Entanglement production due to quench dynamics of an anisotropic *XY* chain in a transverse field

K. Sengupta[1,2] and Diptiman Sen[3]

[1]*Theoretical Physics Division, Indian Association for the Cultivation of Sciences, Jadavpur, Kolkata 700032, India*
[2]*TCMP Division, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India*
[3]*Center for High Energy Physics, Indian Institute of Science, Bangalore 560012, India*
(Received 10 April 2009; revised manuscript received 5 June 2009; published 2 September 2009)

We compute concurrence and negativity as measures of two-spin entanglement generated by a power-law quench (characterized by a rate τ^{-1} and an exponent α) which takes an anisotropic *XY* chain in a transverse field through a quantum critical point (QCP). We show that only spins separated by an even number of lattice spacings get entangled in such a process. Moreover, there is a critical rate of quench, τ_c^{-1}, above which no two-spin entanglement is generated; the entire entanglement is multipartite. The ratio of the entanglements between consecutive even neighbors can be tuned by changing the quench rate. We also show that for large τ, the concurrence (negativity) scales as $\sqrt{\alpha/\tau}\,(\alpha/\tau)$, and we relate this scaling behavior to defect production by the quench through a QCP.

I. INTRODUCTION

The role of entanglement in the theory of quantum phase transitions has been a subject of recent studies [1]. A number of such works have pointed out that entanglement can be used as a tool to characterize quantum phase transitions for both clean and disordered systems [2–6]. These works computed either single-site (or single block) von Neumann entropies or two-spin entanglement measures such as concurrence [7] and negativity [8] and demonstrated that these measures or their derivatives display a peak (discontinuous jump) either near or at the second-order (first-order) quantum critical point (QCP). They can therefore serve as tools for identifying quantum phase transitions in equilibrium quantum critical systems. These studies provide a bridge between quantum information theory and equilibrium quantum critical phenomena which can be useful for several aspects of quantum computations, cryptography, and teleportation [9]. However, in all of these studies, the computed entanglement receives contribution from the ground state of the systems alone; excited states are not probed.

Recently, enormous progress has been made in understanding defect production due to nonequilibrium dynamics of a system passing through a critical point [10–13]. In particular, it was shown that for a slow linear quench through a QCP, the defect density n scales with the quench time τ with an universal exponent: $n \sim \tau^{-\nu d/(z\nu+1)}$, where ν and z are the correlation length and dynamical critical exponents associated with the phase transition and d is the system dimension [11,13]. Such results have been extended to cases where the quench takes the system through a quantum critical surface [14], along a gapless line or a multicritical point [15], and for nonlinear power-law quenches [16]. More recently, the two-spin entanglement properties of a quantum system during time evolution after a sudden quench through a critical point have been studied [17]. However, the nature of two-spin entanglement generation due to a finite rate of quench through a quantum critical point has not been investigated so far although other kinds of entanglement have been studied [18].

In this work, we compute concurrence and negativity as measures of two-spin entanglement generated by a power-law quench characterized by a rate τ and an exponent α, which takes a spin-1/2 *XY* chain in a transverse field through a QCP. Our central results are as follows. First, we show that, in contrast to the studies of entanglement in anisotropic *XY* chains so far [2], such a quench generates entanglement only between spins separated by an even number of lattice spacings (even neighbors); the nearest-neighbor sites are not entangled. Second, a critical quench rate τ_c^{-1} is required to generate two-spin entanglement (unlike other kinds of entanglement which do not seem to require a critical quench rate [18]). For faster quench rates ($\tau < \tau_c$), there is no entanglement between any pair of sites, and the entire entanglement is multipartite which is rather unusual. Third, by tuning the quench rate, one can control the amount of entanglement and tune the ratio of entanglements between a spin and its consecutive even neighbors to take values between zero and 1. Finally, for large quench time τ and a given power α, the concurrence (negativity) scales as $\sqrt{\alpha/\tau}\,(\alpha/\tau)$; this scaling is directly related to defect production by the quench through the QCP. To the best of our knowledge, the scaling behavior of the two-spin entanglement generated by a slow quench through a QCP and its selective generation by tuning the quench rate have not been reported so far. Hence our study constitutes a significant extension of our current understanding of the nature of entanglement in many-body systems. This study may be relevant for the generation of entanglement as a resource for many algorithms of quantum computations, cryptography, and teleportation [9].

The organization of the rest of this paper is as follows. In Sec. II, we compute concurrence and negativity as measures of entanglement for the *XY* model. This is followed by a discussion of our main results in Sec. III. We end with some concluding remarks in Sec. IV.

II. MEASURES OF ENTANGLEMENT

We begin with the Hamiltonian of the spin-1/2 anisotropic *XY* spin chain given by

$$H = \frac{J}{4} \sum_n \left[(1+\gamma) \sigma_n^x \sigma_{n+1}^x + (1-\gamma) \sigma_n^y \sigma_{n+1}^y + h \sigma_n^z \right], \quad (1)$$

where σ^a for $a=x,y,z$ are the Pauli matrices, $J[1+(-)\gamma]/2$ are interaction strengths between $x(y)$ components of the nearest-neighbor spins (we set the lattice spacing $d=1$), h is the magnetic field in units of J [we henceforth set $\hbar=J=1$, and measure all energies (times) in units of J ($\hbar/J$)], and γ is the anisotropy parameter which varies between 0 (isotropic XY chain) and 1 (Ising limit). The phase diagram of the above model can easily be obtained by mapping the spins to fermions via a Jordan-Wigner transformation: $\sigma_i^z = n_i - 1/2$ and $c_i(c_i^\dagger) = \Pi_{j=-\infty}^{i-1} \sigma_j^z (-1)^i \sigma_i^-(\sigma_i^+)$ [19,20]. The fermions can be shown to have an effective two-level Hamiltonian for each pair of momenta $\pm k$ in terms of the states $|0\rangle$ and $|k,-k\rangle = c_k^\dagger c_{-k}^\dagger |0\rangle$ given by $H_k = -[h+2\cos k](I+\tau_z) - 2\tau_y \gamma \sin k$ [19,20], where I, τ_z, and τ_y denote identity and Pauli matrices in the $|0\rangle, |k,-k\rangle$ space. The equilibrium phase diagram for the model is well known; for $|h|>1$, there is a paramagnetic phase with $\langle \sigma_i^x \rangle = 0$, while for $|h|<1$, one finds a ferromagnetic phase with $\langle \sigma_i^x \rangle \neq 0$. At $h=\pm 1$, there is a second-order quantum phase transition with $z=\nu=1$. At the transition at $h=1(-1)$, the fermionic modes are gapless for $k=0(\pi)$. The quench dynamics of the model, for a power-law time variation in the magnetic field $h(t)=h|t/\tau|^\alpha \text{sgn}(t)$, where sgn is the signum function, has also been studied [16,20]. The quench starts with all spins down at $t \to -\infty$ (i.e., the state $|0\rangle$ for all k) and ends at $t \to \infty$ with a state in which the probabilities of $|0\rangle$ and $|k,-k\rangle$ are given by p_k and $1-p_k$, respectively. Here p_k is the defect formation probability and is given by $p_k = \exp(-\pi \tau_{\text{eff}} \gamma^2 \sin^2 k)$, where $\tau_{\text{eff}} = \tau/\alpha$ [16,20].

Armed with these results, we now compute the concurrence and negativity as measures of the two-spin entanglement of the spin chain generated by the quench. We note at the outset that the ground states of the initial and final Hamiltonians, at the beginning and end of such a quench process, are paramagnetic and do not posses any two-spin entanglement. Thus we expect that both for very fast ($\tau \to 0$) and very slow ($\tau \to \infty$) quenches, where the system retains information only about the initial and final ground states, the two-spin entanglement will vanish. Hence any finite entanglement obtained after such a quench with a finite rate τ must be generated by the nonadiabatic quench process and must therefore have contributions from excited states of the system. To compute the concurrence and negativity, we first note that the two-spin density matrix of the spin chain for any two sites i and $j=i+n$ is given by [21]

$$\rho^n = \begin{pmatrix} a_+^n & 0 & 0 & b_1^n \\ 0 & a_0^n & b_2^n & 0 \\ 0 & b_2^{n*} & a_0^n & 0 \\ b_1^{n*} & 0 & 0 & a_-^n \end{pmatrix}, \quad (2)$$

where the matrix elements $a_\pm^n$, a_0^n, and $b_{1,2}^n$ can be expressed in terms of the two-spin correlation functions

$$a_\pm^n = \left\langle \frac{1}{4}(1 \pm \sigma_i^z)(1 \pm \sigma_{i+n}^z) \right\rangle, \quad (3)$$

$$a_0^n = \left\langle \frac{1}{4}(1 \pm \sigma_i^z)(1 \mp \sigma_{i+n}^z) \right\rangle, \quad (4)$$

$$b_{1(2)}^n = \langle \sigma_i^- \sigma_{i+n}^{-(+)} \rangle. \quad (5)$$

The symmetry under $\sigma_i^x \to -\sigma_i^x$, $\sigma_n^y \to -\sigma_n^y$, $\sigma_i^z \to \sigma_i^z$ ensures that all correlation functions such as $\langle \sigma_i^\pm \sigma_{i+n}^z \rangle$ and hence the remaining matrix elements are zero. The nonzero correlation functions for an arbitrary nonlinear quench can be computed by generalizing the method developed for a linear quench in Ref. [20]. We define

$$\alpha_n = \int_0^\pi \frac{dk}{\pi} p_k \cos(nk), \quad (6)$$

and note that since p_k is invariant under $k \to \pi-k$, $\alpha_n=0$ when n is odd. In terms of α_n, the diagonal correlation functions are given by

$$\langle \sigma_i^z \rangle = 1 - 2\alpha_0, \quad (7)$$

$$\langle \sigma_i^z \sigma_{i+n}^z \rangle = \langle \sigma_i^z \rangle^2 - 4\alpha_n^2. \quad (8)$$

Thus, for any two spins separated by odd number of lattice spacings, $\langle \sigma_i^z \sigma_{i+n}^z \rangle = \langle \sigma_i^z \rangle^2$ [20]. The off-diagonal correlators $\langle \sigma_i^a \sigma_{i+n}^b \rangle$ (where a, b can take the values $+$, $-$) can also be computed in terms of α_n. We shall present explicit expressions for these for $n \leq 6$ and provide a qualitative discussion for large n later. We find that $\langle \sigma_i^\pm \sigma_{i+n}^\pm \rangle = b_1^n = 0$ for all n since these involve correlations between two fermionic annihilation or creation operators and hence vanish. Further, $\langle \sigma_i^\pm \sigma_{i+n}^\mp \rangle = b_2^n = 0$ for all odd n since these are odd under the transformation $\sigma_n^x \to (-1)^n \sigma_n^x$, $\sigma_n^y \to (-1)^n \sigma_n^y$, $\sigma_n^z \to \sigma_n^z$ which changes $J_{x,y} \to -J_{x,y}$ and leaves p_k invariant. For even $n \leq 6$, we find

$$\langle \sigma_i^+ \sigma_{i+2}^- \rangle = \alpha_2 \langle \sigma_i^z \rangle, \quad (9)$$

$$\langle \sigma_i^+ \sigma_{i+4}^- \rangle = (\alpha_4 \langle \sigma_i^z \rangle - 2\alpha_2^2) \langle \sigma_i^z \sigma_{i+2}^z \rangle, \quad (10)$$

$$\langle \sigma_i^+ \sigma_{i+6}^- \rangle = [\alpha_6 \langle \sigma_i^z \sigma_{i+2}^z \rangle + 4\alpha_2(\alpha_2^2 + \alpha_4^2 - \alpha_4 \langle \sigma_i^z \rangle)] \\ \times \{\langle \sigma_i^z \rangle [\langle \sigma_i^z \sigma_{i+2}^z \rangle - 4(\alpha_2^2 + \alpha_4^2)] + 16\alpha_2^2 \alpha_4\}. \quad (11)$$

Using these correlation functions, we can find all the nonzero matrix elements of ρ^n for $n \leq 6$ and hence compute the concurrence and negativity. The concurrence is given by $\mathcal{C}^n = \max\{0, \sqrt{\lambda_1^n} - \sqrt{\lambda_2^n} - \sqrt{\lambda_3^n} - \sqrt{\lambda_4^n}\}$, where λ_i^ns are the eigenvalues of $\rho^n(\sigma^y \otimes \sigma^y \rho^{n*} \sigma^y \otimes \sigma^y)$ in decreasing order [7]. The $\sqrt{\lambda_i^n}$ are given by $\sqrt{a_+^n a_-^n}$ (appearing twice) and $a_0^n \pm |b_2^n|$. Thus the spin chain has a nonzero concurrence if $|b_2^n| > \sqrt{a_+^n a_-^n}$ given by

$$\mathcal{C}^n = \max\{0, 2(|b_2^n| - \sqrt{a_+^n a_-^n})\}. \quad (12)$$

To compute the negativity $\mathcal{N}^n$, we need to take a partial transpose of ρ^n with respect to the labels corresponding to the site $j=i+n$ in Eq. (2) [8]. This interchanges $b_1^n \rightleftarrows b_2^n$; the eigenvalues of the resultant matrix $\bar{\rho}^n$ are given by $\tilde{\lambda}_0^n = a_0^n$ (appearing twice) and $\tilde{\lambda}_\pm^n = (1/2)[a_+^n + a_-^n$
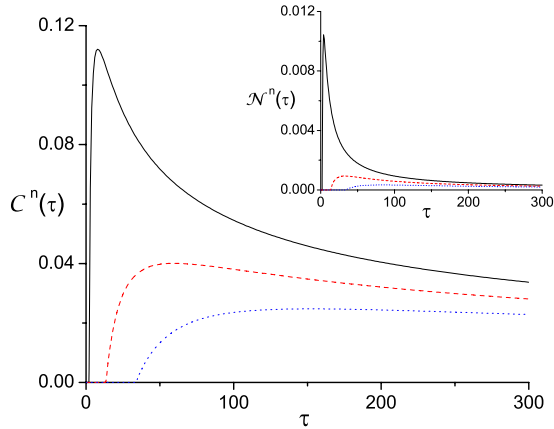
FIG. 1. (Color online) Plots of $\mathcal{C}^n$ as a function of τ for $n=2$ (black solid line), $n=4$ (red dashed line) and $n=6$ (blue dotted line) and for $\gamma=\alpha=1$. The inset shows analogous plots for $\mathcal{N}^n$.

$\pm\sqrt{(a_+^n-a_-^n)^2+4|b_2^n|^2}]$ of which only $\tilde{\lambda}_-^n$ can become negative. This happens when $|b_2^n|>\sqrt{a_+^n a_-^n}$ and yields

$$\mathcal{N}^n=\max\{0,|\tilde{\lambda}_-^n|\}. \tag{13}$$

In the next section, we shall discuss the implications of these measures of entanglement in the context of *XY* model in a transverse field.

III. RESULTS

Equations (12) and (13) are the central results of this work which lead to several conclusions about the two-spin entanglement generated due to the quench for *XY* model in a transverse field. First, for odd n, $\langle\sigma_i^z\sigma_j^z\rangle=\langle\sigma_i^z\rangle^2$ and $\langle\sigma_i^+\sigma_j^-\rangle=0$. Thus all eigenvalues of $\rho^n(\sigma^y\otimes\sigma^y\rho^{n*}\sigma^y\otimes\sigma^y)$ are equal to $\alpha_0(1-\alpha_0)$ leading to $\mathcal{C}^n=0$. All the eigenvalues of $\bar{\rho}$ are also positive; hence $\mathcal{N}^n=0$. Thus the quench generates entanglement only between the even neighbor sites. Second, for large τ_{eff} and $n\ll\sqrt{\tau_{\mathrm{eff}}}$, α_n scales similarly to the defect density: $\alpha_n\sim\sqrt{\alpha/\tau}$. Using this, we find from Eqs. (7)–(11) that $b_2^n\sim\sqrt{\alpha/\tau}$ and $a_+^n a_-^n\sim\alpha/\tau$ which leads to $\mathcal{C}^n\sim\sqrt{\alpha/\tau}$ and $\mathcal{N}^n\sim\alpha/\tau$. Thus, for slow quenches, $\mathcal{C}^n$ ($\mathcal{N}^n$) scales with the same (twice the) universal exponent as the defect density [11,13,16]. This result relates two-spin entanglement generation for slow quenches to defect production by such a process. Third, both the concurrence and the negativity become nonzero for a finite critical quench rate $(\tau_c^n)^{-1}$ (which is the solution of $|b_2^n|^2=a_+^n a_-^n$) above which there is no entanglement between a site and its nth neighbor. Solving this equation numerically, we find $\gamma^2\tau_c^n=1.96$, 13.6, and 33.8 for $n=2$, 4, and 6 when $\alpha=1$. We shall shortly see that for $\tau\leq\tau_c^2$, the entanglement is entirely multipartite.

We now plot $\mathcal{C}^n$ and $\mathcal{N}^n$ as a function of τ for $n=2,4,6$ in Fig. 1 for $\gamma=\alpha=1$. From Fig. 1, we find that $\mathcal{C}^n$ and $\mathcal{N}^n$ becomes nonzero between $\tau=\tau_c^n$ and ∞. Further, the ratios $\mathcal{C}^4/\mathcal{C}^2$ or $\mathcal{N}^4/\mathcal{N}^2$ can be selectively tuned between zero and 1 by tuning τ. The maximum values of both $\mathcal{C}^n$ and $\mathcal{N}^n$ decrease rapidly with n. For large $n\gg\sqrt{\tau}$, using the properties of the Toeplitz determinants used to compute the spin
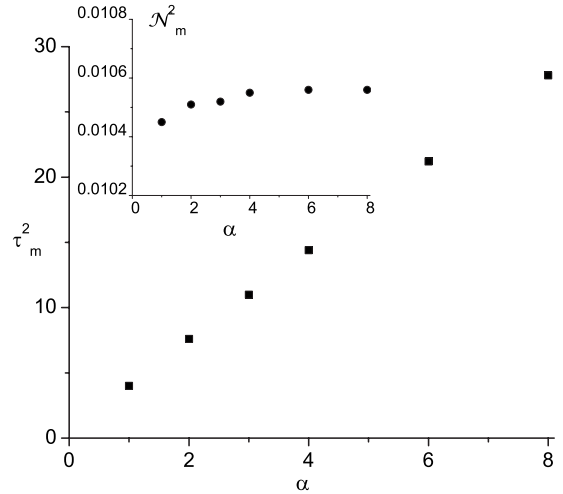

FIG. 2. Variation of the position, τ_m^2, of the maxima of $\mathcal{N}^2$ as a function of α. The inset shows the values of the maxima, $\mathcal{N}_m^2$, as a function of α.

correlators in these systems, it can be shown that $\langle\sigma_i^+\sigma_{i+n}^-\rangle\sim\exp(-n/\sqrt{\tau})$ [20]. Thus we expect the entanglement to vanish exponentially for $n\gg\sqrt{\tau}$. In Fig. 2, we plot the positions τ_m^2 and the magnitudes $\mathcal{N}_m^2$ of the peaks in $\mathcal{N}^2$ as a function of α. We find that τ_m^2 varies linearly with α while $\mathcal{N}_m^2$ are independent of α. This behavior can be understood by noting that $\mathcal{C}^n$ and $\mathcal{N}^n$ depend on α through τ_{eff}. The peak of $\mathcal{C}^n$ ($\mathcal{N}^n$) occurs when $d\mathcal{C}^n/d\tau=\alpha^{-1}d\mathcal{C}^n/d\tau_{\mathrm{eff}}=0$ ($d\mathcal{N}^n/d\tau=\alpha^{-1}d\mathcal{N}^n/d\tau_{\mathrm{eff}}=0$). Thus the position of the maxima, which is a solution to this equation, is given by $\tau_{m\,\mathrm{eff}}^n=\alpha/\tau_m^n$ and hence varies linearly with α. The maximum value of the entanglement $\mathcal{C}_m^n$ ($\mathcal{N}_m^n$) is a function of $\tau_{m\,\mathrm{eff}}^n$ alone; it does not change with α.

Next, we study the time evolution of entanglement by computing $\mathcal{N}^n$ at different times t during the quench for $\alpha=\gamma=1$ and $n=2$. The plots of the scaled negativity $\mathcal{N}^2(t)\tau$ as a function of the scaled time $t/\sqrt{\tau}$ is shown in Fig. 3 for several τ. From this plot, we find that for large enough τ, the generation of the entanglement starts when the system is near
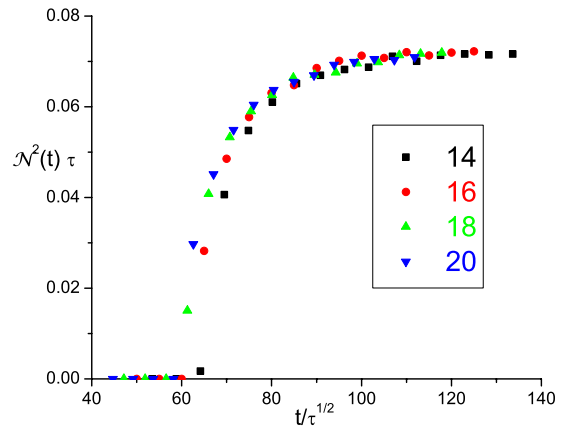

FIG. 3. (Color online) Plot of scaled negativity $\mathcal{N}^2(t)\tau$ as a function of $t/\sqrt{\tau}$ for $\alpha=\gamma=1$ and several $\tau=14,16,18,20$, showing the time evolution of entanglement generated by the quench which started at $t_i=-250$.
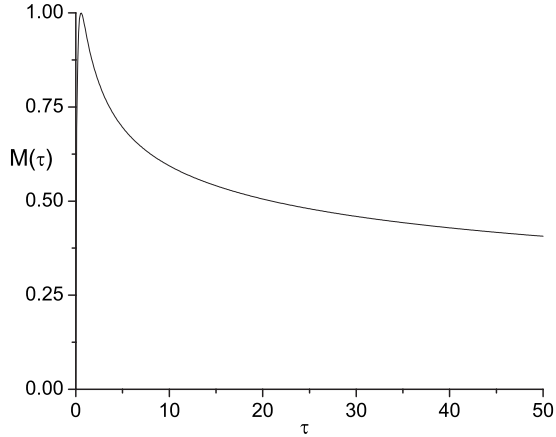
FIG. 4. Plot of M as a function of τ showing the evolution of the multipartite part of the entanglement with the quench time.

the QCP and reaches an asymptotic value $\mathcal{N}^2$ as the quench takes the system away from the quantum critical point. The collapse of all the asymptotic values of the negativity to the same curve for all τ clearly confirms the scaling behavior of the entanglement discussed earlier. Similar characteristics hold for $\mathcal{N}^{4,6}$.

We now discuss the nature of the entanglement generated by the quench. We note that the single-site density matrix of the spin chain is $\rho_i = (I + \sigma^z \langle \sigma_i^z \rangle)/2$. Using Eqs. (3)–(5), we find that the single-site concurrence $\mathcal{C}^{(1)} = \sqrt{4 \det \rho_i}$ $= 2\sqrt{\alpha_0(1-\alpha_0)}$ [22] is finite for all τ except $\tau = 0, \infty$ for which $\alpha_0 = 1, 0$. Since $\mathcal{C}^{(1)}$ is a measure of the entanglement of spin i with all the other spins in the chain, we conclude that the entanglement generated when $\tau \lesssim \tau_c^2$ is entirely multipartite. The multipartite part of the entanglement for any τ can be quantified as [22]

$$M = (\mathcal{C}^{(1)})^2 - \sum_{n \geq 2} (\mathcal{C}^n)^2. \qquad (14)$$

A plot of M as a function of τ is shown in Fig. 4, where we have summed C_n for $n \leq 6$. We find that M decreases rather slowly; hence a determination of the fate of M for large τ appears to be a rather difficult task. We note that for large τ, $(\mathcal{C}^{(1)})^2 \to 0$ as $1/\sqrt{\tau}$, while $(\mathcal{C}^n)^2 \to 0$ as $1/\tau$ for $\tau \gg n^2$. Hence

M may either go to 0 as a power of τ as $\tau \to \infty$ or may vanish at some large τ beyond which all the entanglement becomes bipartite. Differentiating between these two possibilities is left as a subject of future study.

An experimental verification of our theory would involve measuring the two-spin correlation functions of an anisotropic XY chain after performing a quench. There are several compounds such as $K_3Fe(CN)_6$ $(J \simeq -0.23$ K), $(NH_4)_2MnF_5$ $(J \simeq -12$ K), and $RbFeCl_3 \cdot 2H_2O$ $(J \simeq -35$ K) where the Ising limits of these chains are realized [23]. Similar experiments, involving measurement of two-spin correlation functions in equilibrium using neutron scattering, have recently been carried out for square-lattice antiferromagnets, where short-range entanglement between spins has been demonstrated [24].

IV. CONCLUSIONS

To conclude, we have shown that a controlled amount of entanglement can be generated in an anisotropic XY spin chain by performing a power-law quench; such a process generates two-spin entanglement only between sites which are even neighbors. The generated entanglement is entirely multipartite when the quench is faster than a critical rate; such states which have only multipartite entanglement are quite uncommon. The entanglement between even neighbors shows a scaling behavior for slow quenches, similar to the scaling of defects. We note here that a similar calculation can be performed for the two-dimensional Kitaev model [25] since the exact two-spin correlations functions of the model during a quench process with an arbitrary rate τ have been computed in Ref. [14]. Such a calculation shows that the Kitaev model has zero bipartite entanglement ($\mathcal{C}^n = \mathcal{N}^n = 0$ for all n) for all quench times τ, and the entire entanglement is always multipartite. This leaves us with the question of the relation between bipartite entanglement generated during a quench and the integrability and number of conserved quantities of the underlying system; this would be an interesting subject for future studies.

[1] A. Osterloh, L. Amico, G. Falci, and R. Fazio, Nature (London) **416**, 608 (2002); T. J. Osborne and M. A. Nielsen, Phys. Rev. A **66**, 032110 (2002); R. Somma, G. Ortiz, H. Barnum, E. Knill, and L. Viola, *ibid.* **70**, 042311 (2004).

[2] G. Vidal, J. I. Latorre, E. Rico, and A. Kitaev, Phys. Rev. Lett. **90**, 227902 (2003); L.-A. Wu, M. S. Sarandy, and D. A. Lidar, *ibid.* **93**, 250404 (2004); X. Jia, A. R. Subramaniam, I. A. Gruzberg, and S. Chakravarty, Phys. Rev. B **77**, 014208 (2008); R. Orus, Phys. Rev. Lett. **100**, 130502 (2008).

[3] L. Amico, R. Fazio, A. Osterloh, and V. Vedral, Rev. Mod. Phys. **80**, 517 (2008).

[4] M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen De, and U. Sen, Adv. Phys. **56**, 243 (2007).

[5] A. Sen(De), U. Sen, J. Dziarmaga, A. Sanpera, and M. Lewenstein, Phys. Rev. Lett. **101**, 187202 (2008); H. Wichterich, J. Molina-Vilaplana, and S. Bose, Phys. Rev. A **80**, 010304(R) (2009).

[6] A. Tribedi and I. Bose, Phys. Rev. A **79**, 012331 (2009); **77**, 032307 (2008); **75**, 042304 (2007); I. Bose and E. Chattopadhyay, *ibid.* **66**, 062320 (2002).

[7] S. Hill and W. K. Wootters, Phys. Rev. Lett. **78**, 5022 (1997); W. K. Wootters, *ibid.* **80**, 2245 (1998).

[8] A. Peres, Phys. Rev. Lett. **77**, 1413 (1996).

[9] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993); C. H. Bennett and D. P. DiVincenzo, Nature (London) **404**, 247 (2000); M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Communication* (Cambridge University Press, Cambridge, 2000).

[10] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976); W. H. Zurek, Nature (London) **317**, 505 (1985).

[11] N. D. Antunes, L. M. A. Bettencourt, and W. H. Zurek, Phys. Rev. Lett. **82**, 2824 (1999).

[12] W. H. Zurek, U. Dorner, and P. Zoller, Phys. Rev. Lett. **95**, 105701 (2005); J. Dziarmaga, *ibid.* **95**, 245701 (2005).

[13] A. Polkovnikov, Phys. Rev. B **72**, 161201(R) (2005); A. Polkovnikov and V. Gritsev, Nat. Phys. **4**, 477 (2008).

[14] K. Sengupta, D. Sen, and S. Mondal, Phys. Rev. Lett. **100**, 077204 (2008); S. Mondal, D. Sen, and K. Sengupta, Phys. Rev. B **78**, 045101 (2008).

[15] F. Pellegrini, S. Montangero, G. E. Santoro, and R. Fazio, Phys. Rev. B **77**, 140404(R) (2008); U. Divakaran, A. Dutta, and D. Sen, *ibid.* **78**, 144301 (2008); U. Divakaran, V. Mukherjee, A. Dutta, and D. Sen, J. Stat. Mech. (2009) P02007.

[16] D. Sen, K. Sengupta, and S. Mondal, Phys. Rev. Lett. **101**, 016806 (2008); S. Mondal, K. Sengupta, and D. Sen, Phys.

Rev. B **79**, 045128 (2009); R. Barankov and A. Polkovnikov, Phys. Rev. Lett. **101**, 076801 (2008).

[17] A. Sen(De), U. Sen, and M. Lewenstein, Phys. Rev. A **72**, 052319 (2005); H. Wichterich and S. Bose, *ibid.* **79**, 060302(R) (2009).

[18] L. Cincio, J. Dziarmaga, M. M. Rams, and W. H. Zurek, Phys. Rev. A **75**, 052321 (2007); S. Deng, L. Viola, and G. Ortiz, *Proceeding of the 14th International Conference on Recent Progress in Many-Body Theories*, Series on Advances in Many-Body Theory Vol. 11 (World Scientific, Singapore, 2008), pp. 387–397.

[19] V. Mukherjee, U. Divakaran, A. Dutta, and D. Sen, Phys. Rev. B **76**, 174303 (2007).

[20] R. W. Cherng and L. S. Levitov, Phys. Rev. A **73**, 043614 (2006).

[21] O. F. Syljuasen, Phys. Rev. A **68**, 060301(R) (2003).

[22] V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A **61**, 052306 (2000).

[23] G. P. Berman, G. D. Doolen, D. D. Holm, and V. I. Tsifrinovich, Phys. Lett. A **193**, 444 (1994).

[24] N. B. Christensen, H. M. Ronnow, D. F. McMorrow, A. Harrison, T. G. Perring, M. Enderle, R. Coldea, L. P. Regnault, and G. Aeppli, Proc. Natl. Acad. Sci. U.S.A. **104**, 15264 (2007).

[25] A. Kitaev, Ann. Phys. **321**, 2 (2006).