d), the filled circles and squares correspond to $\gamma_T=0$ and $\gamma_T=\Omega$, respectively.

We point out that although a typical absorptive polarizer can be used to obtain a V-polarized transmitted light from an unpolarized or mixed-polarized light beam, it would give a zero transmission for the physical situation considered here. This is because the H polarized is completely blocked and loss. In contrast, our configuration enables an appreciable probability T_V. Here $T_V \approx 22\%$ for $\gamma_T=\Omega$ in Fig. 5(d). We observe that the loss somehow saturates when $N>5$, and hence further increasing N does not diminish the transmission but it will improve the fidelity [Fig. 5(c)]. For a general incident polarization, it is useful to compare our results with the familiar Malus's law: $\cos^2\alpha$, where α is the angle between the incident polarization and the transmission axis of the polarizer. In our system the transmission probability is $\cos^2\alpha + T_V \sin^2\alpha$, which is higher than the constraint imposed by the Malus's law.

V. CONCLUSION

To conclude, the polarization control of a single photon inside a waveguide has been investigated in a Raman model with an array of Λ atoms. We have derived analytic expressions of eigenvectors of the Hamiltonian, which reveal the transmission and reflection properties. By exploiting strong coupling near atomic resonance, we have shown that a photon with an unknown polarization can be converted into a specified one with a small number of atoms ($N \approx 10$) with a high fidelity, and the transmission is higher than ordinary polarizers obeying Malus's law. In particular, the interatomic distance at $L=(m+1/4)\lambda$ is found to be most effective. Since our study has assumed that all atoms are initially polarized in the H state, the rich features of the scattering problem have not been fully explored. This is because there are 2^N possible degenerate atomic ground states in the Λ model, which is in sharp contrast to a single ground state in two-level problems. An interesting and difficult issue is how various atomic ground states affect the scattering properties, and how the final state of scattered photon could reveal signatures of different initial atomic states. These interesting issues are out of the scope of this paper, and they are open for future investigations.

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