

## Phase noise and laser-cooling limits of optomechanical oscillators

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(Received 15 June 2009; published 14 September 2009)

The noise from laser phase fluctuation sets a major technical obstacle to cool the nanomechanical oscillators to the quantum region. We propose a cooling configuration based on the optomechanical coupling with two cavity modes to significantly reduce this phase noise by  $(2\omega_m/\gamma)^2$  times, where  $\omega_m$  is the frequency of the mechanical mode and  $\gamma$  is the decay rate of the cavity mode. We also discuss the detection of the phonon number when the mechanical oscillator is cooled near the quantum region and specify the required conditions for this detection.

DOI: 10.1103/PhysRevA.80.033821

PACS number(s): 42.50.Wk, 07.10.Cm, 03.65.Ta

Cooling of the motion of nanomechanical oscillators has attracted strong interest recently [1–4]. When cooled to the quantum region, this system has many potential applications, such as for mechanical sensors [5], precision measurements [6], or quantum information processing [7,8]. The nanomechanical oscillators can be coupled to the cavity modes in optical resonators and cooled through the sideband laser cooling [9,10]. For the sideband cooling, the bandwidth of the cavity mode needs to be narrow compared with the oscillation frequency of the mechanical oscillator to resolve the sidebands [9–11]. Impressive experimental progress has been reported along this direction, which pushes the mean phonon number to the order of 100 [12–15]. A technical factor that limits the current temperature of the oscillator is from the laser phase noise. The cooling laser is typically red detuned from the cavity, and its inevitable phase fluctuation will induce the photon number fluctuation in the cavity mode. This fluctuation is equivalent to a thermal bath coupled to the mechanical oscillator and seriously limits the temperature of the latter. If one assumes white noise model for the laser phase fluctuation, to achieve the ground-state cooling of the mechanical oscillator, the result estimate has shown that the laser bandwidth has to be extremely narrow, on the order of  $10^{-4}$ – $10^{-3}$  Hz, which is almost impossible to achieve in this configuration [16]. When one takes into account the final correlation time of the laser phase fluctuation, this requirement gets significantly relaxed [17]. However, under practical laser bandwidth, the estimated mean phonon number for the mechanical oscillator is still on the order of 10–100 [17], which is in agreement with the experimental observation [13–15]. This shows that the laser phase noise is still a major factor that limits the current temperature of the mechanical oscillator in experiments.

In this paper, we propose a cooling configuration to significantly reduce the influence of the laser phase noise. We exploit a configuration where the mechanical oscillator is coupled to two cavity modes, with the frequency splitting of the latter equal to the mechanical oscillator frequency. A laser is resonantly driving on the cavity mode with lower frequency. Because of anti-Stokes scattering, phonons in mechanical oscillator are transformed into photons in the other cavity mode with higher frequency. The photons leak out of the cavity and the mechanical oscillator is cooled down. If cavity decay rate  $\gamma$  is much less than the mechanical oscil-

lator frequency  $\omega_m$ , the same cooling rate can be realized with much lower driving power than single cavity mode schemes. With a detailed calculation, we show that the phase noise effects can be suppressed by  $(2\omega_m/\gamma)^2$  times. Besides, as long as the cooling laser driving strength  $\Omega_c$  is less than mechanical frequency  $\omega_m$ , the laser phase noise can be treated independent of the driving power. Similar configurations have been investigated in order to generate Einstein-Podolsky-Rosen (EPR) beams with very high entanglement in the room temperature [18], to optimize the energy transferring from phonon to photon in sideband cooling, to generate entanglement between phonons and photons [19,20], and to enhance the displacement sensitivity and the quantum back-action of mechanical oscillator [21]. Considering both phase noise and mechanical quality factor  $Q$  induced cooling limits, we find that it is possible to cool the mechanical oscillator down to the quantum regime by double cavity modes scheme under the present experimental conditions. At last, we discuss how to measure the mean thermal photon number of the oscillator by measuring the blue and the red sideband spectra. Similar to the sideband cooling of trapped ions [22,23], there will be a large imbalance between the blue and the red sideband output spectra, when the mechanical mode is cooled down to the quantum regime ( $\bar{n}_m < 1$ ).

As shown in Fig. 1, there are two cavity modes  $a_1$  and  $a_2$  involving in the cooling process. The frequencies of the modes are  $\omega_1$  and  $\omega_2$ , respectively. They are coupling with a mechanical mode  $a_m$  with frequency  $\omega_m$ . The condition  $\omega_2 - \omega_1 = \omega_m$  is fulfilled by tuning either the mechanical mode frequency or the cavity mode splitting. A laser is resonantly

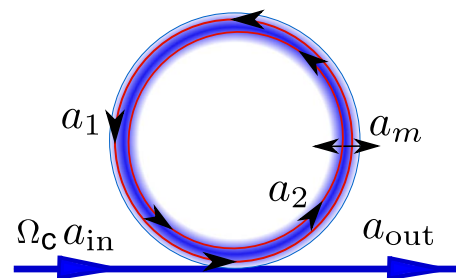


FIG. 1. (Color online) Double cavity mode scheme setup. There are two cavity modes  $a_1$  and  $a_2$  couple with a mechanical mode  $a_m$ .

driving on the cavity mode  $a_1$ . The present setup can be realized in Fabry-Perot cavities, due to the degeneracy of higher-order modes [19,20], or in microsphere cavities whose closely spaced azimuthal cavity modes or forward and backward cavity mode splitting is tuned to match the mechanical frequency [18,24–26]. The Hamiltonian of the system is  $H=H_0+H_L+H_I$  [9,10,21], where

$$H_0 = -\Delta_L a_1^\dagger a_1 + (-\Delta_L + \omega_m) a_2^\dagger a_2 + \omega_m a_m^\dagger a_m, \quad (1)$$

$$H_L = \frac{\Omega_c e^{i\phi}}{2} (a_1 + a_2) + \text{H.c.}, \quad (2)$$

$$H_I = \sum_{i,j=1,2} \eta \omega_m a_i^\dagger a_j (a_m^\dagger + a_m). \quad (3)$$

Here,  $\Omega_c$  is the driving strength of the cooling laser,  $\omega_L$  is the laser frequency,  $\eta$  is the coupling parameter between the cavity modes  $a_{1,2}$  and the mechanical mode  $a_m$ , and  $\phi$  is the random-phase noise [16]. The dimensionless parameter  $\eta$  is defined as  $\eta = (\omega_1/\omega_m)(x_m/R)$ , with  $x_m = \sqrt{\hbar/m\omega_m}$  as the zero-point motion of the mechanical resonator mode  $\omega_m$ ,  $m$  as its effective mass, and  $R$  as the cavity radius. In a typical system, the coupling constant  $\eta$  is on the order of  $10^{-4}$ . We denote detuning as  $\Delta_L = \omega_L - \omega_1$ . The cavity modes and the mechanical mode are all weakly dissipating with rates  $\gamma_1$ ,  $\gamma_2$ , and  $\gamma_m$ , which are much less than  $\omega_m$ . We get quantum Langevin equations

$$\dot{a}_j = -i[a_j, H] - \frac{\gamma_j}{2} a_j + \sqrt{\gamma_j} \phi_j, \quad \text{for } j = 1, 2, m. \quad (4)$$

The driving and the decay terms in Eq. (4) will be balanced when time approaches infinity. The system approaches a classical steady state plus a quantum fluctuation. The latter one is our main interest. To discuss the driving phase noise effects and the quantum fluctuations, we apply transformations  $a_j \rightarrow a_j e^{-i\phi}$  and  $a_j = \alpha_j + a_j$  for  $j=1, 2$ ,  $a_m = a_m + \beta$ , respectively, where  $\alpha_j$  and  $\alpha_m$  are the solutions of classical steady states and  $a_j$  and  $a_m$  are the quantum fluctuation operators. For the steady states, the following conditions need to be fulfilled:

$$i\Delta_L \alpha_1 - i\eta \omega_m (\alpha_1 + \alpha_2) (\beta + \beta^*) - \frac{\gamma_1}{2} \alpha_1 - i \frac{\Omega_c}{2} = 0,$$

$$-i(\omega_m - \Delta_L) \alpha_2 - i\eta \omega_m (\alpha_1 + \alpha_2) (\beta + \beta^*) - \frac{\gamma_2}{2} \alpha_2 - i \frac{\Omega_c}{2} = 0,$$

where  $\beta = -\eta |\alpha_1 + \alpha_2|^2$  and  $\Delta_L = \eta \omega_m (\beta + \beta^*)$ . Because  $\omega_m \gg \gamma_1$ , it is easy to find that  $|\alpha_1| \gg |\alpha_2|$ . We find that  $\beta \approx -\eta |\alpha_1|^2$ ,  $\alpha_1 \approx i\Omega_c/\gamma_1$ , and  $\alpha_2 = (\Omega_c + 2\eta^2 \omega_m \alpha_1 |\alpha_1|^2)/(2i\omega_m + \gamma_2)$ . We find  $\alpha_1/\alpha_2 \approx \gamma_1/(2\omega_m)$ . The Langevin equations (4) become

$$\dot{a}_2 = -\left(i\omega_m + \frac{\gamma_2}{2}\right) a_2 - i\eta \omega_m \alpha_1 (a_m^\dagger + a_m) + i\alpha_2 \dot{\phi} + \sqrt{\gamma_2} a_2^{\text{in}},$$

$$\dot{a}_m = -\left(i\omega_m + \frac{\gamma_m}{2}\right) a_m - i\eta \omega_m (\alpha_1 a_2^\dagger + \alpha_1^* a_2) + \sqrt{\gamma_m} a_m^{\text{in}}. \quad (5)$$

In order to get Eqs. (5), at first we neglect  $\alpha_2$  terms in the coupling strength because it is much less than  $\alpha_1$ . Then, as  $\eta(\beta + \beta^*)\omega = 2\eta^2 |\alpha_1|^2 \omega_m \ll \omega_m$ , we neglect the coupling between  $a_1$  and  $a_2$ . As  $\omega_m \gg \eta \omega_m$ , there is no effective coupling between  $a_1$  and  $a_m$  modes. Therefore, we neglect the  $a_1$  mode in Eq. (4).

The phase noise term in Eqs. (5) induces the photon number fluctuation, which heats the mechanical oscillator. Let us briefly discuss the heating effects. In order to make the phase noise effects more evident, we neglect the coupling between the thermal bath and the mechanical oscillator. In the limit  $\omega_m \gg \gamma_2 \gg \eta \omega_m \alpha_1$ , we can adiabatically eliminate the  $a_2$  mode and get

$$\dot{a}_m = -\frac{\tilde{\gamma}}{2} a_m + \sqrt{\tilde{\gamma}} a_2^{\text{in}} - \sqrt{\tilde{\gamma}} \frac{\alpha_2}{\sqrt{\gamma_2}} \dot{\phi}, \quad (6)$$

where  $\tilde{\gamma} = 4\eta^2 \omega_m^2 |\alpha_1|^2 / \gamma_2$ . The quantum noise term  $a_2^{\text{in}}$  comes from the vacuum bath with correlation  $\langle a_2^{\text{in}\dagger}(t) a_2^{\text{in}}(s) \rangle = \delta(t-s)$ . If we choose white noise model, the phase noise correlation is  $\langle \dot{\phi}(t) \dot{\phi}(s) \rangle = 2\Gamma_L \delta(t-s)$ , where  $\Gamma_L$  is the linewidth of the driving laser [16]. We can treat the phase noise term  $\dot{\phi}$  the same as the vacuum noise term  $a_2^{\text{in}}$ . In order to cool the oscillator down to the ground state, we need to make sure that the heating strength of the phase noise term is much less than the cooling effect of the vacuum noise term. Therefore, we find  $|\alpha_2|^2 \Gamma_L \ll \gamma_2$ , where  $|\alpha_2|^2 = n_2$  is the mean photon number in the cavity mode  $a_2$ . This condition is equivalent to the one in the single cavity mode cooling scheme [16], with the mean cavity photon number reduced by a factor  $(\gamma_1/2\omega_m)^2$ , leaving other parameters unchanged. In the resolved sideband regime,  $\gamma_1$  is much less than  $\omega_m$ . Therefore, the phase noise heating effect is suppressed by  $(2\omega_m/\gamma_1)^2$  times. If  $\Omega_c < \omega_m$ , the mean photon number in the cavity mode  $a_2$  is less than 1. The ground-state cooling condition becomes  $\Gamma_L \ll \gamma_2$ , which is the same as the one used in the sideband cooling of atoms. Besides, the same cooling rate can be realized by  $(\gamma_1/2\omega_m)^2$  times less the driving power than the single cavity mode scheme, which is consistent with the results in Refs. [19,20].

Now we briefly discuss the cooling limit related to the driving phase noise and the mechanical quality  $Q$  based on the current experimental conditions. The experimental available parameters are  $\Gamma_L \approx 10^3$  Hz,  $\gamma_{1,2}/2\pi \sim 1$  MHz, and  $\omega_m/2\pi \sim 100$  MHz [12]. Practically,  $\alpha_2$  is much less than  $\gamma_1/(\omega_m \eta) \sim 100$ . We choose proper laser driving power, which makes  $|\alpha_2|^2 < 10^3$ . We find that  $|\alpha_2|^2 \Gamma_L < \gamma_2$ . So, for the white noise model, the limitation of thermal phonon number is below 1, which is already in the quantum regime. To be more rigorous, we can choose Gaussian noise model with finite correlation time  $\gamma_c^{-1}$  other than white noise model with zero correlation time [17]. The correlation function of the phase noise is  $\langle \dot{\phi}(t) \dot{\phi}(s) \rangle = \Gamma_L \gamma_c e^{-\gamma_c |t-s|}$ . In Ref. [17], it was found that for the finite correlation noise model, the effects of the phase noise reduce by  $(\omega_m^2 + \gamma_c^2)/\gamma_c^2$  times, com-

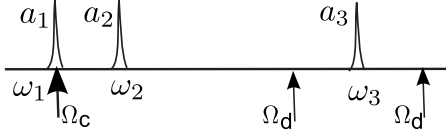


FIG. 2. Measurement setup.

pared with white noise model. In the limit  $\gamma_c \ll \omega_m$ , we can conclude that the phase noise effect is negligible at this time as  $|\alpha_2|^2 \Gamma \gamma_c^2 / (\omega_m^2 + \gamma_c^2) \ll \gamma_2$ .

The cooling limit is also related to the mechanical quality factor  $Q$ . It is found that the limit of cooling is  $n_{mf} > \gamma_m n_{mi} / \gamma_2 > n_{mi} / Q = k_B T / (\hbar \omega_m Q)$  [10,11], where  $T$  is the environment temperature,  $n_{mf}$  is the phonon number after laser cooling, and  $n_{mi}$  is the bath phonon number. In order to cool oscillator to quantum regime, we should make sure that the initial thermal phonon number  $n_{mi}$  is much less than  $Q$ . Therefore, it is necessary to either use high-frequency and high-quality  $Q$  oscillators or cool the environment temperature before laser cooling. Currently, the initial environment temperature is cooled down to 1.65 K and  $Q$  is about 2000 for mechanical oscillator with frequency  $\omega_m = 62$  MHz [14]. So the limit of  $n_{mf}$  is  $k_B T / (\hbar \omega_m Q) = 0.28$ . It is also found that  $Q \sim \omega_m / T^3$  for very low temperature [27]. Therefore,  $Q$  is about  $2 \times 10^4$  for the temperature around 600 mK, which is still possible for  $^3\text{He}$  cooling. The limit of  $n_{mf}$  could be 0.01. Combining the cooling limit set up by phase noise effects and the mechanical quality factor  $Q$ , we conclude that the present scheme greatly decreases phase noise effects and makes cooling optomechanical oscillator down to the quantum regime possible based on the current experimental conditions.

To verify the ground-state cooling of the mechanical oscillator, we need to directly measure the mean thermal phonon number  $n_{mf}$ . Although the phonon number can be measured by displacement noise spectrum [12–15], here, we propose another measurement scheme by measuring the output light intensity. We will compare the two schemes later. As shown in Fig. 2, we choose the third cavity mode  $a_3$  with frequency  $\omega_3$ . By weakly driving the red and the blue detuning sidebands of the cavity mode  $a_3$ , we can measure the mean thermal phonon number after the sideband cooling. The measurement can be processed simultaneously and independently with the sideband cooling. The measurement scheme is similar to the one used in ion trap [22,23]. However, in the present setup, we need to make sure that the measurement process has negligible effect on the cooling process. We will derive the conditions of the driving laser strength. The Hamiltonian involved with the measurement is

$$H_M = -\Delta_{L'} n_3 + \omega_m n_m + \left( \frac{\Omega_d}{2} a_3 + \text{H.c.} \right) + \eta \omega_m n_3 (a_m + a_m^\dagger), \quad (7)$$

where  $n_3 = a_3^\dagger a_3$  and  $n_m = a_m^\dagger a_m$ ,  $\Omega_d$  is the driving strength of the detection laser, and  $\Delta_{L'} = \omega_{L'} - \omega_3$  is the detuning between the driving laser and the cavity mode  $a_3$ . We suppose that  $a_3$  weakly decays with the rate  $\gamma'_3$ . The Langevin equations are similar to Eq. (4) by replacing  $H$  with  $H_M$ . We apply the

transformation  $a_3 = a_3 + \alpha_3$  and  $a_m = a_m + \beta'$ . The classical steady state satisfies

$$\begin{aligned} -\Delta_{L'} \alpha_3 - i \eta \omega_m \alpha_3 (\beta' + \beta'^*) - \frac{\gamma'_3}{2} \alpha_3 - i \frac{\Omega_d}{2} &= 0, \\ \Delta_{L'} + 2 \eta^2 \omega_m |\alpha_3|^2 &= -\omega_m, \\ \beta' &= -\eta |\alpha_3|^2. \end{aligned}$$

Here, we choose  $\Delta_{L'} + 2 \eta^2 \omega_m |\alpha_3|^2 = -\omega_m$ , which represents the blue sideband driving. In the limit  $|\alpha_3|^2 \gg |\langle a_3 \rangle|^2$ , we can linearize the Langevin equations as

$$\dot{a}_j = -i \eta \omega_m \alpha_3 (a_k + a_k^\dagger) - i \omega_m a_j - \frac{\gamma'_j}{2} a_j + \sqrt{\gamma'_j} a_j^{\text{in}}, \quad (8)$$

with  $j, k = 3, m$ . Here, we suppose that the mechanical oscillator couples with an effective thermal bath with mean thermal number  $n_{mf}$  and effective coupling strength  $\gamma'_m$  when laser cooling is spontaneously processing. When the quantum regime approaches, the effective coupling strength  $\gamma'_m = \gamma_m + \tilde{\gamma}$ , where  $\tilde{\gamma}$  is defined in Eq. (6). Before continuing, we need to make sure that the classical steady state exists. Therefore, the Routh-Hurwitz criterion must be fulfilled [28],

$$\begin{aligned} 2 \gamma'_m \gamma'_3 \{ (\gamma'_3{}^2 + 4 \omega_m^2) \gamma'_3{}^2 + \gamma'_m [(\gamma'_m + 2 \gamma'_3)(\gamma'_3{}^2 + \omega_m^2) \\ + 2 \gamma'_3 \omega_m^2] \} > \omega_m^2 (\eta |\alpha_3| \omega_m)^2 (\gamma'_m + 2 \gamma'_3)^2. \end{aligned}$$

In the limit  $\gamma'_m \ll \gamma'_3 \ll \omega_m$ , we find the condition  $2 \gamma'_m \gamma'_3 > \eta^2 \omega_m^2 |\alpha_3|^2$ .

We change the energy reference by transformation  $a_3 \rightarrow e^{-i \omega_m t} a_3$  and  $a_m \rightarrow e^{-i \omega_m t} a_m$ . In the limit  $\omega_m \gg \gamma'_3, \gamma'_m, \eta \alpha_3 \omega_m$ , Eq. (8) can be simplified by the rotating wave approximation

$$\dot{a}_j = -i \eta \omega_m \alpha_3 a_k^\dagger - \frac{\gamma'_j}{2} a_j + \sqrt{\gamma'_j} a_j^{\text{in}}, \quad (9)$$

with  $j, k = 3, m$ . We define the cavity or mechanical operator in the frequency domain by Fourier transformation  $a(t) = \frac{1}{\sqrt{2\pi}} \int a(\omega) e^{-i \omega (t-t_0)} d\omega$ . With standard method [29], we can solve the Langevin equations (9) and get

$$a_3(\omega) = \frac{\sqrt{\gamma'_3}}{\Delta(\omega)} \left( \frac{\gamma'_m}{2} - i \omega \right) a_3^{\text{in}}(\omega) + \frac{i \eta \omega_m \alpha_3}{\Delta(\omega)} \sqrt{\gamma'_m} a_m^{\text{in} \dagger}(-\omega),$$

where  $\Delta(\omega) = [(\gamma'_m/2) - i \omega_m][(\gamma'_3/2) - i \omega] - \eta^2 \omega_m^2 \alpha_3^2$ . We calculate the output mode by the boundary condition  $a_3^{\text{in}} + a_3^{\text{out}} = \sqrt{\gamma'_3} a_3$ ,

$$\begin{aligned} a_3^{\text{out}}(\omega) &= \left[ -1 + \frac{\gamma'_3}{\Delta} \left( \frac{\gamma'_m}{2} - i \omega \right) \right] a_3^{\text{in}}(\omega) \\ &\quad + \frac{i \eta \omega_m \alpha_3 \sqrt{\gamma'_m \gamma'_3}}{\Delta} a_m^{\text{in} \dagger}(-\omega), \end{aligned}$$

We suppose that the mechanical oscillator is continuously cooled when the measurement is processed. The cooling results can be treated as an effective thermal bath with mean phonon number  $n_{mf}$ . Therefore, we have  $\langle a_m^{\text{in} \dagger}(-\omega) a_m^{\text{in}}(\omega') \rangle = n_{mf} \delta(\omega - \omega')$  and  $\langle a_m^{\text{in}}(\omega) a_m^{\text{in} \dagger}(-\omega') \rangle = (n_{mf} + 1) \delta(\omega - \omega')$ . The peak strength of the output field is

$$I_b = \langle a_3^{\text{out}\dagger}(0) a_3^{\text{out}}(0) \rangle = \frac{\eta^2 \omega_m^2 \alpha_3^2}{\Delta^2(0)} \gamma'_m \gamma'_3 (n_{mf} + 1).$$

Similarly, if we choose diving at the red sideband with  $\Delta_L + 2\eta^2 \omega_m |\alpha_3|^2 = \omega_m$  and with the same driving power, the peak strength of the output field is

$$I_r = \langle a_3^{\text{out}\dagger}(0) a_3^{\text{out}}(0) \rangle = \frac{\eta^2 \omega_m^2 \alpha_3^2}{\Delta'^2(0)} \gamma'_m \gamma'_3 n_{mf}.$$

where  $\Delta'(\omega) = [(\gamma'_m/2) - i\omega][(\gamma'_3/2) - i\omega] + \eta^2 \omega_m^2 \alpha_3^2$ . In the limit  $(\eta \omega_m \alpha_3)^2 \ll \gamma'_m \gamma'_3 / 8$  (the stable condition  $2\gamma'_m \gamma'_3 > \eta^2 \omega_m^2 |\alpha_3|^2$  is automatically fulfilled), we get  $\Delta(0) \approx \Delta'(0)$ . The ratio between the red and the blue sideband output central peak strengths is  $I_r/I_b = n_{mf}/(n_{mf} + 1)$ . Therefore, we can measure the final thermal phonon number by measuring the ratio of two sideband field strengths. If we can cool the mechanical mode to the ground state with  $n_{mf} \rightarrow 0$ , we will find that the ratio  $I_r/I_b$  approaches zero.  $\alpha_3$  is on the order of 10 for practical parameters [12], which is much less than the cooling field amplitude  $\alpha_1 \sim 10^3$  or more. Therefore, the measurement has negligible effects on the cooling process.

Before conclusion, we compare the thermal phonon measurement schemes between ours and those used in the current experiments [12–15]. The currently used measurement schemes compare the initial and the final displacement noise spectra and get the final thermal phonon number. Therefore, the bath temperature is needed to calculate the final thermal phonon number. The measurement precision is related to the bath temperature measurement and noise spectrum measurement precision [15]. Because of background noise, the scheme is less and less precise when the system approaches

the quantum regime. In the scheme that we proposed in this paper, both the noise spectrum and the bath temperature are not needed to get  $n_{mf}$ . We only need to measure the output red and blue sideband intensity spectra. Besides, our scheme is reliable only when  $n_{mf}$  is comparable to 1. Therefore, our scheme is much more accurate in the quantum regime than in the classical regime. In fact, the large imbalance between the red and the blue sideband spectra is the direct signal that the oscillator is cooled down to the quantum regime.

In conclusion, we have proposed a double cavity modes scheme to eliminate the driving phase noise in sideband cooling of optomechanical oscillators. We show that phase noise effects are suppressed by  $(2\omega_m/\gamma)^2$  times. The cooling limit from the laser phase noise is already in the quantum regime for the present experimental parameters. Combining the limits by the phase noise and the mechanical quality factor  $Q$ , we conclude that it is possible to cool down to the quantum regime at present. At last, we discuss how to detect the thermal phonon number by measuring the red and the blue sideband spectra when the mechanical oscillator is cooled near the quantum regime. We specify the required conditions for this measurement. We compare the measurement scheme to the currently used ones.

*Note added.* Recently, we found that a related work was published [30], which rigorously discussed how to suppress the phase noise by the double cavity modes scheme.

We thank Lu-ming Duan for helpful discussions and valuable comments on the paper, and Xiao-shun Jiang and Tongcang Li for useful suggestions. Z.Y. was supported by the Government of China through CSC (Contract No. 2007102530).

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