Tunneling times in metamaterials with saturable nonlinearity

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Tunneling times of electromagnetic planar wave propagating through a barrier made of split-ring resonators and long metallic wires embedded into a dielectric with saturable nonlinearity are analyzed. For such structure, analytical expressions for the tunneling times and relation between the group delay and the dwell time are obtained. The electric-field distribution inside the barrier is numerically evaluated and a detailed study of the influence of nonlinearity on the spectral structures of the tunneling times is performed. It is demonstrated that intensity variation in the incident field may change the wave propagation direction, the sign of the barrier's refractive index, and group delay.

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I. INTRODUCTION

Although a typically quantum-mechanical phenomenon [1–3], tunneling can be represented as a problem of classical electromagnetics owing to the deep analogy between the Schrödinger equation and the Helmholtz equation. Expressions that describe the tunneling times, i.e., the dwell time and the group delay, that occur while electromagnetic (EM) wave travels through a barrier placed inside the waveguide with different index of refraction were first derived by Winful [4]. This model included a linear nondispersive obstacle and waveguide with positive refraction indices. Recently, tunneling times have been calculated for linear dispersive media [5] and nondispersive media with Kerr nonlinearity [6].

The refractive index of a material n is the key parameter which describes interactions between the material and the EM wave. Recent studies have verified the existence of artificial complex media, namely, left-handed metamaterials (LHMs) that exhibit negative values of both the dielectric permittivity ϵ and the magnetic permeability μ within a certain range of frequencies. This property of negative refraction was predicted by Veselago [7] in 1968 and it has led to a number of peculiar properties: inverse Snell's law, inverse Doppler shift effect, backward Cerenkov radiation, etc. The first experimental realization of LHMs was performed by Smith [8], who combined a periodic array of metallic wires with a regular array of split-ring resonators (SRRs), which had negative index of refraction in the microwave range of frequencies. Subsequent papers [9,10] have shown the existence of left-handed characteristics for the THz range, too.

Nonlinear behavior of the LHMs was analyzed by observing the example of a lattice of SRRs and wires embedded in a Kerr-type nonlinear dielectric [11,12] or inserting certain nonlinear elements in each SRR slit [13]. Both propositions lead to effectively field-dependent values of ϵ and μ and the same phenomenon has been demonstrated for the case of saturable nonlinearity by using nonlinear photorefractive crystals such as GaAs and $LiNbO_3$ [14,15]. Numerous papers and experiments show that nonlinear effects in metamaterials enable second harmonic generation in SRR-based magnetic metamaterials on glass substrate [16] as well as formation of solitons and discrete breathers [17–19].

In this work we present theoretical and numerical results obtained for the tunneling times described above, i.e., the group delay and the dwell time, in case of the wave packet tunneling through a metamaterial barrier designed using circular SRRs and long metallic wires, embedded in a dielectric with saturable nonlinearity. We presume propagation of TE modes only. In contrast to previously presented expressions for the tunneling times, obtained only for linear dispersive and nondispersive media [4,5] and for nondispersive media with Kerr-type nonlinearity [6], we offer a more generalized approach. This model encompasses both the linear media and all the media with third-order nonlinearities of the electric susceptibility, the Kerr-type nonlinearity being just a specific case of the latter (and described here by a more accurate expression for the EM energy density). In addition, we provide a comparative analysis of the considered effects in terms of varying the structural parameters and the light intensity.

The paper is structured as follows: the model and the theoretical considerations are formulated in Sec. II; a numerical method used for calculating quantities of interest is described in Sec. III; numerical results and discussion of tunneling times are presented in Sec. IV. The paper is concluded by Sec. V.

II. THEORETICAL CONSIDERATIONS

The model, i.e., a planar dielectric waveguide and an absorptive barrier made of circular SRRs and long metallic wires embedded into a material with saturable nonlinearity, placed inside the waveguide, is shown in Fig. 1. The thickness of the barrier is L. The waveguide is made of a linear material whose permittivity and permeability are purely real and denoted by ϵ_b and μ_b, respectively. The effective permittivity and permeability of the barrier are given by [20]

$$\epsilon_s = \epsilon_{LHM} + \epsilon_{NL} = \epsilon_\infty\left(1 - \frac{\omega_p^2}{\omega^2 - \omega_r^2 + i\Gamma_e\omega}\right) + \epsilon_{NL}, \quad (1)$$

$$\mu_s = 1 - \frac{F\omega_0^2}{\omega^2 - \omega_0^2 + i\Gamma_m\omega}, \quad (2)$$

respectively. Here, ϵ_∞ is the background dielectric constant, ω_p is the plasma frequency, measure of the strength of inter-

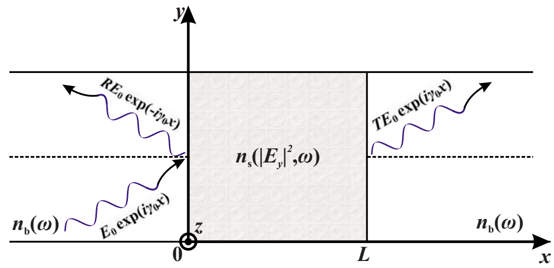

FIG. 1. (Color online) The model: SRRs and long metallic wires embedded into a barrier made of dielectric with saturable nonlinearity. x is the propagation axis.

action between the oscillators and the electric field, ω is the frequency of incident EM wave, ω_r is the resonance frequency of the electric dipole oscillators, ω_0 is the resonance frequency of the magnetic dipole oscillators, while Γ_e and Γ_m stand for the damping frequencies of the electric and the magnetic field, respectively. Finally, F is a measure of the strength of interaction between the oscillators and the magnetic field and $\epsilon_{NL}=\epsilon_{NL}(|E_y|^2)$ represents the intensity-dependent part of the permittivity given by [20]

$$\epsilon_{NL} = \epsilon_{D0} + \alpha\frac{|E_y|^2/E_c}{1+\kappa|E_y|^2/E_c}, \tag{3}$$

where ϵ_{D0} is the linear dielectric permittivity, E_c is a characteristic electric field, α is the sign of the nonlinearity, i.e., $\alpha=+1$ for focusing and $\alpha=-1$ for defocusing nonlinearity, and κ is the saturation strength. The second term of permittivity is a nonlinear function of the electric-field intensity which describes saturable type of nonlinearity and will be noted as ϵ_{SNL}. We presume that $\alpha\neq\alpha(\omega)$, $E_c\neq E_c(\omega)$, and $\kappa\neq\kappa(\omega)$. The dwell time is defined as the time spent by a wave packet in a given region of space [2,4,21] defined by

$$\tau_d = \frac{W}{P_{in}}, \tag{4}$$

where W stands for the stored EM energy inside the barrier and P_{in} is the time-averaged incident power.

In order to calculate the stored EM energy inside the barrier, it is first necessary to define its density w, which can be represented as a sum of the linear w_{LIN} and the nonlinear part w_{NL}. For a dispersive and absorptive linear medium the stored EM energy density is given by [5]

$$w_{LIN} = \frac{\epsilon_0}{4}\epsilon_{effLIN}|E|^2 + \frac{\mu_0}{4}\mu_{effLIN}|H|^2, \tag{5}$$

where ϵ_0 and μ_0 are the vacuum permittivity and permeability, respectively, $\epsilon_{effLIN}=\epsilon_{D0}+\mathrm{Re}(\epsilon_{LHM})+2\omega\,\mathrm{Im}(\epsilon_{LHM})/\Gamma_e$ and $\mu_{effLIN}=\mathrm{Re}(\mu_s)+2\omega\,\mathrm{Im}(\mu_s)/\Gamma_m$. In a nonlinear dispersive medium, the nonlinear part of the time-averaged stored EM energy density is given by [22]

$$w_{NL} = \frac{\epsilon_0}{4}\frac{d(\omega\epsilon_{SNL})}{d\omega}|E|^2 = \frac{\epsilon_0}{4}\epsilon_{effNL}|E|^2 \tag{6}$$

assuming the imaginary part of permittivity to have very small value compared to its real part, as well as $\epsilon_{NL}=\epsilon_{NL}(|E_y|^2)$ and $E_y=E_y(\omega)$. Since only the TE modes propagate through the barrier, the total stored EM energy in the barrier may be derived, by using the Helmholtz equation, in

$$W = -\frac{S\mu_{eff}\gamma_0}{2\omega|\mu_s|^2\mu_0}\mathrm{Im}(R)|E_0|^2 - \frac{S\mu_{eff}}{\omega^2|\mu_s|^2\mu_0}S_1 + \frac{\epsilon_0 S}{4}\left[\int_0^L \epsilon_{eff}|E_y|^2dx + \frac{\mu_{eff}}{|\mu_s|^2}\int_0^L \mathrm{Re}(\epsilon_s\mu_s)|E_y|^2dx\right], \tag{7}$$

where S is the cross-section surface, $|E_0|$ is the incident electric-field amplitude, $\mu_{eff}=\mu_{effLIN}$, $\epsilon_{eff}=\epsilon_{effLIN}+\epsilon_{effNL}$, $\gamma_0=\sqrt{k_0^2\epsilon_b\mu_b-\beta^2}$, k_0 stands for the wave vector in vacuum, $\beta=\sqrt{k_0^2\epsilon_b\mu_b}\sin^2(\theta)$ is the propagation constant, θ is the angle of incidence and

$$S_1 = \mathrm{Re}\left[(\mu_s-\mu_b)\left(E_y^*\frac{E_y'}{\mu(x)}\right)\bigg|_{x=0}\right] + \mathrm{Re}\left[(\mu_b-\mu_s)\left(E_y^*\frac{E_y'}{\mu(x)}\right)\bigg|_{x=L}\right]. \tag{8}$$

The time-averaged incident power, obtained from the complex Poynting theorem, is $P_{in}=\gamma_0\epsilon_0c^2S|E_0|^2/(2\omega)$. The equation describing the dwell time now obtains the following form:

$$\tau_d = \frac{\omega}{2\gamma_0c^2|E_0|^2}\left[(\epsilon\mu)_{eff}\int_0^L|E_y|^2dx + \int_0^L\left(\frac{d(\omega\epsilon_{SNL})}{d\omega}+\frac{\mu_{eff}}{|\mu_s|^2}\mathrm{Re}(\mu_s)\epsilon_{SNL}\right)|E_y|^2dx\right] - \frac{\mu_{eff}}{|\mu_s|^2\omega}\mathrm{Im}(R) - \frac{2\mu_{eff}}{|\mu_s|^2\omega\gamma_0|E_0|^2}S_1. \tag{9}$$

Here, c stands for the speed of light in vacuum and $(\epsilon\mu)_{eff}=\epsilon_{effLIN}+\mu_{effLIN}\mathrm{Re}[(\epsilon_{LHM}+\epsilon_{D0})\mu_s]/|\mu_s|^2$.

In order to derive the relation between the dwell time and the group delay τ_g, it is necessary to find the dependence of $\int_0^L|E_y|^2dx$ on τ_g. Subtracting the conjugate of the Helmholtz equation multiplied by $dE_y/d\omega$ from the derivative of the Helmholtz equation with respect to ω, multiplied by E_y^*, and integrating this expression along the barrier, leads to the following relation:

$$\int_0^L|E_y|^2dx = \frac{\left[2\gamma_0\tau_g+2\frac{d\gamma_0}{d\omega}\mathrm{Im}(R)\right]|E_0|^2 - S_2 - S_3}{\frac{2\omega}{c^2}\epsilon_{LIN}+\frac{\omega^2}{c^2}\frac{d\epsilon_{LIN}}{d\omega}} - \frac{\frac{2\omega\,\mathrm{Re}(\mu_s)}{c^2}\int_0^L\left[\epsilon_{SNL}+\frac{\omega}{2}\frac{d\epsilon_{SNL}}{d\omega}\right]|E_y|^2dx}{\frac{2\omega}{c^2}\epsilon_{LIN}+\frac{\omega^2}{c^2}\frac{d\epsilon_{LIN}}{d\omega}}, \tag{10}$$

where

$$S_2 = \mathrm{Re}\left\{ (\mu_s - \mu_b)\frac{E_y'}{\mu(x)}\frac{dE_y^*}{d\omega} - \frac{d}{d\omega}\left[(\mu_s - \mu_b)\frac{E_y'}{\mu(x)}\right]E_y^* \right\}_{x=0}^{x=L}, \tag{11}$$

$$S_3 = 2k_0^2\int_0^L \mathrm{Im}(\epsilon_s\mu_s)\mathrm{Im}\left(E_y\frac{dE_y^*}{d\omega}\right)dx, \tag{12}$$

$$\epsilon_{LIN} = \mathrm{Re}[(\epsilon_{LHM} + \epsilon_{D0})\mu_s] - \epsilon_b\mu_b\sin^2(\theta), \tag{13}$$

and

$$\tau_g = |T|^2\frac{d\varphi_0}{d\omega} + |R|^2\frac{d\varphi_r}{d\omega}, \tag{14}$$

which represents the group delay, with $R=|R|\exp(i\varphi_r)$ and $T=|T|\exp(i\varphi_t)$ being the reflectance and the transmittance, respectively, and $\varphi_0=\varphi_t+\gamma_0 L$ the phase of the incident wave. In the case of lossless barrier, the terms $d\varphi_0/d\omega$ and $d\varphi_r/d\omega$ become equal, hence, $\tau_g=d\varphi_0/d\omega=d\varphi_r/d\omega$.

By using expressions (9) and (10), the relation between the group delay and the dwell time is obtained:

$$\tau_g = \tau_{deff} + \tau_{ieff} + \tau_{NL} + \tau_{loss} + \tau_{interface}, \tag{15}$$

where τ_{deff} is the effective dwell time and τ_{ieff} is the effective self-interference time which represents the time that the incident wave spends interfering with its reflected part on the cross-section surface of the barrier. Further, τ_{NL} originates from the nonlinearity of the barrier, τ_{loss} is the consequence of the losses in the barrier, while $\tau_{interface}$ occurs due to permeability difference between the waveguide and the barrier. The explicit expressions for all five defined times read

$$\tau_{deff} = \tau_d\frac{\tilde{\epsilon}_{LIN}}{(\epsilon\mu)_{eff}}, \tag{16}$$

$$\tau_{ieff} = \left[\frac{\tilde{\epsilon}_{LIN}}{(\epsilon\mu)_{eff}}\frac{\mu_{eff}}{\omega|\mu_s|^2} - \frac{1}{\gamma_0}\frac{d\gamma_0}{d\omega}\right]\mathrm{Im}(R), \tag{17}$$

$$\begin{aligned}\tau_{NL} = {} & \frac{\omega\,\mathrm{Re}(\mu_s)}{2\gamma_0 c^2}\int_0^L \tilde{\epsilon}_{SNL}\left|\frac{E_y}{E_0}\right|^2 dx \\ & - \frac{\tilde{\epsilon}_{LIN}}{(\epsilon\mu)_{eff}}\frac{\omega}{2\gamma_0 c^2}\left[\int_0^L \frac{d(\omega\epsilon_{SNL})}{d\omega}\left|\frac{E_y}{E_0}\right|^2 dx \right. \\ & \left. + \int_0^L \frac{\mu_{eff}}{|\mu_s|^2}\mathrm{Re}(\mu_s)\epsilon_{SNL}\left|\frac{E_y}{E_0}\right|^2 dx\right],\end{aligned} \tag{18}$$

$$\tau_{loss} = \frac{S_3}{2\gamma_0|E_0|^2}, \tag{19}$$

$$\tau_{interface} = \frac{S_2}{2\gamma_0|E_0|^2} + \frac{\tilde{\epsilon}_{LIN}}{(\epsilon\mu)_{eff}}\frac{2\mu_{eff}S_1}{\omega|\mu_s|^2\gamma_0|E_0|^2}, \tag{20}$$

where $\tilde{\epsilon}_{LIN/SNL}=2[\epsilon_{LIN/SNL}+(\omega/2)(d\epsilon_{LIN/SNL}/d\omega)]$.

III. NUMERICAL METHOD

Once the electric field inside the barrier is determined, all the other time-related quantities defined in Sec. II may be calculated. The distribution of this field depends on parameters that characterize the incident field, such as E_0, ω, θ, and on characteristics of the barrier itself. The starting equation for the calculation of electric field inside the barrier is the Helmholtz equation:

$$\frac{d^2E_y}{dx^2} - \frac{1}{\mu_s}\frac{d\mu_s}{dx}\frac{dE_y}{dx} + (k_0^2\epsilon_s\mu_s - \beta^2)E_y = 0. \tag{21}$$

Presuming that the permeability is homogenous inside the barrier, the second term on the left-hand side of the previous equation becomes equal to 0. By inserting the expressions for permittivity and permeability in the Helmholtz equation, we derive a second-order nonlinear differential equation:

$$\frac{d^2E_y}{dx^2} + [k_0^2(\epsilon_{D0} + \epsilon_{LHM})\mu_s - \beta^2]E_y + k_0^2\mu_s\alpha\frac{\dfrac{|E_y|^2}{E_c^2}}{1 + \kappa\dfrac{|E_y|^2}{E_c^2}}E_y = 0. \tag{22}$$

In order to calculate the distribution of the electric field inside the barrier, it is necessary to solve the differential equation described above taking into consideration the appropriate boundary conditions. This typical boundary value problem can be solved by the shooting method. In this case, the initial conditions are the values of the electric field and its derivative at the left boundary of the barrier:

$$E_y(x=0) = E_0 + E_0(R_r + iR_i), \tag{23}$$

$$\left.\frac{dE_y}{dx}\right|_{x=0} = \frac{\mu_s}{\mu_b}[i\gamma_0E_0 - i\gamma_0E_0(R_r + iR_i)], \tag{24}$$

where R_r and R_i are the variational parameters which represent the real and the imaginary part of reflectivity, respectively. The shooting method starts with solving the Helmholtz equation by using the fifth-order Runge-Kutta method for arbitrary initial values of R_r and R_i and calculating the electric field at the right boundary of the barrier $x=L$. The next step is the calculation of the expression $F(R_r,R_i)$ $=(dE_y/dx-i\gamma_0E_y)|_{x=L}$ which should be minimized by variation in the parameters R_r and R_i in order to satisfy the Sommerfeld radiation condition [23]:

$$\left.\left(\frac{dE_y}{dx} - i\gamma_0E_y\right)\right|_{x=L} = 0. \tag{25}$$

The minimization of the function $F(R_r,R_i)$ is based on the Broyden's method [24]. This method was used to calculate the distribution of the electric field and its derivative, as well as the reflectivity, transmittivity, and the absorption inside the barrier. Then, it was straightforward to calculate the quantities that stem from these variables such as the dwell time and the group delay.

IV. NUMERICAL RESULTS AND DISCUSSION

We presume that the barrier is placed in vacuum, with the effective index of refraction defined by n_{eff}

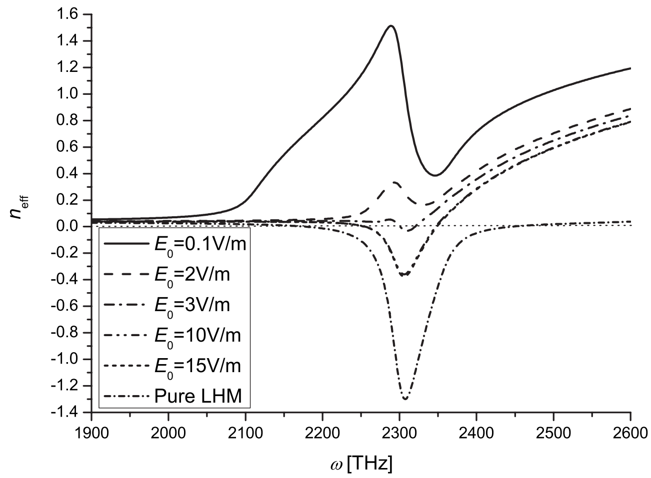

FIG. 2. Index of refraction n_{eff} versus incident frequency ω for $L=10^{-7}$ m, for a barrier placed in vacuum. The increment of the incident field amplitude shifts the peak of the refractive index toward higher frequencies.

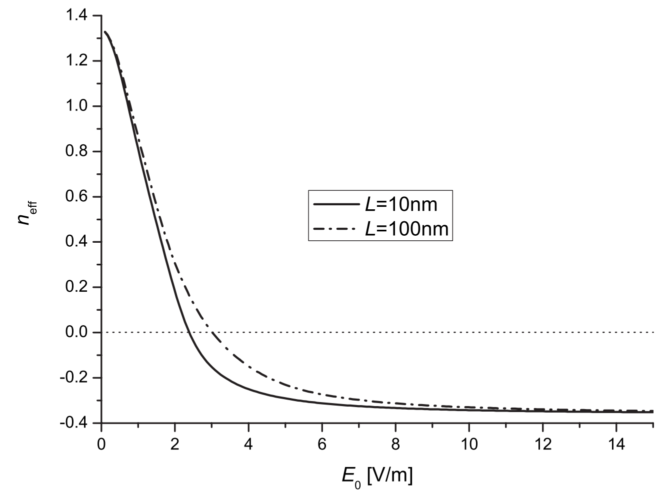

FIG. 3. Refractive index n_{eff} versus incident field amplitude E_0 for two different barrier lengths: $L=10$ nm and $L=100$ nm.

$=\mathrm{Re}(\int_0^L n(x)dx/L)$. In each of the following cases, the barrier parameters are chosen so to obtain τ_d, τ_g, and n_{eff} when the effects of nonlinearity and the left-handed property are the most pronounced, but also to describe realistic metamaterials [9]. We present numerical results for τ_d, τ_g, and n_{eff} as functions of ω for different values of the amplitude of the incident wave E_0, comparing two cases: SRRs and long metal wires embedded into dielectric with Kerr and with saturable nonlinearity. The mathematical model describing the propagation of EM waves in surroundings with Kerr nonlinearity is already included in the corresponding model for materials with saturable nonlinearity when the saturation strength vanishes ($\kappa=0$).

The refractive index is calculated from an expression $n=\sqrt{\epsilon\mu}$ if $\epsilon(\omega\to\infty)>0$ and $\mu(\omega\to\infty)>0$ as it has been derived in [25]. Assuming the following dependences: $\epsilon_s=|\epsilon_s|\exp(i\Phi_\epsilon)$, $\mu_s=|\mu_s|\exp(i\Phi_\mu)$, where $(\Phi_\epsilon,\Phi_\mu\in[0,\pi])$, since Γ_e, $\Gamma_m>0$, index of refraction can be derived as $n_s=\pm\sqrt{|\epsilon_s||\mu_s|}\exp[i(\Phi_\epsilon+\Phi_\mu)/2]$. In our case, it is evident from the Helmholtz equation that for $\omega\to\infty$, $E_y\to 0$, therefore $\epsilon_{NL}\to\epsilon_{D0}>0$. Also, it is obvious that $\epsilon_{LHM}(\omega\to\infty)\to\epsilon_\infty>0$ and $\mu_s(\omega\to\infty)\to 1>0$, thus, for all frequencies the refractive index can be represented by $n_s=\sqrt{\epsilon_s\mu_s}$. Consequently, it can be shown that the sign of the real part of refractive index is directly proportional to the sign of the expression $\mathrm{Re}(\epsilon_s)\mathrm{Im}(\mu_s)+\mathrm{Im}(\epsilon_s)\mathrm{Re}(\mu_s)$ [25].

The parameters of the observed material are $\epsilon_\infty=3.1$, $\omega_p=2700$ THz, $\omega_r=0$, $\Gamma_e=35$ THz, $F=0.052$, $\omega_0=2300$ THz, $\Gamma_m=35$ THz, $E_c=1$ V/m, $\epsilon_{D0}=2$, and the angle of incidence is $\theta=\pi/6$. A pure metamaterial (not embedded into a nonlinear dielectric) with these parameters has negative index of refraction in the range of 2178 THz$\leq\omega\leq$2445 THz (Fig. 2). However, taking into account ϵ_{D0}, the refractive index becomes positive for all incident frequencies. By embedding SRRs and metal wires into a material with self-defocusing saturable nonlinearity ($\alpha=-1$, $\kappa=1$), one can change its index of refraction by modifying the incident electric field. For small values of E_0, the change in refractive index is negligible, since the nonlinear part of dielectric permittivity is significantly smaller than its linear part. However, the increase in the incident field leads to a rapid growth of nonlinear part of dielectric constant, which leads to a decrease in the refractive index due to the self-defocusing property of the nonlinear dielectric. At a certain point, for sufficiently strong incident fields (around 3 V/m), the index of refraction becomes negative within a range of frequencies. Further increase in the incident field widens the range of frequencies in which the refractive index has negative values, again up to a point (around 10 V/m) after which the nonlinear contribution of dielectric constant saturates, which leads to saturation of the refractive index. This example shows that $n_{eff}(\omega)$ may be varied significantly by changing the incident field amplitude: from positive, over extremely small, up to saturation values. In case of self-focusing type of nonlinearity, the opposite effect occurs. By increasing E_0, n_{eff} increases for all frequencies, up to some value, after which it saturates. In this case ($\alpha=1$), n_{eff} remains positive for all frequencies and intensities of the incident field, which is not of interest for this observation.

The previously described effect can also be seen in Fig. 3, where the dependence of the refractive index on the incident field is depicted for two different barrier lengths for $\omega=2300$ THz. In the linear case, i.e., for small values of incident field amplitude, refractive index is not a function of the barrier length L, since the dielectric permittivity and magnetic permeability are not functions of electric field $E_y(x)$, thus of x coordinate in the barrier. On the other hand, if the incident field amplitude is too large ($E_0\geq 10$ V/m), the contribution of the dielectric permittivity that describes saturable nonlinearity is independent of $E_y(x)$ ($\epsilon_{SNL}\approx -1$), thus the whole dielectric permittivity is independent of x. This implies that the refractive index is not a function of the barrier length for extremely small or high values of the incident field. However, between these two cases, when 2 V/m $\leq E_0\leq 10$ V/m, saturable part of dielectric permittivity strongly depends on $E_y(x)$, thus on L. This dependence of ϵ_{SNL} on L clearly causes n_{eff} to be a function of L for the specified interval of incident field amplitudes. Therefore, it can be noted that the smaller the barrier length, the smaller the incident field is required to achieve the negative index of refraction. For the range of incident field amplitudes between 2 and 10 V/m, nonlinear contribution of the dielectric constant depends significantly on E_0, thus n_{eff} also has a strong dependence on E_0: from 1.4, it reduces to −0.35.

For nearly all frequencies of interest, the dwell time is increased by embedding SRRs and metal wires into a saturable nonlinearity dielectric (Fig. 4). However, around the frequency where the metamaterial becomes double-negative, i.e., where the dwell time reaches maximum, the nonlinearity decreases τ_d. Around the upper limit frequency of the metamaterial's double-negativity region, there is a local maximum which is a consequence of the nonzero angle of incidence and which increases by placing the metamaterial into a material with saturable nonlinearity. The absorption has a similar dependence on ω as the dwell time; therefore, it has the largest value around the frequencies at which the refractive index changes its sign. Also, the behavior of these quantities (the absorption and the dwell time) is in agreement with the Hartman effect; i.e., they saturate with the increase in the barrier length [26].

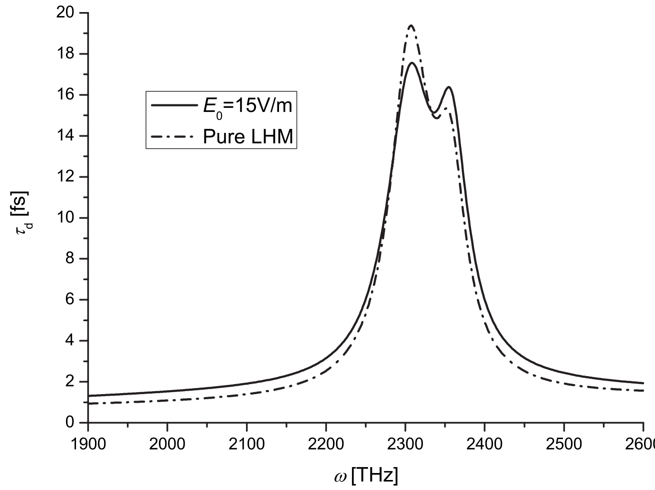

FIG. 4. Dwell time as a function of the incident wave frequency in case of a barrier made of SRRs and metal wires (dash-dotted line) and SRRs and metal wires embedded into a material with saturable nonlinearity (solid line). In both cases the barrier is placed in vacuum and $L=10^{-7}$ m.

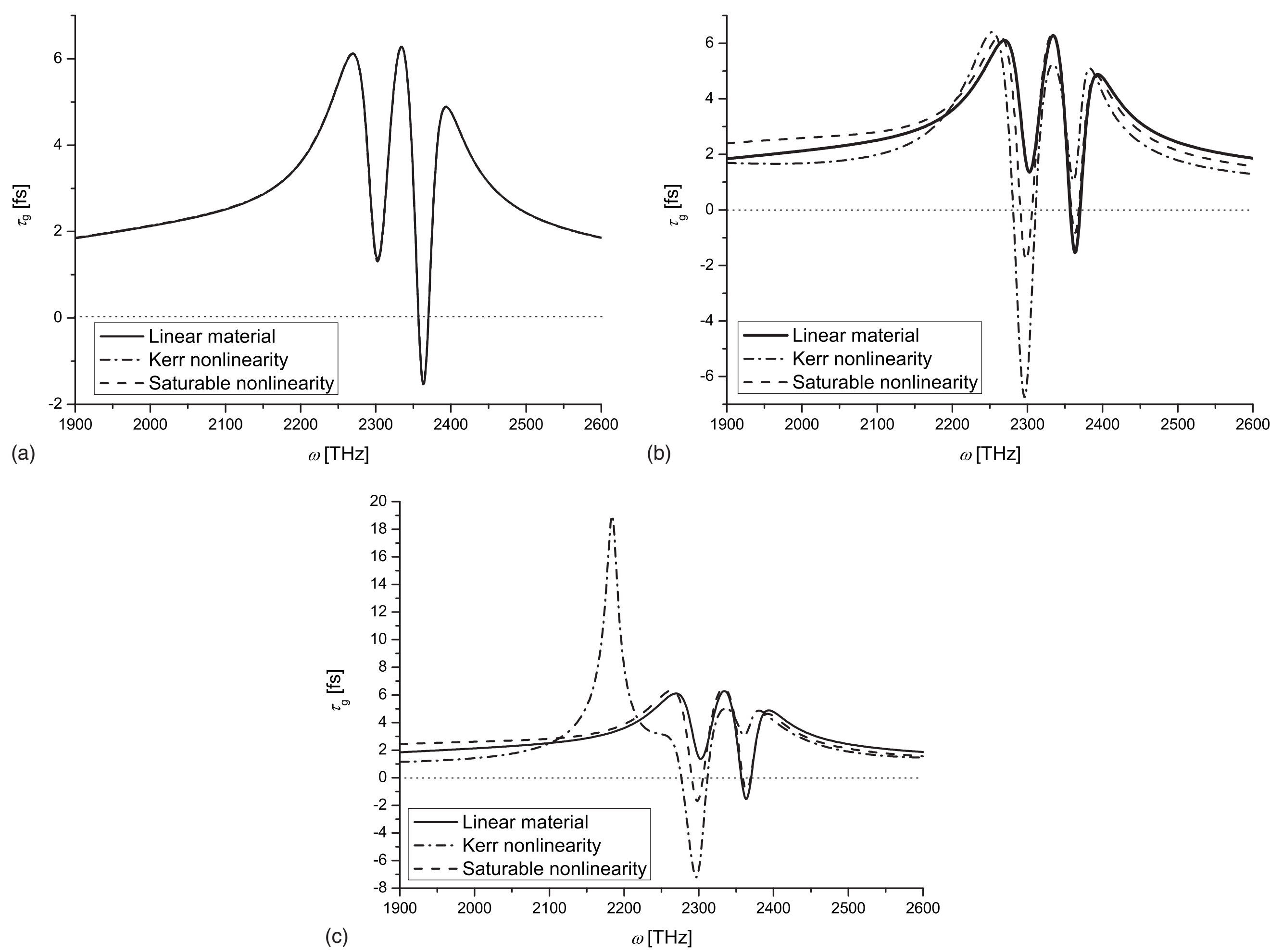

FIG. 5. Dependence of the group delay on incident wave frequency for a barrier made of SRRs and metal wires embedded into linear dielectric with permittivity $\epsilon_{D0}=2$ (solid line); a barrier made of SRRs and metal wires embedded into dielectric with Kerr nonlinearity (dash-dotted line) and a barrier made of SRRs and metal wires embedded into dielectric with saturable nonlinearity (dashed line) for three different values of incident field amplitude: (a) $E_0=0.1$ V/m, (b) $E_0=2$ V/m, and (c) $E_0=4$ V/m.

For low incident power, i.e., when $E_0=0.1$ V/m, τ_g is equal for all three media, since the nonlinear term of permittivity, in both the Kerr and the saturable media, becomes negligible compared to the linear part (Fig. 5). With the increase in amplitude of the incident field, the group delay in nonlinear materials becomes smaller and even negative at its first local minimum, which corresponds to the resonant frequency of the magnetic dipole oscillators. This may seem to contradict the classical causality principle, which states that the particle cannot exit a region before entering it. In that sense, the group delay must always have a positive value. This counterintuitiveness and apparent ambiguity in interpretation of the negative group delay stems from the intricacies involved in extending the group delay concept to the quantum domain [27]. Nevertheless, the negative group delay for a particle traversing a quantum well, subject to specific restrictions imposed on the particle energy and the well thickness, has been reported in recent works [28]. Because of the apparent analogy between the Schrödinger and the Helmholtz equations, one expects to find evidence of this phenomenon in electromagnetic wave propagation as well and such assumption has been verified by experimental data [29].

Also, the first local maximum of the group delay in nonlinear media is shifted toward lower frequencies and intensified. It is important to mention that the second local minimum for the Kerr medium, which is a consequence of the nonzero angle of incidence, becomes positive. This implies that in this type of media, at some frequencies, it is possible to manipulate the direction of wave propagation simply by changing the incident field amplitude. For $E_0=4$ V/m, τ_g is significantly smaller and negative only at its first local minimum, especially for the Kerr medium, while for the saturable nonlinear material it reaches its saturable value. Furthermore, the first peak for the Kerr medium reaches three times its previous value and shifts significantly for about 100 THz.

V. CONCLUSION

Analytic expressions for the tunneling times, in case of a barrier made of SRRs and long metal wires embedded in a material with saturable or Kerr nonlinearity, are derived. It is shown that the group delay can be represented as a sum of five contributions, among which are the dwell time and the self-interference time. The electric-field distribution inside the barrier is numerically calculated by the shooting method. Based on these calculations, a detailed numerical study of the influence of the nonlinearity on the spectral structure of the tunneling times is presented. Due to the nonlinear response of the metamaterial it is possible to change the sign of the refractive index and the direction of wave propagation by changing only the incident field amplitude, which strongly affects the spectral profiles of the tunneling times. The dwell time and the absorption have the highest values in frequency regions in which the refractive index sign and the wave propagation direction can be changed. The nonlinearity leads to the creation of two frequency regions in which it is possible to obtain negative values of the group delay. The effects of the negative group delay are more pronounced for the metamaterial with Kerr nonlinearity than the one with saturable nonlinearity. The negative group delay concept does not violate the causality principle; moreover, in microelectronics negative group delays can apparently be used to cancel out the positive group delays introduced by, e.g., transistor latency [30].

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