Fast high-fidelity information transmission through spin-chain quantum wires

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Spin chains have been proposed as quantum wires for information transfer in solid-state quantum architectures. We show that huge gains in both transfer speed and fidelity are possible using a minimalist control approach that relies only on a single local on-off switch actuator. Effective switching time sequences can be determined using optimization techniques for both ideal and disordered chains. Simulations suggest that effective optimization is possible even in the absence of knowledge of the Hamiltonian or accurate models.

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The dynamical evolution of a chain of coupled spins is well suited to the job of a data bus, transferring quantum information over the short distances between quantum registers. This is particularly desirable in architectures where internal quantum communication is required but the use of photons is impractical [1,2]. A practical scalable data bus should achieve fast high fidelity of information transfer without the need for intricate manipulation of its parts. Ideally, the data should flow along the bus without any external control [3,4]. However, without any dynamic control perfect information transfer for most spin chains is possible only if the couplings are precisely engineered [5–7]. Although it often suffices to engineer the couplings near the end of the chain [8,9], fabrication of chains with such precisely engineered couplings is a significant challenge. Perfect state transfer for any length of chain is also possible in principle if we have full dynamic control over individual couplings between adjacent spins, using either dynamic or adiabatic passage schemes [10,11]. However, these schemes require control of all couplings and thus multiple control electrodes, which aside from increasing system complexity may also be sources of noise contributing to decoherence, an effect potentially amplified by the high-field strengths and longer transfer times required for adiabatic passage.

Recent work has shown that many systems are controllable even if we can only modify the dynamics of a small part of the system. In particular, the dynamics on the first excitation subspace of many spin chains is controllable if we can vary the coupling at one end of the chain [12], for instance. It has also been shown that this global controllability can be exploited to improve the transfer fidelities in certain types of spin chains by repeatedly applying certain unitary operations at one end of the chain [13], and in the limit of instantaneous gate operations and fast repetition, it can be shown that arbitrarily high transfer fidelities can be achieved. Theoretical controllability results, however, suggest that one could do much better. Almost any local perturbation of the Hamiltonian theoretically suffices to effectively control the system, and any objective can be achieved in finite time by simply switching this perturbation on and off at specific

times. Here we present a systematic approach to finding the correct switching times to achieve fast high-fidelity information transfer for various spin chains, including both uniform and disordered spin chains. We further demonstrate that effective bang-bang switching sequences can be found even in the absence of a precise model of the system using adaptive closed-loop experiments. This is important as the Hamiltonians for a particular physical realization of a spin-chain quantum wire are usually at best approximately known and subject to variation due to fabrication tolerances, although this problem can potentially be overcome by experimental system identification [14].

For proof-of-principle simulations we consider spin chains of N spin-$\frac{1}{2}$ particles with a coupling Hamiltonian

$$H_I = \frac{1}{2}\sum_{n,m} J_{mn}^x \sigma_m^x \sigma_n^x + J_{mn}^y \sigma_m^y \sigma_n^y + J_{mn}^z \sigma_m^z \sigma_n^z, \qquad (1)$$

where σ_n^* for $* \in \{x,y,z\}$ are the usual Pauli matrices for the nth spin and $J_{mn} = J_{nm}$ are the coupling constants. This model covers common types of chains, including XY, Heisenberg, and dipole-coupled spin chains. For applications such as information transfer, it is desirable to restrict the dynamics by ensuring that H_I commutes with the total spin operator $S = \sum_{n=1}^{N} \sigma_n^z$, and thus conservation of the total number of excitations. This condition is equivalent to xy isotropy, i.e., $J_{mn}^x = J_{mn}^y$, and ensures that the Hamiltonian decomposes into excitation subspaces. As usual in work on spin chains for information transfer we assume $J_{mn}^x = J_{mn}^y$ and work in the first excitation subspace E_1. We have dim $E_1 = N$ and the off-diagonal elements of the Hamiltonian restricted to E_1 are $H_{mn}^{(1)} = J_{mn}^x = J_{mn}^y$ for $m \neq n$. The diagonal elements depend on J_{mn}^z. Most simulations will be based on isotropic XY or Heisenberg chains, where $J_{mn}^x = J_{mn}^y$ and $J_{mn}^z = 0$ in the former case and $J_{mn}^x = J_{mn}^y = J_{mn}^z$ in the latter. For chains with nearest-neighbor coupling (NNC) $J_{mn} = 0$ except for $n = m \pm 1$. An NNC chain is uniformly nearest-neighbor coupled (UNNC) if $J_{n,n+1} = J$ for all n, and choosing time in units of J^{-1}, we may assume $J = 1$ in this case.

The objective of optimizing information transmission through a spin-chain quantum wire is to achieve high-fidelity transfer in a minimum amount of time. We aim to achieve this by optimizing the switching time sequence
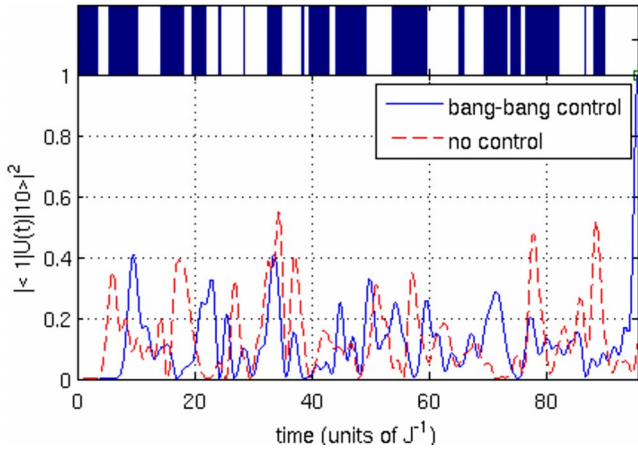
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FIG. 1. (Color online) Population of tenth spin for length-10 disordered NNC *XYZ* chain without control (dashed red line) and with optimized bang-bang control (solid blue line). Although the actuator only effects a local perturbation of the coupling between spins 1 and 2, it significantly changes the evolution of the entire system, resulting in "constructive interference" of the populations at the 10th spin at the target time $T = 95.4740$.

$\mathbf{t} = (t_1, \ldots, t_K)$ of a simple binary switch actuator, such as a local control electrode. The evolution of the system subject to this bang-bang control is given by

$$U(\mathbf{t}) = U^{(1)}(t_1) U^{(2)}(t_2) \cdots U^{(1)}(t_{K-1}) U^{(2)}(t_K), \qquad (2)$$

where $U^{(m)}(t_k) = \exp(-i t_k H_m)$. $U^{(1)}(t_k)$ corresponds to free evolution under $H_1 = H_I$ (actuator off), and $U^{(2)}(t_k)$ corresponds to evolution under the perturbed Hamiltonian $H_2 = H_I + H_C$ for t_k time units. The idea is that although the actuator changes the Hamiltonian only locally in a fixed way, by switching this perturbation on and off at different times, we can realize infinitely many effective Hamiltonians that give rise to different evolutions of the system. Most of these will not produce a desirable evolution, but some are likely to have desirable characteristics such as a near unit-height peak in the population of a particular state after a short time as shown in Fig. 1. The optimization tries to find such desirable cases by systematically exploring the parameter space, in our case the possible switching time sequences $\mathbf{t}$. Specifically, we seek $\mathbf{t}$ that minimize the average transmission error for a quantum bit propagating through the chain [3], $E(\mathbf{t}) = 1 - |\langle N | U(\mathbf{t}) | 1 \rangle|^2$, subject to the constraint that the transmission time $T = \sum_{k=1}^{K} t_k$ and number of switches remain below thresholds $T_{\max}$ and $K_{\max}$.

If the chain Hamiltonian H_I and the perturbation H_C induced by the actuator are known then we can easily solve the optimization problem. For fixed K the gradient of the objective function is $\nabla E = (\partial_1 E, \ldots, \partial_K E)$, where the partial derivatives are

$$\partial_k E = \frac{\partial E}{\partial t_k} = -2 \, \mathrm{Im}[\langle N | U^{(k)}(\mathbf{t}) | 1 \rangle \langle 1 | U(\mathbf{t})^\dagger | N \rangle], \qquad (3)$$

with $U^{(k)}(\mathbf{t}) = U_1 \cdots H_k U_k \cdots U_K$, where U_ℓ is the ℓth factor in Eq. (2). Equipped with this gradient information we can use either a gradient descent algorithm to find $\mathbf{t}_{\mathrm{opt}}$ or we can
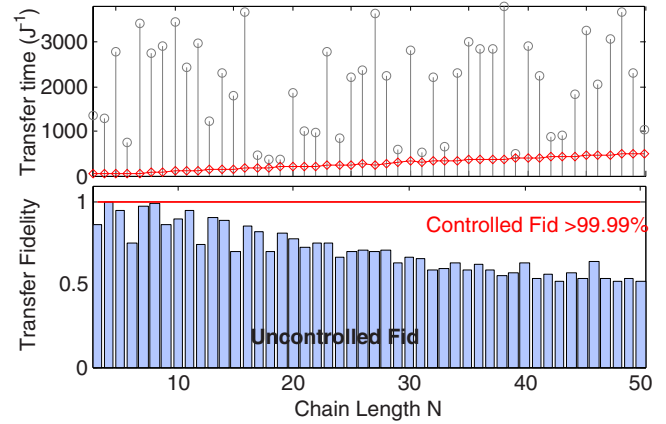

FIG. 2. (Color online) Transfer times and fidelities for Heisenberg spin chain with uniform nearest-neighbor coupling. Best fidelities achievable according to [3] in at most 4000 time units without control (shaded bars or circles) and with our optimized bang-bang control (red diamonds or line).

similarly calculate the Hessian matrix of second derivatives $H = (H_{k\ell})$ with $H_{k\ell} = \partial_k \partial_\ell E(\mathbf{t})$ and use the more efficient Newton method with an adjustable step size $\gamma > 0$ to iteratively update $\mathbf{t}$

$$\mathbf{t}_{s+1} = \mathbf{t}_s - \gamma \{H[E(\mathbf{t}_s)]\}^{-1} \nabla E(\mathbf{t}_s) \qquad (4)$$

until convergence is achieved. In practice there are some additional complications as we have to impose constraints on the switching times (e.g., $t_k \geq t_{\min}$, $\Sigma_k t_k \leq T_{\max}$, etc.), but these can easily be incorporated in the algorithm [15]. The resulting $\mathbf{t}_{\mathrm{opt}}$ depends on the initial guess $\mathbf{t}_0$, and as with virtually all optimization algorithms, we can only guarantee convergence to a local optimum, but the method is generally both effective and very efficient.

We first tested if we are indeed able to achieve fast high-fidelity information transfer using a simple binary switch actuator for UNNC *XY* and *XYZ* chains. For such chains perfect information transfer without control or engineered couplings is possible only for $N = 3$, and, as Fig. 2 for the Heisenberg chain shows, the maximum transfer fidelity in a limited amount of time decreases quickly, and the peak transfer times vary erratically with the chain length N. Assuming a simple local actuator that switches off the coupling between the first two spins, we were able to find switching sequences $\mathbf{t}$ that achieved transfer fidelities > 0.9999 for both *XY* and *XYZ* chains up to length 50 with transfer times $T(N) \approx 10N$ and at most $K = 4N$ switches. Even when the precision of the switching times t_k was limited to 10^{-4}, i.e., four decimal digits, the error remained below threshold except for *XYZ* chains with $N = 48, 50$, where the error was slightly above threshold (1.7×10^{-4} and 5.6×10^{-4}). Surprisingly, we were able to achieve transfer fidelities > 0.9999 in many cases even when the switching times were limited to only three decimal digits accuracy, and for shorter chains even fewer digits appeared sufficient in many cases although at the expense of somewhat increased transfer times. This is significant as it shows that we do not require "double preci-
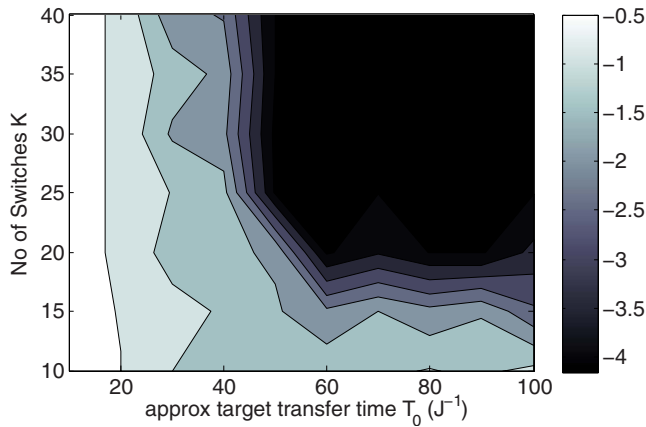
FIG. 3. (Color online) Error $\log_{10}[E(\mathbf{t}_{opt})]$ as a function of the number of switches and target transfer times T_0, where $E(\mathbf{t}_{opt})$ is the minimum error over 10 runs with random $\mathbf{t}_0$.

sion" control of the switching times, which would be experimentally infeasible.

Although the optimal transfer times of $T \approx 10N$ are impressive compared with those for the uncontrolled chain, especially considering the transfer fidelities, the ease with which the algorithm appeared to be able to find solutions suggested that both K and T could be further reduced. To investigate this, we systematically varied the number of switches and approximate target transfer times T_0 for a benchmark case of an XYZ UNNC chain of length 10. The contour plot of the error on a logarithmic scale [$\log_{10} E(\mathbf{t}_{opt})$] as a function of K and T_0 (Fig. 3) suggests that for this system near-perfect transfer could be accomplished in as little as 50 time units (measured in units of J^{-1}) with as few as 22–24 switches. Pushing the limits tends to slow down the optimization, increasing the number of iterations and possibly requiring several runs with different initial guesses $\mathbf{t}_0$. Yet, for the simple benchmark problem, all runs completed in a few seconds (average 1.3 s, max 4.6 s) on a standard laptop.

While chains with uniform nearest neighbor coupling provide nice models, in practice the coupling constants J_{mn} are likely to be subject to variation, and direct coupling may not be limited to nearest neighbors. It is thus crucial to investigate the performance of the bang-bang control scheme and optimization algorithm for perturbed chains. As Fig. 4 shows, with very few exceptions the algorithm had no difficulty in finding solutions that achieved threshold transfer fidelities of 99.99% with transfer times of $T \approx 100$ and $K \leq 40$ switches for moderately disordered NNC Heisenberg chains of length 10. Similar results were achieved for other chains, e.g., XY chains. Preliminary simulations suggest that substantial gains in fidelities and transfer times are possible even for highly disordered chains although in some cases finding solutions becomes challenging and target transfer times and/or the number of switches may need to be increased.

These results are promising in that the simulations suggest substantial improvements of both transfer fidelities and transfer times are possible with very limited control and very simple actuators even for disordered chains. However, our
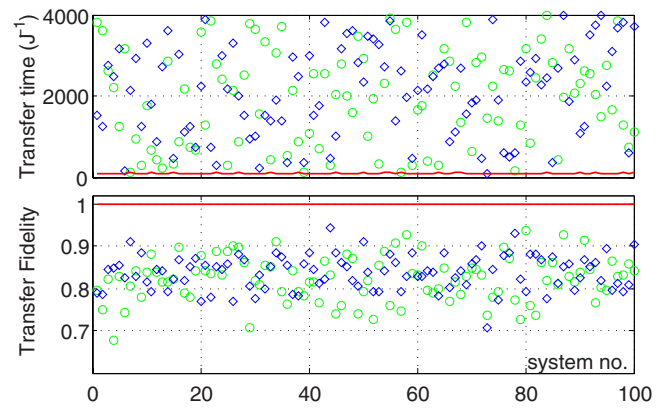

FIG. 4. (Color online) Transfer times T and fidelities $\mathcal{F}$ for 100 (NNC) Heisenberg chains of length 10 with randomly perturbed couplings $J_{n,n+1} = 1 + \epsilon\xi$ [ξ Gaussian random variable, $\langle\xi\rangle = 0$, $\sigma(\xi) = 1$] for 1% and 10% disorder. Without control, the maximum transfer fidelities achievable in ≤ 4000 time units and transfer times T are scattered with $\langle\mathcal{F}\rangle = 83.74\%$, $\langle T\rangle = 2,120$ for $\epsilon = 0.01$ (blue diamonds) and $\langle\mathcal{F}\rangle = 82.21\%$, $\langle T\rangle = 1,956$ for $\epsilon = 0.1$ (green circles). With optimized bang-bang control, near-perfect transfer $\langle\mathcal{F}\rangle \geq 0.9999$ for $\epsilon = 0.01$ and $\langle\mathcal{F}\rangle \geq 0.9995$ for $\epsilon = 0.1$ can be achieved for all 100 test systems in approximately 100 time units (red lines).

optimization procedures assumed knowledge of the chain's Hamiltonian and the perturbation induced by the actuator. A valid objection to the practical feasibility of the approach is that this information is often simply not available for a particular physical system. Furthermore, unknown environmental effects may perturb the evolution. Hence, it is crucial to consider if we can determine a set of optimal switching times for a given system without recourse to an idealized model using adaptive closed-loop experiments. In this case, the figure of merit, in our case the transfer fidelity, for a particular switching sequence is evaluated experimentally as follows: initialize the system, create an excitation at one end, apply the control sequence, and measure the spin at the other end of the chain. Assuming a simple binary-outcome projective measurement, this experiment is repeated until we have accumulated sufficient data to estimate the transfer fidelity to a desired number of significant digits. In this setting the optimization must find an optimal time sequence based only on the limited-precision fidelity measurement data.

To assess if we can still find effective switching pulse sequences we simulated this situation, i.e., we choose a model system to calculate the fidelities, but instead of providing the optimization routine with information about the actual model as before, it now only had access to an estimate for the fidelity with D significant digits for each time sequence $\mathbf{t}$. For our benchmark problem (a UNNC XYZ chain of length 10) we compared for four types of algorithms: genetic, pattern search, Nelder-Mead simplex, and Newton iteration with discrete gradients derived from the limited-precision (simulated) measurement data. The results are shown in Table I. Pattern search was abandoned due to extremely poor performance and convergence issues. Most notably, the standard genetic algorithm, despite being a popular choice for closed-loop optimization experiments in some areas, failed completely for this problem. Out of 100 trials with

TABLE I. Performance of various optimization algorithms in closed-loop experiment simulations for benchmark problem.

	% Success	$\langle \#\mathcal{F} \rangle$	$\langle t_{\text{exe}} \rangle$	$\langle T \rangle$	$T_{\min}$
Genetic	0	12275.5	35.4497	71.6552	
Simplex	75	8696.3	21.7677	99.9145	94.9778
Newton-1	100	1379.8	3.6295	99.5020	74.6144
Newton-2	100	96.3	0.7087	99.6101	74.6268

different initial populations (50 individuals), not one came close to reaching threshold fidelity, and the number of fidelity evaluations ($\#\mathcal{F}$) and hence experiments required was huge. The simplex algorithm did significantly better in that 75% of the trials succeeded in finding solutions above threshold fidelity. Application of the quasi-Newton-type optimization procedure outlined above requires that the analytical gradients be replaced by discrete gradients calculated from the fidelity measurements. Although the limited precision is a challenge here, with careful tuning of the parameters in the discrete gradient estimation, the quasi-Newton routine (Newton-1) far outperformed all other choices in terms of success rate, number of fidelity evaluations, execution times, and best transfer times. For comparison we have included results for our model-based Newton iteration (Newton-2) using (partially) analytic gradients. As expected, the model-based iteration is more efficient in terms of function evaluations and execution times, but there is no difference in the success rates or average transfer times.

As estimating the fidelities to a large number of significant digits is costly in that it requires many experiment repetitions for each fidelity evaluation, we further investigated the effect of limiting the precision of the fidelity measurements to a few significant (decimal) digits. We note that four digits is the minimum accuracy required if we wish to achieve transfer fidelities $\geq 99.99\%$. We found that with suitable choice of the algorithmic parameters, especially with regard to the discrete gradient estimation, the quasi-Newton-type algorithm generally continued to find solutions with error probabilities $\leq 10^{-4}$ in almost all cases even when the accuracy of the fidelities was limited to five to eight digits, and in some cases the algorithm was able to find time sequences achieving transfer fidelities ≥ 0.9999 for our benchmark problem even when both the accuracy of the fidelity measurements and the switching times t_k were limited to four decimal digits. This suggests that threshold fidelities can be achieved even with limited-precision measurements and control.

We have shown that minimal control using a binary switch actuator that induces a local perturbation to a fixed Hamiltonian holds considerable promise to improve information transmission through spin-chain quantum wires. The effectiveness of the technique for disordered chains and the possibility of model-free optimization using closed-loop experiments with limited precision measurements further enhance its potential appeal. Furthermore, the technique is not limited to information transfer in spin chains with nearest-neighbor coupling. Preliminary simulations suggest that the technique is still effective for more complex spin networks with non-nearest-neighbor couplings and potentially many other types of systems and that it can be applied to implement control objectives other than state transfer, including nonlocal unitary operations.

[1] D. Kielpinski, C. Monroe, and D. J. Wineland, Nature (London) **417**, 709 (2002).

[2] A. J. Skinner, M. E. Davenport, and B. E. Kane, Phys. Rev. Lett. **90**, 087901 (2003).

[3] S. Bose, Contemp. Phys. **48**, 13 (2007).

[4] S. Bose, Phys. Rev. Lett. **91**, 207901 (2003).

[5] C. Albanese, M. Christandl, N. Datta, and A. Ekert, Phys. Rev. Lett. **93**, 230502 (2004).

[6] P. Karbach and J. Stolze, Phys. Rev. A **72**, 030301(R) (2005).

[7] A. Kay, Phys. Rev. A **73**, 032306 (2006).

[8] A. Wojcik, T. Luczak, P. Kurzynski, A. Grudka, T. Gdala, and M. Bednarska, Phys. Rev. A **72**, 034303 (2005).

[9] A. O. Lyakhov and C. Bruder, Phys. Rev. B **74**, 235303 (2006).

[10] T. Ohshima, A. Ekert, D. K. L. Oi, D. Kaslizowski, and L. C. Kwek, e-print arXiv:0702.0019.

[11] K. Eckert, O. Romero-Isart, and A. Sanpera, New J. Phys. **9**, 155 (2007).

[12] S. G. Schirmer, I. C. H. Pullen, and P. J. Pemberton-Ross, Phys. Rev. A **78**, 062339 (2008).

[13] D. Burgarth, V. Giovannetti, and S. Bose, Phys. Rev. A **75**, 062327 (2007).

[14] D. Burgarth, K. Maruyama, and F. Nori, Phys. Rev. A **79**, 020305(R) (2009).

[15] S. P. Han, J. Optim. Theory Appl. **22**, 297 (1977).