Dynamics of the inhomogeneous Dicke model for a single-boson mode coupled to a bath of nonidentical spin-1/2 systems

Oleksandr Tsyplyatyev and Daniel Loss
Department of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland
(Received 14 November 2008; published 5 August 2009)

We study the time dynamics of a single boson coupled to a bath of two-level systems (spins 1/2) with different excitation energies, described by an inhomogeneous Dicke model. Analyzing the time-dependent Schrödinger equation exactly, we find that at resonance the boson decays in time to an oscillatory state with a finite amplitude characterized by a single Rabi frequency if the inhomogeneity is below a certain threshold. In the limit of small inhomogeneity, the decay is suppressed and exhibits a complex (mainly Gaussian-like) behavior, whereas the decay is complete and of exponential form in the opposite limit. For intermediate inhomogeneity, the boson decay is partial and governed by a combination of exponential and power laws.

DOI: 10.1103/PhysRevA.80.023803 PACS number(s): 42.50.Ct

I. INTRODUCTION

Coherent interaction between light and matter [1] continues to receive strong interest due to significant experimental progress in various areas of physics. Prime examples are the achievement of Bose-Einstein condensation of cold-atom gases in electromagnetic traps [2], which made possible the coherent coupling of 10^5 atoms to a single photon of an optical resonator [3,4]. The time dynamics of quantum optical systems has received particular attention [5,6] due to fast optical probing techniques, especially in the context of quantum metrology based on cavity-QED systems containing atomic ensembles [7,8]. Advances in solid-state technology enabled the fabrication of optical microcavities in semiconductors where electron-hole excitations in quantum wells are strongly coupled to a photon eigenmode of the cavity [9,10]. Strong coupling of a transmission-line resonator to a Cooper-pair box [11] as well as coupling of a cavity to a single semiconductor quantum dot has been demonstrated [12,13]. Several schemes for quantum computing based on light-matter interaction have been proposed [14–18].

The theoretical understanding of all these coupled light-matter systems is based on a model introduced long ago by Dicke [19], which describes N two-level systems ("spin bath") with excitation energies ϵ_j coupled to a single-boson mode ω of the quantized light field [see Eq. (1) below]. For the special case of identical atoms ($\epsilon_j=\epsilon$) and constant coupling constant g_j between boson and spin bath, this model has been diagonalized [20], and the time dynamics has been obtained exactly [21]. For inhomogeneous g_j (but still constant ϵ_j) the boson was shown to oscillate with a single Rabi frequency $\Omega=\sqrt{N\langle g^2\rangle}$, where $\sqrt{\langle g^2\rangle}$ is an effective spin coupling. Also perturbative [22] and numerical [23] approaches to the time dynamics were considered.

In this paper we solve the quantum time dynamics of a single-boson mode coupled to a bath of *nonidentical* spin 1/2 characterized by inhomogeneous energy ("Zeeman") splittings ϵ_j with bandwidth Δ. In condensed-matter systems such energy inhomogeneities are generally expected, with a typical example being the exciton-polariton system where such inhomogeneities arise from the unavoidable disorder in a semiconductor [24]. In quantum optical systems atomic levels are usually quite perfect ($\epsilon_j\equiv\epsilon$); however, for example, in cold-atom QED systems such inhomogeneities can play a role as a trap that induces spatial variation in the magnetic field [25].

Analyzing the time-dependent Schrödinger equation exactly, we find that the bosonic occupation number decays only partially if the inhomogeneity Δ is below a threshold given by a single Rabi frequency Ω. Below the threshold the boson decays to an oscillatory state determined by Ω and a reduced amplitude that decreases with an increasing ratio Δ/Ω. The time decay is exponential for large spin-bath inhomogeneity $\Delta\gg\Omega$, is complex (mainly Gaussian-like) in the opposite limit $\Delta\ll\Omega$, and is a combination of exponential and power-law behaviors in the intermediate regime $\Delta\simeq\Omega$. These results are valid if the boson energy is tuned in resonance with the average spin excitation energy $\langle\epsilon\rangle-\omega=0$. With an increasing detuning $|\langle\epsilon\rangle-\omega|\gg\max\{\Omega,\Delta\}$, the time dynamics of the boson becomes suppressed.

The paper is organized as follows. In Sec. II we analyze the time-dependent Schrödinger equation and derive the exact solution in the Laplace domain. In Sec. III we consider rectangular and Gaussian distribution functions of ϵ_j in resonance with the boson mode, $\langle\epsilon\rangle=\omega$, to obtain the time evolution of the wave functions. Section IV contains the analysis and the discussion of a finite detuning, $\langle\epsilon\rangle\neq\omega$. In Appendixes A and B we give the details of the calculations in Secs. III and IV.

II. INHOMOGENEOUS DICKE MODEL

The Hamiltonian for the Dicke model governing the dynamics of a single-boson mode coupled to N two-level systems is given by

$$H=\omega b^{\dagger}b+\sum_{j=1}^{N}\epsilon_j S_j^z+\sum_{j=1}^{N}g_j(S_j^+b+S_j^-b^{\dagger}),\qquad(1)$$

where S_j^{α} are spin-1/2 operators, $S_j^{\pm}=S_j^x\pm iS_j^y$, and b ($b^{\dagger}$) is the standard Bose annihilation (creation) operator [26]. The total number of excitations, $L=n+\Sigma_j S_j^z$, is conserved in the Dicke model, where $n=b^{\dagger}b$ is the bosonic occupation number. The eigenvalues c of L are the so-called cooperation numbers, given by $c=\langle L\rangle$, where $\langle\cdots\rangle$ denotes the expectation value.

In the following we assume that the spin bath can be prepared in its ground state with all spins down, e.g., either dynamically or by thermal cooling [27]. Also, the mode ω is assumed to be empty or occupied by one boson only. The nonequilibrium dynamics of a single-boson excitation can then be initiated by a short radiation pulse from an external source. The dissipation of the boson mode, e.g., through leakage of photons through the mirrors that define an optical cavity, can be used to detect the dynamics if the cavity escape time exceeds the internal time scales of the system dynamics. In a multishot experiment [5,6] the probability of detecting a leaking photon at a given time is proportional to the boson expectation value. Next, we note that if initially the system has only one excitation, either in the spin or in the boson subsystem, the subsequent time evolution is restricted to this subspace and described by the general state

$$|\Psi(t)\rangle = \alpha(t)|\Downarrow,1\rangle + \sum_{j=1}^{N}\beta_j(t)|\Downarrow\uparrow_j,0\rangle \qquad (2)$$

with $c=-N/2+1$ and where $\alpha(t)$ and $\beta_j(t)$ are normalized amplitudes, $|\alpha(t)|^2+\Sigma_j|\beta_j|^2=1$, of finding either a state with one boson and no spin excitations present or a state with no boson and the jth-spin excited (flipped) [28].

The time evolution within this subspace is determined by the interaction term in Eq. (1) that transfers back and forth the excitations between the spin bath and the boson. Inserting $|\Psi(t)\rangle$ into the time-dependent Schrödinger equation, we obtain

$$-i\frac{d\alpha(t)}{dt} = -\sum_j \frac{(\epsilon_j-\omega)}{2}\alpha(t) + \sum_j g_j\beta_j(t),$$

$$-i\frac{d\beta_k(t)}{dt} = \sum_j (\epsilon_j-\omega)\left(\delta_{jk}-\frac{1}{2}\right)\beta_k(t) + g_k\alpha(t). \qquad (3)$$

In the above derivation we have subtracted the integral of motion ωL from the Hamiltonian (1) as it leads only to an overall phase of $|\Psi\rangle$ with no observable effect. The initial conditions $\alpha(0)=1$, $\beta_j(0)=0$ assumed in the following correspond to a singly occupied boson mode. The physical observable of interest is the time-dependent expectation value of the boson occupation number, which can be expressed in terms of the amplitude α as $\langle n(t)\rangle=\langle\Psi(t)|n|\Psi(t)\rangle=|\alpha(t)|^2$.

The set of equations, Eq. (3), is equivalent to the one obtained in the Weisskopf-Wigner theory in the study of bosonic systems [29] in contrast to spin 1/2 considered here. We solve Eq. (3) by making use of the Laplace transform, $\alpha(s)=\int_0^\infty dt\,\alpha(t)e^{-st}$, where $\Re s>0$. In the Laplace domain we obtain then a system of linear algebraic equations. By solving them, we find

$$\alpha(s) = \frac{i}{is + \frac{N\langle\omega-\epsilon_j\rangle}{2} - \left\langle\frac{g_j^2N}{is+\langle\omega-\epsilon_j\rangle N/2-\omega+\epsilon_j}\right\rangle}, \qquad (4)$$

where $\langle\cdots\rangle=(\Sigma_j\cdots)/N$. The sum over j depends on the particular form of the inhomogeneities of ϵ_j and g_j. To be specific, we consider the following limiting cases when ϵ_j varies on a much longer or shorter length scale than g_j, which also includes the case with either ϵ_j or g_j being constant. In this case and for large N the sum can be substituted by an integral, $(\Sigma_j\cdots)/N\to\int d\epsilon dg\, P(\epsilon)Q(g)$, where $P(\epsilon)$ and $Q(g)$ are independent normalized distribution functions of the excitation energies and the coupling constants, respectively. The integral over g in Eq. (4) separates and gives an effective coupling $\sqrt{\langle g^2\rangle}$ [30]. Further, we assume that the boson mode ω is tuned in resonance with the spin bath, i.e., $\omega-\langle\epsilon\rangle=0$.

III. INVERSE LAPLACE TRANSFORM

The inverse Laplace transform of Eq. (4) depends on the particular form of $P(\epsilon)$ that determines the analytical structure of $\alpha(s)$. We will analyze several cases below. If the spin bath is homogeneous then $P(\epsilon)=\delta(\epsilon-\omega)$, and $\alpha(s)$ has two poles on the imaginary axis at $s=\pm i\sqrt{N\langle g^2\rangle}$, with the associated residues of 1/2. In the time domain these poles give $\alpha(t)=\cos(\Omega t)$, where $\Omega=\sqrt{N\langle g^2\rangle}$ is the collective Rabi frequency due to all N spins. This agrees with the result obtained from exact diagonalization [21].

Next, we consider an inhomogeneous spin bath with excitation energies spread over a band of width Δ, for which we have $P(\epsilon)=\theta(-\epsilon+\omega+\Delta/2)\theta(\epsilon-\omega+\Delta/2)/\Delta$, where $\theta(x)$ is the step function. This case is realized e.g., for $\epsilon_j=j\Delta/N$, where $-N/2\le j\le N/2$, i.e., spins in a magnetic field with constant gradient. The integral over ϵ in Eq. (4) gives

$$\alpha(s) = \frac{i}{is + \frac{N\langle g^2\rangle}{\Delta}\ln\left(\frac{is-\Delta/2}{is+\Delta/2}\right)}. \qquad (5)$$

Note that the inverse Laplace transform of Eq. (4) is in principle a quasiperiodic function of t. Therefore, Eq. (5) is correct up to the Poincaré recurrence time t_p, which we can estimate as follows. We evaluate the discrete sum over ϵ_j exactly, expand it in $1/N$, and estimate the time at which corrections to the logarithmic term in Eq. (5) (due to discreteness of the sum) become important to be $t_p=N/\Delta$. Thus, the following time behavior is valid for times less than t_p $=N/\Delta$. For small N it is more convenient to find the few poles of Eq. (4) directly and analyze $\alpha(t)$ numerically as a sum of few harmonic modes rather than to use Eq. (5).

We discuss now the analytical structure of $\alpha(s)$ in Eq. (5). There are two branch points at $s=\pm i\Delta/2$ due to the logarithm. We choose the branch cut as a straight line between these two points. In addition, there are two poles at s $=\pm is_0$ given by the zeroes of the denominator where s_0 is a real and positive solution of

$$\exp\left(-\frac{s_0\Delta}{N\langle g^2\rangle}\right) = \frac{s_0-\Delta/2}{s_0+\Delta/2}. \qquad (6)$$

In the time domain, the amplitude α has two contributions, $\alpha=\alpha_p+\alpha_c$. One is given by the poles

$$\alpha_p(t) = \frac{2}{1 + N\langle g^2\rangle/(s_0^2 - \Delta^2/4)}\cos(s_0 t). \tag{7}$$

This contribution describes a residual oscillation at long times with amplitude that is reduced from the initial value $\alpha(0)=1$. The other one is given by the integral enclosing the branch cut,

$$\alpha_c(t) = \int_0^1 dy \frac{(4N\langle g^2\rangle/\Delta^2)\cos(y\Delta t/2)}{\left[y - \frac{2N\langle g^2\rangle}{\Delta^2}\ln\left(\frac{1+y}{1-y}\right)\right]^2 + \left(\frac{2\pi N\langle g^2\rangle}{\Delta^2}\right)^2}. \tag{8}$$

This contribution describes the decay that occurs due to destructive interference of many modes forming a continuous spectrum (for large N).

The integral in Eq. (8) can be approximated quite accurately for $t \gg 2/\Delta$. Due to the fast oscillating cosine, the main contribution to the integral comes from $y \lesssim 2/\Delta t \ll 1$. Expansion of the logarithm in Eq. (8) for small y permits us to evaluate the integral in terms of the integral sine and cosine. An expansion of these special functions for $\Delta t/2 \gg 1$ gives

$$\alpha_c(t) = \frac{\Delta^2}{N\langle g^2\rangle}\left(\frac{Ae^{-A\Delta t/2}}{2\pi} + \frac{A^2\sin(\Delta t/2)}{\pi^2(1+A^2)\Delta t/2}\right), \tag{9}$$

where $A=\pi/2/|1-\Delta^2/4N\langle g^2\rangle|$. Note that, for vanishing coupling g, $\alpha_p(t)$ vanishes and $\alpha_c(t)$ tends to 1. Further, the integrand in Eq. (8) can be expanded for $\Delta^2/N\langle g^2\rangle$ for $\Delta^2 \ll N\langle g^2\rangle$. The leading term is linear in $\Delta^2/N\langle g^2\rangle$ and the remaining integral in the prefactor is a complicated decaying function of t, which we approximate qualitatively. First, we perform a change of variable: $y=\tanh(x)$ turning the denominator into $1/f=\exp[-\ln(f)]$, where we expand $\ln(f)$ up to x^2 and linearize the argument of the cosine in x for $x \ll 1$. Finally, as a result of the Gaussian integral over x, we obtain a Gaussian decay law

$$\alpha_c(t) = \frac{\Delta^2}{2\pi N\langle g^2\rangle}\sqrt{\frac{\pi}{\pi^2+4}}\exp\left(-\frac{\pi^2\Delta^2 t^2}{4(\pi^2+4)}\right). \tag{10}$$

This approximation agrees reasonably well with Eq. (8) when evaluated numerically for $t < 6/\Delta$ but breaks down for $t > 6/\Delta$ where Eq. (9) is valid (see Fig. 1).

The time dynamics of $\langle n(t)\rangle = |\alpha(t)|^2$ can be classified in terms of the ratio Ω/Δ, with Rabi frequency $\Omega=\sqrt{N\langle g^2\rangle}$. If the inhomogeneity of the spin bath is small, $\Delta \ll \Omega$, the boson oscillates with a single frequency like in the homogeneous case. The main contribution to $\alpha(t)$ comes from poles (7) with $s_0=\Omega+\Delta^2/24\Omega$, which is shifted with respect to the homogeneous system. The amplitude of $\alpha(t)$ is only slightly reduced from its initial value, $1-\Delta^2/12\Omega^2$. The decay law to this value is mainly Gaussian-like, Eq. (10), with the decay time $t_1 \approx 2.4/\Delta$ (see Fig. 1). If the spin bath is strongly inhomogeneous, $\Delta \gg \Omega$, the boson mode decays completely from $\alpha(0)=1$ to 0. The main contribution to $\alpha(t)$ comes from the branch cut, Eq. (9), with $A \approx 2\pi\Omega^2/\Delta^2$, whereas the pole contribution is exponentially small. The decay behavior is mainly exponential with time scale $t_2 \approx \Delta/\pi\Omega^2$. At long times $t \gg t_2$ the second term in Eq. (9) becomes dominant, exhibiting a slow power-law decay.

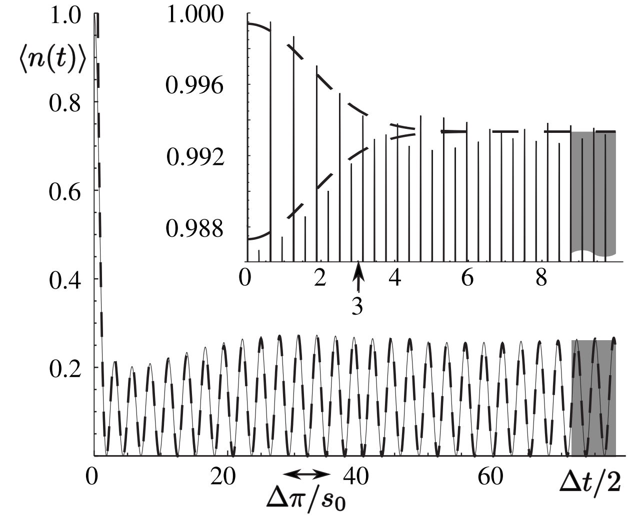

FIG. 1. Time evolution of the boson $\langle n(t)\rangle = |\alpha(t)|^2$ obtained from numerical evaluation of Eqs. (7) and (8) (full lines). Period of oscillation is $T=2\pi/s_0$ and gray bars are $|\alpha_p(0)|^2$. Main plot illustrates the intermediate regime with a partial decay and a combination of exponential and power decay laws: $\Delta/\Omega=2.2$; dashed line is the asymptote, $|\alpha_p(t)+\alpha_c(t)|^2$, from Eqs. (7) and (9), $s_0=0.57\Delta$, and $|\alpha_p(0)|^2=0.26$. Inset illustrates the small inhomogeneity regime with a parametrically small decay and a Gaussian-like decay law: $\Delta/\Omega=0.2$; dashed line is the asymptote, $|\alpha_p(0)\pm\alpha_c(t)|^2$, from Eqs. (7) and (10). The decay of $|\alpha(t)|^2$ is small, ($|\alpha_p(0)|^2=1-\Delta^2/6\Omega^2$), and the main contribution comes from α_p.

In the intermediate regime, $\Delta \simeq \Omega$, the time decay is only partial, with the amplitude of the residual oscillation of $\alpha(t)$ being less than unity but staying constant in time. Its precise value can be found from the numerical solution of Eqs. (6) and (7). The decay displayed in Eq. (9) is governed by a combination of exponential and power-law behaviors. As $A \simeq 1$ and $s_0 \simeq \Omega \simeq \Delta$ there is no clear separation of time scales coming from the exponential, the inverse power-law, and the oscillatory contributions (see Fig. 1). Note that in case of $\Delta=2\Omega$ the first term in Eq. (9) vanishes; thus, the decay in this particular case is purely power law. The nonstandard dynamics, in particular the nonexponential decay in the intermediate regime, is a manifestation of the quantum nature of the system. For other models with non-Markovian decay, see e.g., [31,32].

For a Gaussian distribution $P(\epsilon)=\exp[-(\epsilon-\omega)^2/\Delta^2]/\sqrt{\pi}\Delta$, the dynamics we find is qualitatively the same as the one obtained before for the rectangular distribution (see Appendix A). The ϵ integral in Eq. (4) leads to the complex error function of s. In Laplace space, $\alpha(s)$ exhibits one branch cut along the imaginary axis that vanishes at $\pm i\infty$. In the time domain, $\alpha(t)$ is given by an integral around this branch cut. In the limit of $\Delta \ll \Omega$, we recover the previous result for the homogeneous spin bath. In the opposite limit of strong inhomogeneity, $\Delta \gg \Omega$, we obtain the same result as in Eq. (9) up to numerical prefactors $\sqrt{\pi}$.

The physical interpretation of the decay is as follows. The boson flips, say, spin j, and then this spin precesses for some time with a frequency ϵ_j before this excitation gets transferred back to the boson. The acquired phase of the boson is thus different for each particular spin. The sum over these random phases eventually leads to a destructive interference (for $N \gg 1$) and thus to a decay.

IV. FINITE DETUNING

Next, we analyze the effect of finite detuning. If the spin bath is homogeneous, a small detuning $|\omega-\epsilon| \ll \Omega$ forces $\alpha(t)$ to oscillate with two distinct frequencies $(N-1)(\omega-\epsilon)/2 \pm \sqrt{(\epsilon-\omega)^2+\Omega}/2$ instead of only one Ω. A large detuning $|\omega-\epsilon| \gg \Omega$ suppresses the dynamics of $\alpha(t)$. The phase of the wave function oscillates with frequency $N(\omega-\epsilon)/2$, but the amplitude stays constant at the initial value of $\alpha(0)=1$ up to a small correction on the order of $\Omega^2/(\omega-\epsilon)^2$.

In the inhomogeneous case we perform a similar calculation as for zero detuning and obtain $\alpha(s)$ with an analytical structure similar to Eq. (5) (see Appendix B). There are two poles on the imaginary axis and a branch cut that is responsible for the relaxation. Explicit expressions for $a_p(t)$ and $a_c(t)$, that are similar to Eqs. (7) and (8), are obtained in Appendix B [see Eqs. (B5) and (B6)]. For small detuning $|\langle\epsilon\rangle-\omega| \ll \Omega$, two poles that emerge are not complex conjugates of each other and thus lead to two distinct frequencies of the residual oscillations of $\alpha(t)$. Large detuning $|\langle\epsilon\rangle-\omega| \gg \max\{\Delta,\Omega\}$ suppresses the relaxation and any long-time dynamics. The main contribution to $\alpha(t)$ comes from one of the poles with residue $1-\Omega^2/(\omega-\langle\epsilon\rangle)^2$. Thus, the initial value $\alpha(0)=1$ remains almost unaltered under evolution independent of the ratio Ω/Δ.

The dynamics at large detuning can also be analyzed using perturbation theory. Applying a Schrieffer-Wolff transformation to the Dicke Hamiltonian, the boson-spin coupling can be removed to lowest order in g and thereby an effective XY spin coupling within the spin bath is obtained [18]. As a result, the boson number n and the z component of the total spin $\Sigma_j S_j^z$ are conserved separately by this effective Hamiltonian. Thus, again, the initially excited boson mode will remain unaltered under the evolution in the leading order of the perturbation. However, there is a virtual boson process that induces the dynamics within the spin bath.

V. CONCLUSIONS

In conclusion, we analyzed the dynamics of a single-boson mode coupled to an inhomogeneous spin bath exactly and found a complex decay behavior of the boson. While we focused in this work on particular inhomogeneities of the spin-bath excitation energies, it is straightforward to apply the approach presented here to other cases.

ACKNOWLEDGMENTS

We thank M. Duckheim and M. Trif for discussions. We acknowledge support from the Swiss NSF, the NCCR Nanoscience Basel, the JST ICORP, and the DARPA Quest.

APPENDIX A: GAUSSIAN DISTRIBUTION OF THE SPINS' SPLITTING ENERGIES

Here we derive the time dynamics resulting from the Gaussian distribution function of ϵ, $P(\epsilon)=\frac{1}{\sqrt{\pi}\Delta}e^{-(\epsilon-\omega)^2/\Delta^2}$. Performing the integral over ϵ in Eq. (4), we obtain

$$\alpha(s) = \frac{1}{s + \frac{\sqrt{\pi}N\langle g^2\rangle}{\Delta}\varpi\left(\imath\frac{s}{\Delta}\right)}, \tag{A1}$$

where $\varpi(z)$ is defined in the upper and the lower complex half planes separately as

$$\varpi(z) = \begin{cases} w(z), & \mathrm{Im}\, z \geq 0 \\ -w(-z), & \mathrm{Im}\, z < 0, \end{cases} \tag{A2}$$

and where $w(z)=e^{-z^2}\mathrm{erfc}(-\imath z)$ is the error function. The function $\varpi(z)$ has a branch cut along the real axis, $\lim_{\delta\to 0}\varpi(\pm\imath\delta)=\pm 1$, which vanishes at infinity,

$$\lim_{x\to\pm\infty}\lim_{\delta\to 0}\varpi(x\pm\imath\delta) = \lim_{x\to\pm\infty} e^{-x^2}[\pm 1+\mathrm{erf}(\imath x)] = 0. \tag{A3}$$

The inverse Laplace transform is given by an integral around the entire imaginary axis

$$\alpha(t) = -\frac{N\langle g^2\rangle}{2\sqrt{\pi}\Delta^2}\int_{-\infty}^{\infty} dy \frac{e^{-\imath yt/2\Delta}e^{-4y^2}}{\left(\imath y - \frac{\sqrt{\pi}N\langle g^2\rangle}{2\Delta^2}w(2y)\right)^2 + \frac{\sqrt{\pi}N\langle g^2\rangle e^{-4y^2}}{\Delta^2}\left(\imath y - \frac{\sqrt{\pi}N\langle g^2\rangle}{2\Delta^2}w(2y)\right)}, \tag{A4}$$

where the substitutions $s=\imath 2\Delta y$ and $\omega(-z)=2e^{-z^2}-\omega(z)$ were used.

For $\Delta \ll \sqrt{N\langle g^2\rangle}$, Eq. (A1) can be expanded in the small parameter Δ. The leading term has an analytical structure similar to Eq. (5). There are two symmetric poles on the imaginary axis and a finite length branch cut between $s=\pm\imath 2\Delta$. The contribution from the poles is

$$\alpha_p(t) = \frac{2\cos(s_0 t)}{1-N\langle g^2\rangle/s_0^2}. \tag{A5}$$

Using the large z asymptotics of the error function $\mathrm{erfc}(z) = \frac{e^{-z^2}}{\sqrt{\pi}z}(1-\frac{1}{2z^2})$, the two poles are given by $s_0=\pm\imath\sqrt{N\langle g^2\rangle}$. The residues at this poles are

$$\operatorname*{Res}_{s=s_0} \alpha(s)e^{st} = \frac{e^{s_0 t}}{2}. \tag{A6}$$

Thus, the contribution from the poles is dominant. In this limit we recover the noninteracting case, a single Rabi oscillation,

$$\alpha(t) = \cos(\sqrt{N\langle g^2\rangle}t). \tag{A7}$$

In the opposite regime $\Delta \gg \sqrt{N\langle g^2\rangle}$ there are no poles and there is just a single branch cut. The long-time asymptotics can be evaluated by expanding the denominator for small y and approximating $e^{-4y^2} \approx 1$ in the numerator,

$$\alpha_c(t) = \frac{N\langle g^2\rangle}{\sqrt{\pi}\Delta^2}\int_0^1 dy \frac{\cos(yt/2\gamma)}{y^2 + \left(\frac{\sqrt{\pi}N\langle g^2\rangle}{2\Delta^2}\right)^2}. \tag{A8}$$

This integral, up to a numerical factor, is the same as in Eq. (8) in this limit.

APPENDIX B: CALCULATION FOR $\langle \epsilon \rangle \neq \omega$

Here we assume that the detuning is finite $\gamma = \langle \epsilon \rangle - \omega \neq 0$. We repeat the same steps as before, and similarly to the zero detuning case we obtain in the Laplace domain

$$\alpha(s) = \frac{\imath}{\imath s + \frac{N\gamma}{2} + \frac{N\langle g^2\rangle}{\Delta}\ln\left(\frac{\imath s + (N-2)\gamma/2 - \Delta/2}{\imath s + (N-2)\gamma/2 + \Delta/2}\right)}. \tag{B1}$$

This function is characterized by two poles and one branch cut.

The two poles are given by zeroes of the denominator $s = \imath(N\gamma/2 + s_{1,2})$, where $s_{1,2}$ are the solutions of

$$\exp\left(-\frac{s\Delta}{N\langle g^2\rangle}\right) = \frac{s-\gamma-\Delta/2}{s-\gamma+\Delta/2}. \tag{B2}$$

This equation is not symmetric with respect to $s \to -s$; thus, the two poles are not symmetric. The residues of the poles are given by

$$\operatorname*{Res}_{s} \alpha_s e^{st} = \frac{1}{1 + \frac{N\langle g^2\rangle}{(s_{1,2}-\gamma)^2 - \Delta^2/4}} e^{st}. \tag{B3}$$

Performing the inverse Laplace transformation, we obtain similarly to Eq. (6)

$$\alpha_p(t) = \sum_{k=1,2} \frac{e^{\imath N\gamma t/2 + \imath s_k t}}{1 + \frac{N\langle g^2\rangle}{(s_k-\gamma)^2 - \Delta^2/4}}. \tag{B4}$$

The branch points are $s = \imath(N\gamma/2 \pm \Delta/2)$. Similarly, to the case of zero detuning, the contribution from the branch cut is given by the integral

$$\alpha_c(t) = \frac{2N\langle g^2\rangle e^{\imath[(N-2)\gamma+\Delta]t/2}}{\Delta^2}\int_{-1}^{1} dy \times \frac{e^{\imath y\Delta t/2}}{\left[y - \frac{2\gamma}{\Delta} - \frac{2N\langle g^2\rangle}{\Delta^2}\ln\left(\frac{1+y}{1-y}\right)\right]^2 + \left(\frac{2\pi N\langle g^2\rangle}{\Delta^2}\right)^2}. \tag{B5}$$

At small detuning $|\omega - \langle\epsilon\rangle| \ll \max(\Delta/2, N\langle g^2\rangle)$ there are two distinct frequencies in Eq. (B4); thus, the final state oscillates with two frequencies. For a large detuning $|\omega - \langle\epsilon\rangle| \gg \max(\Delta/2, N\langle g^2\rangle)$, the relaxation is suppressed. In the limit of strong detuning the roots of Eq. (B2) are given by $s_1 = -2N\langle g^2\rangle/\gamma$ and $s_2 = \gamma - \Delta/2$. The residue at s_2 is exponentially small and the contribution from the poles is given only by the pole s_1,

$$\alpha_p(t) = \frac{e^{-\imath 2N\langle g^2\rangle t/\Delta + \imath N\gamma t/2}}{1 + \frac{N\langle g^2\rangle}{(2N\langle g^2\rangle/\gamma + \gamma)^2 - \Delta^2/4}} \approx e^{\imath N\gamma t/2}. \tag{B6}$$

In this result the amplitude of $\alpha(t)$ remains constant in time. From the initial condition $\alpha_p(0) + \alpha_c(0) = 1$, the contribution from the branch cut is negligible, and therefore there is no decay for sufficiently strong detuning. The corrections to this result are small and on the order of $\max(\Delta/2, N\langle g^2\rangle)/|\omega - \langle\epsilon\rangle|$.

[1] C. W. Gardiner and P. Zoller, *Quantum Noise* (Springer, New York, 2004).
[2] K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[3] T. Aoki, B. Dayan, E. Wilcut, W. P. Bowen, A. S. Parkins, T. J. Kippenberg, K. J. Vahala, and H. J. Kimble, Nature (London) **443**, 671 (2006).
[4] F. Brennecke, T. Donner, S. Ritter, T. Bourdel, M. Köhl, and T. Esslinger, Nature (London) **450**, 268 (2007).
[5] G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987).
[6] M. Greiner, O. Mandel, T. W. Hänsch, and I. Bloch, Nature (London) **419**, 51 (2002).
[7] D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, Phys. Rev. A **50**, 67 (1994).
[8] A. K. Tuchman, R. Long, G. Vrijsen, J. Boudet, J. Lee, and M. A. Kasevich, Phys. Rev. A **74**, 053821 (2006).
[9] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szymańska, R. André, J. L. Staehli, V. Savona, P. B. Littlewood, B. Deveaud, and Le Si Dang, Nature (London) **443**, 409 (2006).
[10] R. Balili, V. Hartwell, D. Snoke, L. Pfeiffer, and K. West, Science **316**, 1007 (2007).
[11] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature

(London) **431**, 162 (2004).

[12] J. Berezovsky, M. H. Mikkelsen, N. G. Stoltz, L. A. Coldren, and D. D. Awschalom, Science **320**, 349 (2008).

[13] K. Hennessy, A. Badolato, M. Winger, D. Gerace, M. Atatüre, S. Gulde, S. Fält, E. L. Hu, and A. Imamoğlu, Nature (London) **445**, 896 (2007).

[14] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).

[15] A. Imamoğlu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. **83**, 4204 (1999).

[16] L. Childress, A. S. Sorensen, and M. D. Lukin, Phys. Rev. A **69**, 042302 (2004).

[17] G. Burkard and A. Imamoğlu, Phys. Rev. B **74**, 041307(R) (2006).

[18] M. Trif, V. N. Golovach, and D. Loss, Phys. Rev. B **77**, 045434 (2008).

[19] R. H. Dicke, Phys. Rev. **93**, 99 (1954).

[20] M. Tavis and F. W. Cummings, Phys. Rev. **170**, 379 (1968).

[21] F. W. Cummings and A. Dorri, Phys. Rev. A **28**, 2282 (1983).

[22] M. Kozierowski, A. A. Mamedov, and S. M. Chumakov, Phys. Rev. A **42**, 1762 (1990); I. Sainz, A. B. Klimov, and S. M. Chumakov, J. Opt. B: Quantum Semiclassical Opt. **5**, 190 (2003).

[23] C. E. Lopez, H. Christ, J. C. Retamal, and E. Solano, Phys. Rev. A **75**, 033818 (2007).

[24] For an exciton-polariton system, the spin is electron-hole excitation. In [9], $\Omega \simeq 26$ meV and $\omega \simeq 1.7$ eV. The disorder in a semiconductor can be 0.1–50 meV.

[25] For a cold gas system, single spin is a hyperfine state of an atom. In [4], $\Omega \simeq 2$ GHz and $\omega \simeq 7$ GHz. In [8], $\Omega \simeq 10$ MHz and $\omega \simeq 0.7$ GHz. From 10 T, gradient of the magnetic field inhomogeneity can be estimated as $\Delta \simeq 10$ MHz.

[26] The Dicke model is valid near the resonance between boson and spin-bath energies, $|\omega - \epsilon_j| \ll \omega, \epsilon_j$. Still, if $\Delta \ll \omega$, a relatively large detuning $|\langle \epsilon \rangle - \omega| > \Delta, \Omega$ can also be studied within this model.

[27] For high temperatures, $T \gg N\epsilon$, thermal spin excitations get transferred collectively to the boson. Such a spontaneous high population ($n \gg 1$) of a photon mode leads to the Dicke "superradiance" effect [19].

[28] A similar ansatz is used in the central spin model describing the inhomogeneous isotropic interaction between a single-electron spin and a nuclear-spin bath [31].

[29] Y. Yamamoto and A. Imamoğlu, *Mesoscopic Quantum Optics* (John Wiley and Sons, Inc., New York, 1999).

[30] If g_i and ϵ_i are correlated, e.g., $P(\epsilon, g) = \delta(\epsilon - g)\theta(-\epsilon + \omega + \Delta/2)\theta(\epsilon - \omega + \Delta/2)/\Delta$, the integrals over g and ϵ do not separate in Eq. (4) even for $N \gg 1$.

[31] A. V. Khaetskii, D. Loss, and L. Glazman, Phys. Rev. Lett. **88**, 186802 (2002).

[32] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).