

Robustness of quantum discord to sudden death

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We calculate the dissipative dynamics of two-qubit quantum discord under Markovian environments. We analyze various dissipative channels such as dephasing, depolarizing, and generalized amplitude damping, assuming independent perturbation, in which each qubit is coupled to its own channel. Choosing initial conditions that manifest the so-called sudden death of entanglement, we compare the dynamics of entanglement with that of quantum discord. We show that in all cases where entanglement suddenly disappears, quantum discord vanishes only in the asymptotic limit, behaving similarly to individual decoherence of the qubits, even at finite temperatures. Hence, quantum discord is more robust than the entanglement against decoherence so that quantum algorithms based only on quantum discord correlations may be more robust than those based on entanglement.

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Entanglement is widely seen as the main reason for the computational advantage of quantum over classical algorithms. This view is backed up by the discovery that, in order to offer any speedup over a classical computer, the universal pure-state quantum computer would have to generate a large amount of entanglement [1]. However, quantum entanglement is not necessary for deterministic quantum computation with one pure qubit (DQC1), introduced by Knill and Laflamme [2]. As seen in [3,4], although there is no entanglement, other kinds of nonclassical correlation are responsible for the quantum computational efficiency of DQC1. Such correlations are characterized as quantum discord [5], which accounts for all nonclassical correlations present in a system, being the entanglement a particular case of it. Besides its application in DQC1, quantum discord has also been used in studies of quantum phase transition [6], estimation of quantum correlations in the Grover search algorithm [7], and to define the class of initial system-bath states for which the quantum dynamics is equivalent to a completely positive map [8].

When considering a pair of entangled qubits exposed to local noisy environments, disentanglement can occur in a finite time [9–13] differently from the usual local decoherence in asymptotic time. The occurrence of this phenomenon, named “entanglement sudden death” (ESD), depends on the system-environment interaction and on the initial state of the two qubits. Our goal was to investigate the dynamics of quantum discord of two qubits under the same conditions in which ESD can occur. We show in this Brief Report that even in cases where entanglement suddenly disappears, quantum discord decays only in asymptotic time. Furthermore, this occurs even at finite temperatures. In this sense, quantum discord is more robust against decoherence than entanglement, implying that quantum algorithms based only on quantum discord correlations are more robust than those based on entanglement.

There are various methods to quantify the entanglement between two qubits [14–16] and, even when they give different results for the degree of entanglement of a specific state, all of them result in zero for separable states. There-

fore, under dissipative dynamics where an initial entangled state can disappear suddenly, all of them necessarily agree about the time when the quantum state becomes separable.

Here, to investigate the two-qubit entanglement dynamics we use concurrence as the quantifier [15]. The concurrence is given by $\max\{0, \Lambda(t)\}$, where $\Lambda(t) = \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4$ and $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the square roots of the eigenvalues of the matrix $\rho(t)\sigma_2 \otimes \sigma_2 \rho^*(t)\sigma_2 \otimes \sigma_2$, $\rho^*(t)$ being the complex conjugate of $\rho(t)$ and σ_2 being the second Pauli matrix. The density matrix we use to evaluate concurrence has an X structure [12], defined by $\rho_{12} = \rho_{13} = \rho_{24} = \rho_{34} = 0$, which are constant during the evolution, for the various dissipative channels used here. In this case, the concurrence has a simple analytic expression $C(t) = 2 \max\{0, \Lambda_1(t), \Lambda_2(t)\}$, where $\Lambda_1(t) = |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}$ and $\Lambda_2(t) = |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}$. However, as pointed out above, entanglement is not the only kind of quantum correlation. In quantum information theory, the Von Neumann entropy, $S(\rho) = -\text{Tr}(\rho \log_2 \rho)$, is used to quantify the information in a generic quantum state ρ . The total correlation between two subsystems \mathcal{A} and \mathcal{B} of a bipartite quantum system $\rho_{\mathcal{A}\mathcal{B}}$ is given by the mutual information,

$$\mathcal{I}(\rho_{\mathcal{A}\mathcal{B}}) = S(\rho_{\mathcal{A}}) + S(\rho_{\mathcal{B}}) - S(\rho_{\mathcal{A}\mathcal{B}}), \quad (1)$$

where $S(\rho_{\mathcal{A}\mathcal{B}}) = -\text{Tr}(\rho_{\mathcal{A}\mathcal{B}} \log_2 \rho_{\mathcal{A}\mathcal{B}})$ is the joint entropy of the system [17]. This bipartite quantum state $\rho_{\mathcal{A}\mathcal{B}}$ is a hybrid object with both classical and quantum characteristics and, in order to reveal the classical aspect of correlation, Henderson and Vedral suggested that correlation could also be split into two parts, the quantum and the classical [18]. The classical part was defined as the maximum information about one subsystem that can be obtained by performing a measurement on the other subsystem. If we choose a complete set of projectors $\{\Pi_k\}$ to measure one of the subsystems, say \mathcal{B} , the information obtained about \mathcal{A} after the measurement resulting in outcome k with probability p_k is the difference between the initial and the conditional entropies [18],

$$\mathcal{Q}_{\mathcal{A}}(\rho_{AB}) = \max_{\{\Pi_k\}} \left[S(\rho_{\mathcal{A}}) - \sum_k p_k S(\rho_{\mathcal{A}|k}) \right], \quad (2)$$

where $\rho_{\mathcal{A}|k} = \text{Tr}_{\mathcal{B}}(\Pi_k \rho_{AB} \Pi_k) / \text{Tr}_{\mathcal{A}\mathcal{B}}(\Pi_k \rho_{AB} \Pi_k)$ is the reduced state of \mathcal{A} after obtaining the outcome k in \mathcal{B} . This measurement of classical correlation assumes equal values, irrespective of whether the measurement is performed on the subsystem \mathcal{A} or \mathcal{B} for all states ρ_{AB} such that $S(\rho_{\mathcal{A}}) = S(\rho_{\mathcal{B}})$ [18]. This condition is true of all density operators used in this Brief Report since they can be written as $\rho = \frac{1}{4}[1 + c_0(\sigma_3 \otimes \mathbb{I} + \mathbb{I} \otimes \sigma_3) + \sum_j c_j \sigma_j \otimes \sigma_j]$, where c_i and σ_i ($i=1,2,3$) are real constants and Pauli matrices, respectively. Therefore $\rho_{\mathcal{A}} = \rho_{\mathcal{B}}$.

In this scenario, a quantity that provides information on the quantum component of the correlation between two systems can be introduced as the difference between the total correlations in Eq. (1) and the classical correlation in Eq. (2). This quantity is identical to the definition of quantum discord introduced by Ollivier and Zurek in [5], namely,

$$D(\rho_{AB}) = \mathcal{I}(\rho_{AB}) - \mathcal{Q}(\rho_{AB}), \quad (3)$$

this being zero for states with only classical correlations [5,18] and nonzero for states with quantum correlations. Moreover, quantum discord includes quantum correlations that can be present in states that are not entangled [5], revealing that all the entanglement measurements such as concurrence, entanglement of formation, etc., do not capture the whole of quantum correlation between two mixed separate systems. For pure states, the discord reduces exactly to a measure of entanglement, namely, the entropy of entanglement.

In order to calculate the quantum discord between two qubits subject to dissipative processes, we use the following approach. The dynamics of two qubits interacting independently with individual environments is described by the solutions of the appropriate Born-Markov-Lindblad equations [19], which can be obtained conveniently by the Kraus operator approach [17]. Given an initial state for two qubits $\rho(0)$, its evolution can be written compactly as

$$\rho(t) = \sum_{\mu,\nu} E_{\mu,\nu} \rho(0) E_{\mu,\nu}^\dagger, \quad (4)$$

where the so-called Kraus operators $E_{\mu,\nu} = E_\mu \otimes E_\nu$ [17] satisfy $\sum_{\mu,\nu} E_{\mu,\nu}^\dagger E_{\mu,\nu} = \mathbb{I}$ for all t . The operators $E_{\{\mu\}}$ describe the one-qubit quantum channel effects.

In the cases where the quantum channel induces a disentanglement only in asymptotic time, the quantum discord does not disappear in a finite time since the entanglement is itself a kind of quantum correlation. Therefore, we present below what happens to the discord in the ESD situations for some of the common channels for qubits: dephasing, generalized amplitude damping (GAD) (thermal bath at arbitrary temperature), and depolarizing.

Dephasing. The dephasing channel induces a loss of quantum coherence without any energy exchange [17]. The quantum state populations remain unchanged throughout the time. To examine the two-qubit entanglement and discord dynamics under the action of a dephasing channel, we utilize the Werner state as the initial condition, i.e., $\rho(0) = (1-\alpha)\mathbb{I}/4 + \alpha|\Psi^-\rangle\langle\Psi^-|$, $\alpha \in [0,1]$, and $|\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$. In this

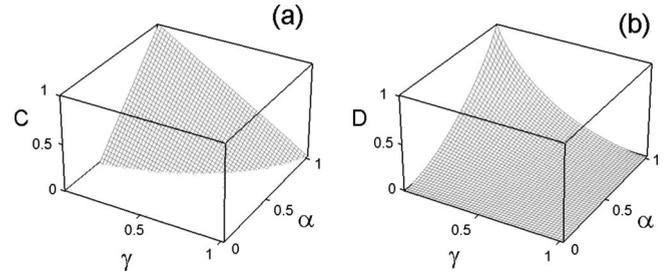


FIG. 1. Dissipative dynamics of (a) concurrence and (b) discord as functions of α and γ , assuming independent dephasing perturbative channels.

case, we can calculate analytically the quantum discord in a situation where the entanglement suddenly disappears. Thus, according to Eq. (4), with the nonzero Kraus operators for a dephasing channel given by $E_0 = \text{diag}(1, \sqrt{1-\gamma})$ and $E_1 = \text{diag}(1, \sqrt{\gamma})$, where $\gamma = 1 - e^{-\Gamma t}$, Γ denoting the decay rate [17], the elements of the density matrix of this system evolve to

$$\rho_{ii}(t) = \rho_{ii}(0), \quad i = 1, \dots, 4,$$

$$\rho_{23}(t) = \rho_{23}(0)(1-\gamma) = \rho_{32}(t).$$

The concurrence for this state is given by $C(\rho) = \alpha(3/2 - 2\gamma) - 1/2$, which reaches zero in a finite time for any $\alpha \neq 1$, as shown in Fig. 1(a). On the other hand, based on the results given in [20], the quantum discord for this state reads $D(\rho) = [F(a+b) + F(a-b)]/4 - F(a)/2$, where $F(x) = x \log_2 x$, $a = (1-\alpha)$, and $b = 2\alpha(1-\gamma)$. As shown in Fig. 1(b), for any α , the quantum discord vanishes [$D(\rho) = 0$] only in the asymptotic limit.

Generalized amplitude damping. The GAD describes the exchange of energy between the system and the environment, including finite temperature aspects. It is described by the Kraus operators $E_0 = \sqrt{q} \text{diag}(1, \sqrt{1-\gamma})$, $E_1 = \sqrt{q\gamma}(\sigma_1 + i\sigma_2)/2$, $E_2 = \sqrt{(1-q)\gamma} \text{diag}(\sqrt{1-\gamma}, 1)$, and $E_3 = \sqrt{(1-q)\gamma}(\sigma_1 - i\sigma_2)/2$, where γ is defined above and q defines the final probability distribution of the qubit when $t \rightarrow \infty$ ($q=1$ corresponds to the usual amplitude damping with $T=0K$) [17].

For the initial condition given by $\rho(0) = |\Phi\rangle\langle\Phi|$ with

$$|\Phi\rangle = \sqrt{1-\alpha}|00\rangle + \sqrt{\alpha}|11\rangle, \quad \alpha \in [0,1], \quad (5)$$

we obtain, according to Eq. (4), the density matrix dynamics,

$$\rho_{11}(t) = \rho_{11}(0)\{1 - \gamma[2(1-q) - \gamma(1-2q)]\} + \gamma^2 q^2,$$

$$\rho_{22}(t) = \rho_{33}(t) = \gamma[\rho_{11}(0)(1-2q)(1-\gamma) + q(1-\gamma q)],$$

$$\rho_{44}(t) = 1 - \rho_{11}(t) - 2\rho_{22}(t),$$

$$\rho_{14}(t) = \rho_{41}(t) = \rho_{14}(0)(1-\gamma).$$

We examine the dissipative dynamics derived from this channel, taking $q=1$ and $q=2/3$. For these cases, we compute the discord numerically and compare it with the concurrence. To calculate the discord, we chose the set of projectors $\{|\psi_1\rangle\langle\psi_1|, |\psi_2\rangle\langle\psi_2|\}$, where $|\psi_1\rangle = \cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle$ and

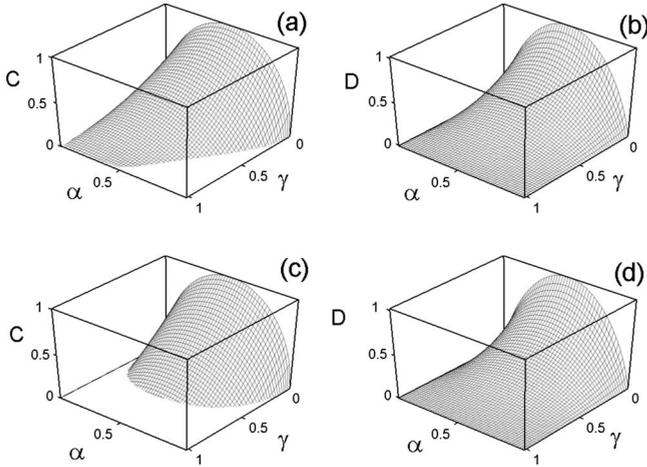


FIG. 2. Dissipative dynamics of (a) and (c) concurrence and (b) and (d) discord as functions of α and γ , assuming independent generalized amplitude damping. (a) and (b) $q=1$ and (c) and (d) $q=2/3$.

$|\psi_2\rangle = -\cos\theta|1\rangle + e^{-i\phi}\sin\theta|0\rangle$, to measure one of the subsystems. The maximum of Eq. (2) is obtained numerically by varying the angles θ and ϕ from 0 to 2π .

For $q=1$, we have $C(t) = \max\{0, \Lambda_1(t)\}$, with $\Lambda_1(t) > 0$ for all t , whenever $\alpha < 1/2$ [see Fig. 2(a)]. On the other hand, for $q \neq 1$, we have $\Lambda_1(t \rightarrow \infty) < 0$ for all α , as shown in Fig. 2(c). In this case, since the concurrence decays monotonically under the Markovian approximation [21], the ESD occurs for any initial superposition state, i.e., for any α different from 0 or 1.

The discord evidently behaves differently from the concurrence [see Figs. 2(b) and 2(d)]. In both situations ($q=1$ and $q=2/3$), the discord decays exponentially and vanishes only asymptotically. For $\alpha=1$ or $\alpha=0$ (pure separable states) the quantum discord is zero all the time, as expected.

Depolarizing. The depolarizing channel represents the process in which the density matrix is dynamically replaced by the maximal mixed state $I/2$, I being the identity matrix of a single qubit. The set of Kraus operators that reproduces the effect of the depolarizing channel is given by $E_0 = \sqrt{1-3\gamma/4}I$, $E_1 = \sqrt{\gamma/4}\sigma_1$, $E_2 = \sqrt{\gamma/4}\sigma_2$, and $E_3 = \sqrt{\gamma/4}\sigma_3$, with γ defined above [17]. Assuming the initial condition given by Eq. (5), we obtain the density matrix element dynamics,

$$\rho_{11}(t) = \rho_{11}(0)(1-\gamma) + \gamma^2/4,$$

$$\rho_{22}(t) = \rho_{33}(t) = \gamma/2(1-\gamma/2),$$

$$\rho_{44}(t) = 1 - \rho_{11}(t) - 2\rho_{22}(t),$$

$$\rho_{14}(t) = \rho_{41}(t) = \rho_{14}(0)(1-\gamma).$$

As in generalized amplitude damping, the ESD occurs for any initial condition since $C(t) = \max\{0, \Lambda_1(t)\}$ and $\Lambda_1(t \rightarrow \infty) < 0$ [Fig. 3(a)]. Again, as shown in Fig. 3(b), the quantum discord does not disappear in a finite time. Here we used

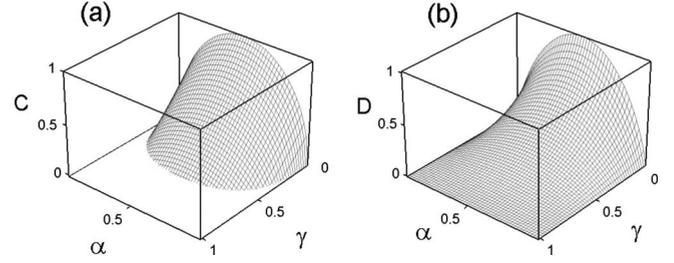


FIG. 3. Dissipative dynamics of (a) concurrence and (b) discord as functions of α and γ , assuming independent depolarizing channels.

the same procedure as above to calculate the discord numerically.

Dephasing plus amplitude damping. In Fig. 4 we plot the concurrence and quantum discord for the case where both qubits interact individually with two distinct reservoirs: those which induce dephasing and amplitude damping. We assume equal decay rates (Γ) for both channels, $q=1$ ($T=0K$) and $\gamma=1-e^{-\Gamma t}$. For the initial condition given by Eq. (5), the dephasing channel alone is not able to induce sudden death of entanglement. On the other hand, in the presence of an amplitude damping channel, the entanglement suddenly disappears for some values of α [Fig. 2(a)], as discussed above. However, when both channels are present, the dynamics of the entanglement is very different, suddenly disappearing for all values of α as we can see in Fig. 4(a). This nonadditivity of the decoherence channels in the entanglement dynamics was first pointed out by Yu and Eberly [11], in contrast to the additivity of the decay rates of different decoherence channels of a single system [11]. But, as shown in Fig. 4(b), the discord still decays asymptotically when both decoherence channels are present, as shown in Fig. 4(b), indicating that the additivity of the decoherence channels is valid for the quantum discord.

In conclusion, we have calculated the discord dynamics for two qubits coupled to independent Markovian environments. We observed that under the dissipative dynamics considered here, discord is more robust than entanglement, even at a finite temperature, being immune to a “sudden death.” This also points to a fact that the absence of entanglement does not necessarily indicate the absence of quantum correlations. Thus, quantum discord might be a better measure of

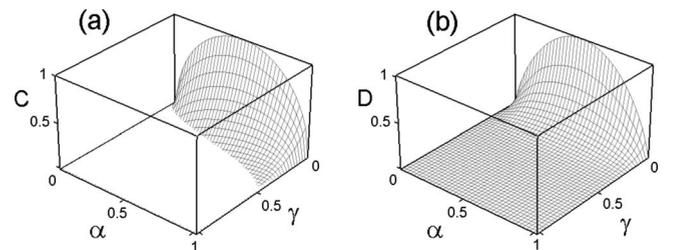


FIG. 4. Dissipative dynamics of (a) concurrence and (b) discord as functions of α and γ , assuming simultaneous action of dephasing and generalized amplitude damping channels in each qubit with $q=1$.

the quantum resources available to quantum information and computation processes. This also suggests that quantum computers based on this kind of quantum correlation, differently from those based on entanglement, are more resistant to external perturbations and, therefore, introduce a new hope of implementing an efficient quantum computer. Moreover, discord may be considered, in this scenario, as a good indicator of classicality [5,22], since it vanishes only in the asymptotic limit, when the coherence of the individual qubits disappears. However, we have not demonstrated that the sud-

den death of discord in a Markovian regime is impossible, and a study of the discord from a geometrical point of view [23], for example, might be useful to address this important question.

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