PHYSICAL REVIEW A **80**, 022315 (2009)

Entanglement properties of non-Gaussian resources generated via photon subtraction and addition and continuous-variable quantum-teleportation improvement

Yang Yang and Fu-Li Li*

MOE Key Laboratory for Nonequilibrium Synthesis and Modulation of Condensed Matter, and Department of Applied Physics, Xi'an Jiaotong University, Xi'an 710049, China

(Received 29 April 2009; published 12 August 2009)

The entanglement properties of non-Gaussian states are investigated, which are obtained by performing the photon addition, photon subtraction, photon-addition-then-subtraction, and photon-subtraction-then-addition operations on the two-mode squeezed vacuum state. We show that the partial von Neumann entropy of all the resulting states is greater than that of the original squeezed state, but only the photon-subtracted states and the photon-added-then-subtracted states have the stronger Einstein-Podolsky-Rosen correlation than the original squeezed state. Quantum teleportation of Braunstein and Kimble protocol is studied for coherent states, squeezed states, and mixed Gaussian states with the non-Gaussian entangled resources. For all the states to be teleported, the fidelity with the photon-subtracted and the photon-added-then-subtracted entangled resources is higher than that with the two-mode squeezed vacuum resource. Based on Bures fidelity, we find that quantum teleportation for mixed and classical single-mode Gaussian states is more faithful than for single-mode Gaussian states with high purity and nonclassicality.

DOI: 10.1103/PhysRevA.80.022315 PACS number(s): 03.67.Hk, 03.67.Mn, 42.50.Dv

I. INTRODUCTION

Gaussian-type entangled resources such as two-mode squeezed states and Gaussian operations that can be implemented by any combination of linear, quadratic optics, and homodyne detection have been widely used in the quantum information processing of continuous variables. However, it has been shown that Gaussian entangled resources and Gaussian operations have some restrictions. For example, quantum speed up is impossible for harmonic oscillators by Gaussian operations with Gaussian inputs [1,2], and entanglement distillation from two Gaussian entangled states is impossible only by Gaussian local operations and classical communication [3–5]. Moreover, it has been proved very recently that Gaussian operations are of no use for protecting Gaussian states against Gaussian errors in quantum communication protocols [6]. Therefore, it is desirable to seek non-Gaussian resources and operations which can be more efficient in the quantum information processing. Performing photon subtraction and addition on a given Gaussian state is one possible approach to generate non-Gaussian entangled resources.

In recent years, the progress in experimental technique has made it possible to perform photon subtraction and addition on a given input state. In fact, photon-subtracted squeezed states [7], photon-added coherent states [8,9], and thermal states [10,11] have been generated in experiments. Recently, photon-added-then-subtracted and photon-subtracted-then-added thermal states have also been generated [11]. Based on the photon-addition and -subtraction experimental success, Kim *et al.* [12] proposed an experiment scheme to directly prove the commutation relation between bosonic annihilation and creation operators. Superposition of quantum states such as coherent states has a very important role for fundamental tests of quantum theory. Marek *et al.* [13] showed that superposition of coherent states can be generated with nearly perfect fidelity by consecutively applying photon subtraction from a squeezed vacuum state. Nonclassicality of the states generated from the photon-addition and -subtraction process has also been investigated both experimentally [7–11] and theoretically [14–18]. It is shown that the photon-subtracted Gaussian state is nonclassical if and only if the original Gaussian state is nonclassical, while the photon-added, photon-added-then-subtracted, and photon-subtracted-then-added Gaussian states are always nonclassical no matter whether the original Gaussian state is nonclassical or not; besides, nonclassicality of these states is enhanced [18]. Since performing photon subtraction and addition on a single-mode Gaussian state can enhance nonclassicality of the given state, one may ask if it is possible to enhance entanglement of a two-mode Gaussian state via photon subtraction and addition. Opatrný *et al.* [19] and Cochrane *et al.* [20] showed that entanglement of the two-mode squeezed vacuum state can be enhanced indeed by performing photon subtraction on both modes, and the fidelity of quantum teleportation for coherent states and squeezed states is improved by use of the photon-subtracted two-mode squeezed state as entangled resource. Even if inconclusive photon subtraction is considered, quantum teleportation can still be improved as well if the squeezing parameter is below a certain threshold [21]. Enhancement of nonlocality was also investigated with such photon-subtracted two-mode squeezed states [22–24]. Enhancement of fidelity of quantum teleportation or nonlocality is closely related to enhancement of entanglement. An entanglement evaluation is performed, using negativity as a computable measure, of photon-subtracted two-mode squeezed state generated by both ideal single-photon subtraction and inconclusive photon subtraction [25]. In a recent experiment, photon-subtracted two-mode squeezed states are generated through the coherent photon subtraction process [26].

*Corresponding author; flli@mail.xjtu.edu.cn

Dell'Anno *et al.* [27] investigated the performance of squeezed Bell-like entangled resources that take photon-subtracted and photon-added two-mode squeezed states as particular instances in continuous-variable quantum teleportation of Braunstein and Kimble protocol [28]. Although photon-subtracted and photon-added two-mode squeezed states have the same degree of entanglement, the former can always enhance the fidelity compared to the two-mode squeezed state. It seems that the quality of quantum teleportation is not determined only by the amount of entanglement of the entangled resource. Besides adding or subtracting one-photon operations, sequential adding (subtracting)-then-subtracting (adding) one-photon operations are also realizable in experiments [11]. In this paper, the entanglement amount and the Einstein-Podolsky-Rosen (EPR) correlation of non-Gaussian states are investigated, which are obtained by performing the photon addition, photon subtraction, photon-addition-then-subtraction, and photon-subtraction-then-addition operations on the two-mode squeezed vacuum state. The performance of these non-Gaussian entangled resources in continuous-variable quantum teleportation of Braunstein and Kimble protocol for coherent states, squeezed states, and mixed Gaussian states is studied. We find that in the weak squeezing region the photon-added-then-subtracted state has the strongest EPR correlation and leads to the highest fidelity.

This paper is organized as follows. In Sec. II the entanglement properties of the non-Gaussian states are investigated. In Sec. III continuous-variable quantum teleportation based on the non-Gaussian entangled resources is studied. In Sec. IV the obtained main results are summarized.

II. ENTANGLEMENT AND EPR CORRELATION OF NON-GAUSSIAN STATES

The two-mode squeezed vacuum state is widely used as an entangled resource in various quantum information processes such as continuous-variable quantum teleportation [28], quantum dense coding [29], and entanglement swapping [30–32], which can be written as

$$|\Psi\rangle = e^{r(a_1^\dagger a_2^\dagger - a_1 a_2)}|0,0\rangle_{12} = \sqrt{1-\lambda^2}\sum_{n=0}^{\infty}\lambda^n|n,n\rangle_{12}, \quad (1)$$

where $a_i(a_i^\dagger)$ is the annihilation (creation) operator of photons in mode i $(i=1,2)$, and $\lambda=\tanh(r)$ with the squeezing parameter r. The two-mode squeezed state has a Gaussian-type Wigner function and possesses the EPR correlation between phase-quadrature components that are analogous to position and momentum operators of a massive particle [33].

By performing the photon-subtracted, photon-added, photon-added-then-subtracted, and photon-subtracted-then-added manipulations on the two-mode squeezed state, one can generate the following non-Gaussian states:

$$|\Psi\rangle_s = N_s a_1 a_2|\Psi\rangle = \sqrt{\frac{(1-\lambda^2)^3}{1+\lambda^2}}\sum_{n=0}^{\infty}\lambda^n(n+1)|n,n\rangle_{12}, \quad (2)$$

$$|\Psi\rangle_a = N_a a_1^\dagger a_2^\dagger|\Psi\rangle = \sqrt{\frac{(1-\lambda^2)^3}{1+\lambda^2}}\sum_{n=0}^{\infty}\lambda^n(n+1)|n+1,n+1\rangle_{12}, \quad (3)$$

$$|\Psi\rangle_{sa} = N_{sa} a_1 a_2 a_1^\dagger a_2^\dagger|\Psi\rangle = \sqrt{\frac{(1-\lambda^2)^5}{1+11\lambda^2+11\lambda^4+\lambda^6}}\sum_{n=0}^{\infty}\lambda^n(n+1)^2|n,n\rangle_{12}, \quad (4)$$

$$|\Psi\rangle_{as} = N_{as} a_1^\dagger a_2^\dagger a_1 a_2|\Psi\rangle = \sqrt{\frac{(1-\lambda^2)^5}{1+11\lambda^2+11\lambda^4+\lambda^6}}\sum_{n=0}^{\infty}\lambda^n(n+1)^2|n+1,n+1\rangle_{12}, \quad (5)$$

where $N_{s,a,as,sa}$ is the normalization constant of $|\Psi\rangle_{s,a,as,sa}$. In current experiments, from the two-mode squeezed vacuum state, the photon-subtracted state can be generated using a beam splitter of low reflectivity [7], the photon-added state can be generated using a parametric down-converter with low gain [8–10], and the photon-added (subtracted)-then-subtracted (added) state can be generated by a sequence of photon addition (subtraction) followed by photon subtraction (addition) [11].

Since these states are all bipartite pure states, their entanglement can be quantified using the partial von Neumann entropy $E(\psi_{12})=S(\rho_1)=-\mathrm{tr}(\rho_1 \ln \rho_1)$, i.e., entanglement of formation [34]. Note that the photon-subtracted and photon-added states have exactly the same degree of entanglement because the two states have the same set of Schmidt coefficients which determine the entanglement. For the same reason, the photon-added-then-subtracted and photon-subtracted-then-added states have the same degree of entanglement too. The entanglement of these states is plotted in Fig. 1. We observe that the entanglement of all the states obtained from the photon-addition and -subtraction manipulation is greater than that of the two-mode squeezed state. Moreover, for a given squeezing degree λ, the photon-added (subtracted)-then-subtracted (added) state has the largest amount of entanglement among the states. It means that the entanglement can be really enhanced by the photon addition and subtraction operation on the two-mode squeezed state.

Besides the degree of entanglement, states (1)–(5) can also be characterized by EPR correlations between phase-quadrature components of the two modes. As counterparts of position and momentum operators of a massive particle, the phase-quadrature operators of each mode are defined as $x_j=\frac{1}{\sqrt{2}}(a_j+a_j^\dagger)$ and $p_j=\frac{1}{i\sqrt{2}}(a_j-a_j^\dagger)$ $(j=1,2)$. In the vacuum state, both the variances $\Delta(x_1-x_2)^2$ and $\Delta(p_1+p_2)^2$ are equal to 1. For any classical two-mode states, both the variances $\Delta(x_1-x_2)^2$ and $\Delta(p_1+p_2)^2$ are larger than 1. In the EPR state [35], $\Delta(x_1-x_2)^2=\Delta(p_1+p_2)^2=0$. It means that x_1 and p_1 of the first mode can be exactly estimated by measured results of x_2 and p_2 of the second mode or vice versa. In this sense, we say the existence of the ideal EPR correlation between the two modes. There may be some two-mode states that

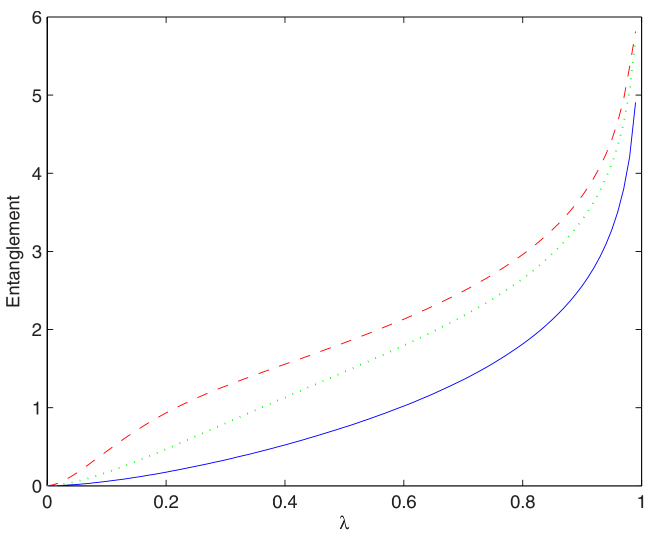

FIG. 1. (Color online) Entanglement of formation as a function of the squeezing parameter λ for (a) the two-mode squeezed state (blue full line), (b) photon-subtracted (added) state (green dotted line), and (c) photon-added (subtracted)-then-subtracted (added) state (red dashed line).

possess the EPR correlation beyond the limit of the vacuum state, i.e., both the variances $\Delta(x_1-x_2)^2$ and $\Delta(p_1+p_2)^2$ are less than one. For example, the two-mode squeezed state has the variances $\Delta(x_1-x_2)^2=\Delta(p_1+p_2)^2=e^{-2r}$. Whenever the squeezing parameter is not equal to zero, the EPR correlation between the two modes exists. This quantum correlation is a key ingredient for the realization of quantum teleportation of continuous variables [28].

For all the states [Eqs. (1)–(5)], $\Delta(x_1-x_2)^2=\Delta(p_1+p_2)^2$. In Fig. 2, the variance $\Delta(x_1-x_2)^2$ of these states is plotted. It is noticed that the variance of the photon-added and photon-subtracted-then-added states is larger than that of the two-

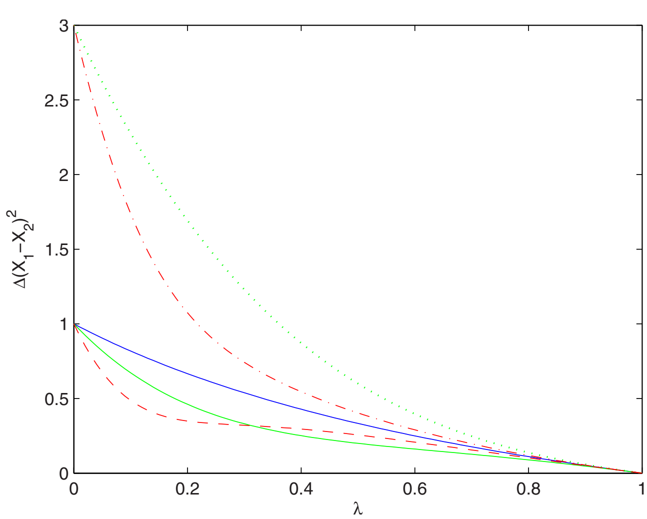

FIG. 2. (Color online) Variance $\Delta(x_1-x_2)^2$ as a function of λ for (a) the two-mode squeezed state (blue full line), (b) the photon-subtracted state (green full line), (c) the photon-added state (green dotted line), (d) the photon-added-then-subtracted state (red dashed line), and (e) the photon-subtracted-then-added state (red dotted-dashed line).

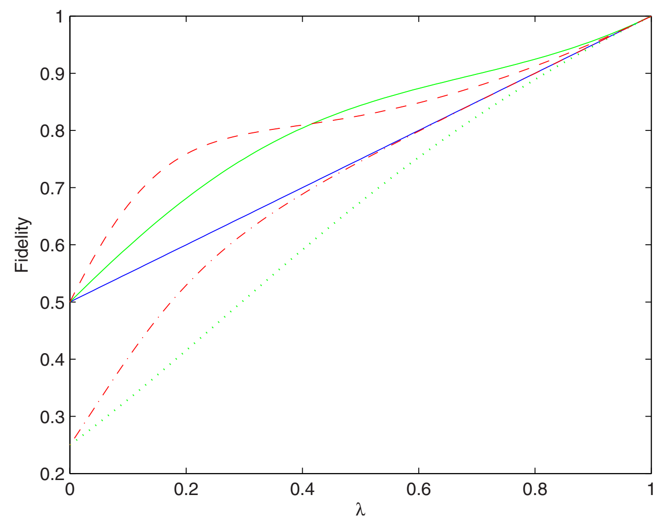

FIG. 3. (Color online) Fidelity as a function of λ for (a) two-mode squeezed state (blue full line), (b) photon-subtracted state (green full line), (c) photon-added state (green dotted line), (d) photon-added-then-subtracted state (red dashed line), and (e) photon-subtracted-then-added state (red dotted-dashed line).

mode squeezed state, and the variance of the photon-subtracted and photon-added-then-subtracted states is smaller than that of the two-mode squeezed state. We also observe that if λ is smaller than a certain value the variance of photon-added-then-subtracted states is even smaller than that of photon-subtracted states. Thus, the EPR correlation can be enhanced only by either the photon-subtracted process or photon-added-then-subtracted process. In Braunstein and Kimble protocol of quantum teleportation of continuous variables [28], the quantum channel is based on the EPR correlations and the fidelity of teleported states is determined by the EPR correlations. Thus, one may expect that the quality of quantum teleportation of continuous-variables can be improved by use of either the photon-subtracted state or the photon-added-then-subtracted state as entangled resource. In fact, Dell'Anno *et al.* [27] showed that the fidelity of continuous-variable quantum teleportation is raised by use of the photon-subtracted state for quantum channel. In the next section, we will show that in the weak squeezing region the fidelity of continuous-variable quantum teleportation can be further raised by use of the photon-added-then-subtracted state as entangled resource.

Comparing Figs. 1 and 2, one may notice that larger amount of entanglement does not always mean stronger EPR correlations. The states possessing large amounts of entanglement such as photon-added states and photon-subtracted-then-added states may not be of benefit to and even harmful to quantum information processing. Thus, from the respect of applications such as continuous-variable quantum teleportation, the amount of entanglement and the EPR correlation may not be the same thing.

III. CONTINUOUS-VARIABLE TELEPORTATION WITH THE NON-GAUSSIAN ENTANGLED STATES

The original idea of teleportation was proposed by Bennett *et al.* [36] in the discrete variable regime, sending an unknown quantum state of a spin-$\frac{1}{2}$ particle to a distant receiver via dual classical and Einstein-Podolsky-Rosen channels. Later Vaidman put forward the idea of continuous-variable teleportation [37]. As an example of quantum teleportation of continuous variables, the quantum-optical protocol for the teleportation of phase-quadrature components of a light field was proposed by Braunstein and Kimble [28], and soon realized by Furusawa and co-workers [38].

In Braunstein and Kimble protocol of quantum teleportation of continuous variables, coherent amplitudes α and β of the entangled light that is described by the Wigner function W_{EPR} are distributed to Alice (sender) and Bob (receiver), respectively. The Wigner function of a state to be teleported is $W_{in}(\gamma)$. The joint Wigner function of the total field under consideration is given by

$$W(\gamma,\alpha,\beta) = W_{in}(\gamma) \otimes W_{EPR}(\alpha,\beta). \tag{6}$$

On Alice's side, the input mode γ and the entangled mode α interfere at a 50:50 beam splitter. The amplitudes of the field appearing from outports of the beam splitter are

$$\mu = (\alpha+\gamma)/\sqrt{2}, \quad \nu = (\alpha-\gamma)/\sqrt{2}. \tag{7}$$

After the beam splitter, the joint Wigner function becomes

$$W(\mu,\nu,\beta) = W_{in}\left(\frac{\mu-\nu}{\sqrt{2}}\right) \otimes W_{EPR}\left(\frac{\mu+\nu}{\sqrt{2}},\beta\right). \tag{8}$$

At the outports, homedyne measurements on x_μ, the real part of the amplitude μ, and p_ν, the imaginary part of the amplitude ν are performed at the same time. Once a result (x_μ,p_ν) occurs, Winger function (8) collapses to

$$W(\beta,z) = 2P^{-1}(z)\int d^2x_\nu d^2p_\mu W_{in}\left(\frac{\mu-\nu}{\sqrt{2}}\right) \otimes W_{EPR}\left(\frac{\mu+\nu}{\sqrt{2}},\beta\right), \tag{9}$$

where

$$P(z) = 2\int d^2\gamma d^2\beta W_{in}(\gamma) \otimes W_{EPR}(z^*-\gamma^*,\beta,t) \tag{10}$$

is the probability of measuring the result (x_μ,p_ν), and $z=\sqrt{2}(x_\mu-ip_\nu)$. Then, Alice sends Bob her measuring result (x_μ,p_ν) through a classical information channel. When receiving the result, Bob performs the displacement $\beta\to\beta-gz$, where g is the gain factor of the classical information channel and can be used to optimize the teleportation process [39]. After the displacement and on average over all the possible measuring results, the Winger function of the teleported field is given by

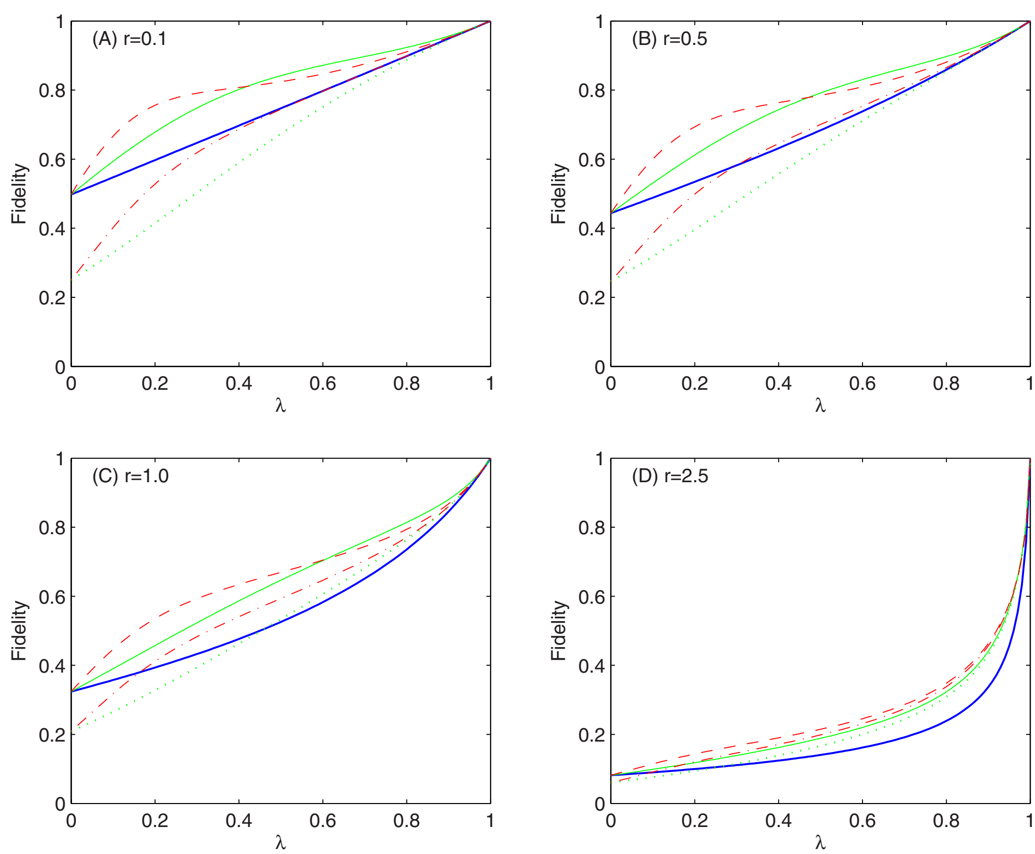

FIG. 4. (Color online) Fidelity as a function of λ for (a) two-mode squeezed state (blue full line), (b) photon-subtracted state (green full line), (c) photon-added state (green dotted line), (d) photon-added-then-subtracted state (red dashed line), and (e) photon-subtracted-then-added state (red dotted-dashed line).

$$W_{out}(\beta) = \int d^2 z P(z) W(\beta - gz, z). \tag{11}$$

The quality of the teleported state is evaluated by the fidelity [40]

$$F = \pi \int d^2\beta W_{in}(\beta) W_{out}(\beta). \tag{12}$$

For a given state, there is a one-to-one correspondence between its Wigner function and symmetrically ordered characteristic function [33]. The above formalism of the Winger function can be reformulated in terms of the corresponding symmetrically ordered characteristic functions. It has been shown that the characteristic function of the teleported state takes the factorized form [41]

$$\chi_{out}(\alpha) = \chi_{in}(\alpha)\chi_{12}(\alpha^*, \alpha), \tag{13}$$

where $\chi_{in}(\alpha)$ and $\chi_{12}(\alpha,\beta)$ are the symmetrically ordered characteristic functions of the input state to be teleported and the entangled resource, respectively, and the gain factor g of the classical information channel is chosen to be unity. In the formalism of the characteristic functions for quantum teleportation of continuous variables, the fidelity can be written as [40]

$$F = \mathrm{tr}(\rho_{in}\rho_{out}) = \frac{1}{\pi}\int d^2\alpha \chi_{in}(\alpha)\chi_{out}(-\alpha). \tag{14}$$

First, let us consider Braunstein and Kimble protocol of quantum teleportation for single-mode coherent states, where entangled states (1)–(5) are used for quantum channel. The symmetrically ordered characteristic functions of two-mode entangled states (1)–(5) are listed in Appendix A. Upon substituting these characteristic functions into Eq. (14), we worked out the fidelity for teleporting a coherent state based on entangled resources (1)–(5),

$$F = \frac{1+\lambda}{2}, \tag{15}$$

$$F_s = \frac{(1+\lambda)^3(\lambda^2 - 2\lambda + 2)}{4(1+\lambda^2)}, \tag{16}$$

$$F_a = \frac{(1+\lambda)^3}{4(1+\lambda^2)}, \tag{17}$$

$$F_{sa} = \frac{(1+\lambda)^5(\lambda^4 - 3\lambda^3 + 5\lambda^2 - 2\lambda + 2)}{4(1+\lambda^2)(\lambda^4 + 10\lambda^2 + 1)}, \tag{18}$$

$$F_{as} = \frac{(1+\lambda)^5(\lambda^2 + \lambda + 1)}{4(1+\lambda^2)(\lambda^4 + 10\lambda^2 + 1)}, \tag{19}$$

where the subscript denotes the corresponding entangled resources. It can be seen that the fidelity is only dependent on the parameter λ of the entangled resources and is independent of amplitude of the coherent state.

The fidelity for teleporting a coherent state is plotted as a function of λ with entangled resources (1)–(5) in Fig. 3. As expected, the fidelity with the photon-subtracted and photon-added-then-subtracted states is higher than that with the original two-mode squeezed state. Although the photon-added and photon-subtracted-then-added states possess larger amounts of entanglement, the fidelity with these entangled resources is smaller than that with the two-mode squeezed state. If the squeezing parameter λ is smaller than a certain value, the fidelity with the photon-added-then-subtracted state is greater than that with the photon-subtracted state. Therefore, instead of pursuing strongly squeezed resources for continuous-variable quantum teleportation of high fidelity, one can realize high-quality quantum teleportation by use of the photon-added-then-subtracted state obtained from the two-mode weak squeezed state.

Next we consider to teleport the single-mode squeezed vacuum state $\exp[(r/2)(a^2 - a^{\dagger 2})]|0\rangle$ whose symmetrically ordered characteristic function reads

$$\chi_{sq}(\alpha) = \exp\left[-\frac{\cosh 2r}{2}|\alpha|^2 - \frac{\sinh 2r}{4}(\alpha^2 + \alpha^{*2})\right]. \tag{20}$$

Upon substituting Eq. (20) into Eq. (14), we can analytically worked out the fidelity with entangled resources (1)–(5). In Fig. 4, the fidelity for the squeezed state with various values of the squeezing parameter and the different entangled resources is shown as a function of λ. We see that unlike the teleportation of coherent states the fidelity for the squeezed state is strongly state dependent. When the squeezing degree of the input state is low, the higher fidelity than that of the two-mode squeezed state can be maintained by making use of the photon-subtracted and photon-added-then-subtracted entangled resources. Moreover, in the low-energy region of the original two-mode squeezed state, the fidelity with the photon-added-then-subtracted entangled resource is higher than that of all the entangled resources under consideration. As the squeezing degree of the input state increases, however, the fidelity decreases. In order to obtain high fidelity for teleporting strongly squeezed states, one needs to have the two-mode squeezed resource with $\lambda \sim 1$ where the energy of the entangled resource becomes infinite. Therefore, it is more difficult to teleport strongly squeezed states than coherent states. We also observe that when the squeezing degree of the input state is large the fidelity with all the non-Gaussian entangled resources obtained from the photon addition and subtraction operations is higher than that of the two-mode squeezed vacuum state, as shown in Fig. 4(d). Among the non-Gaussian entangled resources, the photon-added-then-subtracted entangled state leads to the highest fidelity.

In the above discussion, the states to be teleported are pure Gaussian states. We now consider to teleport the general mixed single-mode Gaussian states characterized by the covariance matrix

$$\mathbf{V} = \begin{pmatrix} n + \frac{1}{2} & m \\ m^* & n + \frac{1}{2} \end{pmatrix}, \tag{21}$$

where $n = \langle a^\dagger a\rangle$, $m = -\langle a^2\rangle$, and the first-order moments are assumed to be zero without loss of generality. The two pa-

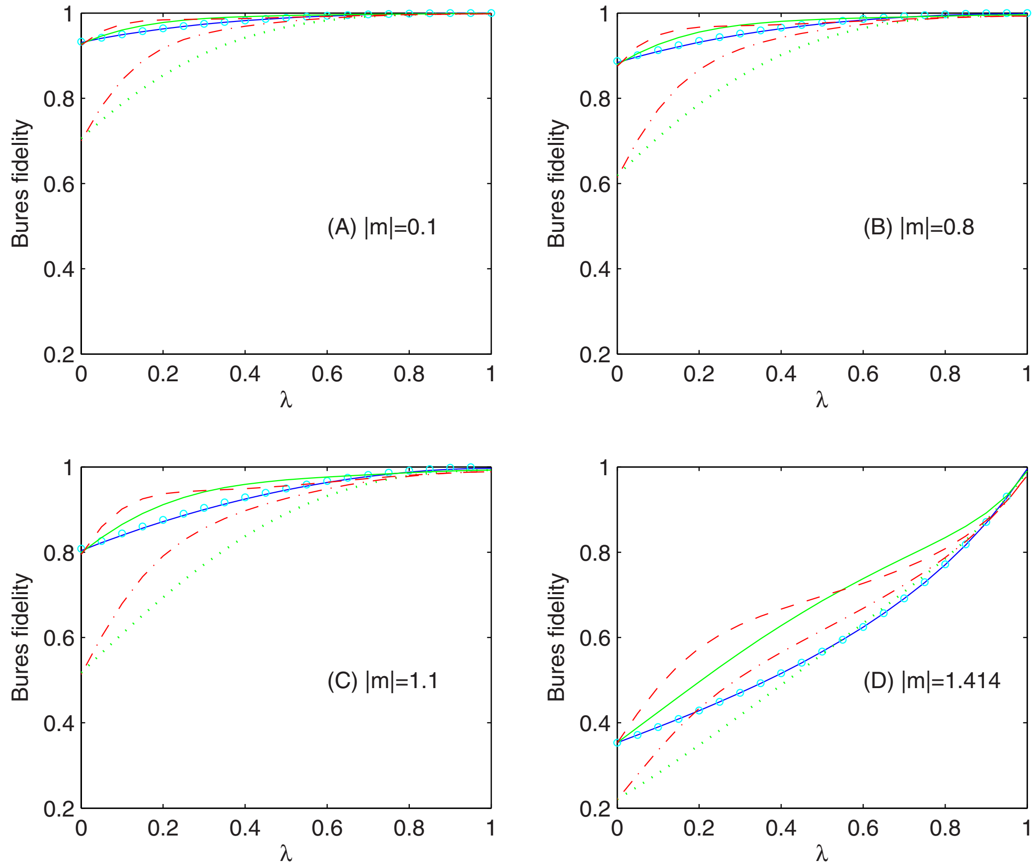

FIG. 5. (Color online) Bures fidelity as a function of λ for (a) two-mode squeezed state (blue full line), (b) photon-subtracted state (green full line), (c) photon-added state (green dotted line), (d) photon-added-then-subtracted state (red dashed line), (e) photon-subtracted-then-added state (red dotted-dashed line), and (f) two-mode squeezed state (cyan circles) using the analytical formula in Ref. [46]. The state to be teleported is a mixed Gaussian state: (a) $|m|=0.1$, (b) $|m|=0.8$, (c) $|m|=1.1$, and (d) $|m|=1.414$, while $n=1$ for all the four cases.

rameters n and m of the physical Gaussian state have to satisfy Heisenberg's uncertainty relation $n(n+1)\geq |m|^2$ [42]. If the equality condition $n(n+1)=|m|^2$ holds the state is pure since the purity is $1/(2\sqrt{(n+1/2)^2-|m|^2})$. For a fixed value of the parameter n, the purity increases as the parameter $|m|$ increases. The nonclassical depth of the Gaussian state is $\max\{0,|m|-n\}$ [43,44]. Thus, the state is classical when $|m| \leq n$. When $|m|>n$ the state becomes nonclassical and its nonclassicality increases as $|m|$ increases. Therefore, we may investigate the influence of purity and nonclassicality of the input state on the fidelity of teleportation by varying values of the parameter m.

If both the input and output states are mixed, the fidelity defined previously by Eq. (14) is no longer appropriate for evaluating the similarity of the teleported state to the input state. For the case of mixed states, the well-known Bures fidelity [45] is used to evaluate the quality of teleportation, which is defined as

$$F(\rho_1,\rho_2)=(\mathrm{tr}\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}})^2. \qquad (22)$$

If both ρ_1 and ρ_2 are pure states or one of them is pure while the other is mixed, Bures fidelity reduces to the fidelity defined by Eq. (14).

If input and output states are Gaussian, the analytical expression of Bures fidelity has been derived out in terms of the covariance matrices [46]. When non-Gaussian entangled resources (2)–(5) are used, the output states are non-Gaussian although the input state is Gaussian type. For this case, no analytical formula of Bures fidelity is available at present. When using the definition Eq. (22) for the calculation of Bures fidelity, we have to deal with squared roots of the matrices. For this reason, the diagonal representation of the input and output states is convenient. On the other hand, the characteristic functions of the input and output states are easily worked out in the case under consideration. Thus, we need the two steps to get the fidelity. First, the density matrices are constructed from the characteristic functions. Then, the density matrices are diagonalized and their squared roots are worked out. In Appendix B, we propose a method of reconstructing the corresponding density matrix in the representation of photon number states from a given symmetrically ordered characteristic function. Based on the reconstructed matrices, with the photon number truncation approximation, we can calculate the Bures fidelity for the input Gaussian state and entangled resources (1)–(5). Although this method is accurate in principle, the calculation is so lengthy and time consuming that we only consider some weak input Gaussian states.

In Fig. 5, we plot the Bures fidelity as a function of λ for

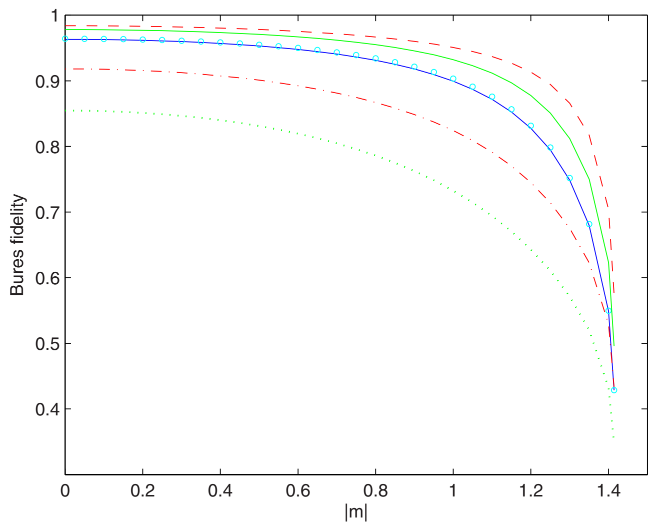

FIG. 6. (Color online) Bures fidelity as a function of $|m|$ for (a) two-mode squeezed state (blue full line), (b) photon-subtracted state (green full line), (c) photon-added state (green dotted line), (d) photon-added-then-subtracted state (red dashed line), (e) photon-subtracted-then-added state (red dotted-dashed line), and (f) two-mode squeezed state (cyan circles) using the analytical formula in Ref. [46] with $n=1$ and $\lambda=0.2$.

the input Gaussian states with different values of $|m|$ and a fixed n using entangled resources (1)–(5). The fidelity for the two-mode squeezed state is calculated in two ways, one by the reconstructed density matrix while the other by the analytical formula from the reference [46]. We find that the both results agree with each other well. As is shown in Figs. 5(a) and 5(b) where the input states are classical and mixed, the fidelity can be maintained in a high level. Once $|m|>n$ and then the input states are nonclassical, the fidelity lowers down rapidly. We notice that in all the cases under consideration the fidelity with the photon-subtracted and photon-added-then-subtracted entangled resources is higher than that of the two-mode squeezed state. Moreover, the highest fidelity is always reached in the weak squeezing region by the photon-added-then-subtracted state. In the limit of $|m|=\sqrt{n(n+1)}$, the input state is pure and its nonclassical depth becomes the largest. As is shown in Fig. 5(d), the fidelity drops down. Thus, compared to the mixed and classical states, it is more difficult to teleport pure and nonclassical states. In order to more clearly see this point, the fidelity is shown for the input states with the fixed value of the parameter n and various allowed values of the parameter $|m|$ and the entangled resources with the fixed value of the squeezing parameter λ in Fig. 6. It is clearly observed that for the mixed classical and nonclassical states the high fidelity can be obtained but the fidelity goes down rapidly as the purity condition is approached.

IV. CONCLUSIONS

The entanglement properties of non-Gaussian states are investigated, which are obtained by performing the photon addition, photon subtraction, photon-addition-then-subtraction, and photon-subtraction-then-addition operations on the two-mode squeezed vacuum state. We show that the partial von Neumann entropy of all the resulting states is greater than that of the original two-mode squeezed state. Among the states, the photon-added (subtracted)-then-subtracted (added) states has the highest amount of entanglement. As for the EPR correlation between phase-quadrature components of the two modes in the states, which signals the existence of entanglement, we find that both the photon-subtracted and photon-added-then-subtracted states have stronger EPR correlation than the original two-mode squeezed vacuum state. Moreover, in the low-energy region of the two-mode squeezed vacuum state, the photon-added-then-subtracted state has the strongest EPR correlation. We also study quantum teleportation of Braunstein and Kimble protocol for coherent states, squeezed states, and mixed Gaussian states with the resulting non-Gaussian states as entangled resources. For coherent states, the analytical expression of fidelity is found. It is noted that the fidelity is independent of the amplitude of coherent states to be teleported. For the other states, the fidelity is state dependent. For all the states to be teleported, we notice that the fidelity with the photon-subtracted and photon-added-then-subtracted entangled resources is higher than that with the two-mode squeezed vacuum resource. In the weak squeezing region of the original two-mode squeezed state, the photon-added-then-subtracted entangled resource can lead to the highest fidelity. Thus, the quality of teleportation is determined by the EPR correlation of the entangled resource instead of the amount of entanglement. When the states to be teleported are mixed, Bures fidelity is numerically calculated from symmetrically ordered characteristic functions of the input and output states. We find that quantum teleportation for states with high purity and nonclassicality is more difficult than for mixed and classical states.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grants No. 60778021 and No. 10674106.

APPENDIX A: CHARACTERISTIC FUNCTIONS

In this appendix, we list symmetrically ordered characteristic functions of states (1)–(5). The characteristic function of the two-mode squeezed state is

$$\chi(\alpha,\beta)=\exp\left[-\frac{(1+\lambda^2)}{2(1-\lambda^2)}(|\alpha|^2+|\beta|^2)+\frac{\lambda}{(1-\lambda^2)}(\alpha\beta+\alpha^*\beta^*)\right]. \quad \text{(A1)}$$

The characteristic functions of the photon-subtracted, photon-added, photon-added-then-subtracted, and photon-subtracted-then-added states can be written in a unified form

$$\chi_x(\alpha,\beta) = N_x^2 e^{-|\alpha|^2+|\beta|^2/2}\Lambda_x(\alpha)\Lambda_x(\beta)[\chi(\alpha,\beta)e^{|\alpha|^2+|\beta|^2/2}], \quad \text{(A2)}$$

where x denotes different states and the corresponding differential operators $\Lambda_x(\alpha)$s are listed below,

$$\Lambda_s(\alpha) = \frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha^*}, \quad \text{(A3)}$$

$$\Lambda_a(\alpha) = -\frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha^*} + \alpha\frac{\partial}{\partial\alpha} + \alpha^*\frac{\partial}{\partial\alpha^*} - \alpha\alpha^* + 1, \quad \text{(A4)}$$

$$\Lambda_{sa}(\alpha) = \frac{\partial^2}{\partial\alpha^2}\frac{\partial^2}{\partial\alpha^{*2}} - \alpha\frac{\partial^2}{\partial\alpha^2}\frac{\partial}{\partial\alpha^*} - \alpha^*\frac{\partial}{\partial\alpha}\frac{\partial^2}{\partial\alpha^{*2}} + (\alpha\alpha^* - 3)\frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha^*} + \alpha\frac{\partial}{\partial\alpha} + \alpha^*\frac{\partial}{\partial\alpha^*} + 1, \quad \text{(A5)}$$

$$\Lambda_{as}(\alpha) = \frac{\partial^2}{\partial\alpha^2}\frac{\partial^2}{\partial\alpha^{*2}} - \alpha\frac{\partial^2}{\partial\alpha^2}\frac{\partial}{\partial\alpha^*} - \alpha^*\frac{\partial}{\partial\alpha}\frac{\partial^2}{\partial\alpha^{*2}} + (\alpha\alpha^* - 1)\frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha^*}, \quad \text{(A6)}$$

while $\Lambda_x(\beta)$s are of the same form.

APPENDIX B: RECONSTRUCTING DENSITY MATRIX FROM CHARACTERISTIC FUNCTION

In this appendix, we propose a method of reconstructing density matrix from characteristic function. The characteristic function of a quantum state is defined as

$$\chi(z) = \mathrm{tr}[D(z)\rho], \quad \text{(B1)}$$

where $D(z)=e^{za^\dagger - z^* a}$ is the displacement operator. The density operator ρ can be expressed as

$$\rho = \frac{1}{\pi}\int d^2z\chi(z)D(-z). \quad \text{(B2)}$$

In the representation of photon number states, the density-matrix element ρ_{nm} is

$$\begin{aligned}\langle n|\rho|m\rangle &= \frac{1}{\pi}\int d^2z\chi(z)\langle n|D(-z)|m\rangle \\ &= \frac{1}{\pi}\int d^2z\chi(z)e^{-|z|^2/2}\langle n|\frac{1}{\pi}\int d^2\alpha|\alpha\rangle \\ &\quad\times\langle\alpha|e^{-za^\dagger}e^{z^*a}\frac{1}{\pi}\int d^2\beta|\beta\rangle\langle\beta|m\rangle \\ &= \frac{1}{\pi}\int d^2z\chi(z)e^{-|z|^2/2}\frac{1}{\pi^2} \\ &\quad\times\int\int d^2\alpha d^2\beta\frac{\alpha^n}{\sqrt{n!}}\frac{\beta^{*m}}{\sqrt{m!}}e^{-|\alpha|^2-|\beta|^2+\alpha^*\beta-z\alpha^*+z^*\beta}.\end{aligned} \quad \text{(B3)}$$

We introduce the integral with auxiliary parameters u and v,

$$\begin{aligned}I_{nm}(u,v) &= \frac{1}{\pi^2}\int\int d^2\alpha d^2\beta \\ &\quad\times\frac{\alpha^n}{\sqrt{n!}}\frac{\beta^{*m}}{\sqrt{m!}}e^{-|\alpha|^2-|\beta|^2+\alpha^*\beta-z\alpha^*+z^*\beta+u\alpha+v\beta^*} \\ &= \frac{1}{\sqrt{n!\,m!}}\frac{\partial^n}{\partial u^n}\frac{\partial^m}{\partial v^m} \\ &\quad\times\left[\frac{1}{\pi^2}\int\int d^2\alpha d^2\beta e^{-|\alpha|^2-|\beta|^2+\alpha^*\beta-z\alpha^*+z^*\beta+u\alpha+v\beta^*}\right] \\ &= \frac{1}{\sqrt{n!\,m!}}\frac{\partial^n}{\partial u^n}\frac{\partial^m}{\partial v^m}e^{-uz+vz^*+uv}.\end{aligned} \quad \text{(B4)}$$

Then ρ_{nm} can be express as

$$\begin{aligned}\langle n|\rho|m\rangle &= \frac{1}{\pi}\int d^2z\chi(z)e^{-|z|^2/2}I_{nm}(0,0) \\ &= \frac{1}{\sqrt{n!\,m!}}\frac{\partial^n}{\partial u^n}\frac{\partial^m}{\partial v^m} \\ &\quad\times\left[\frac{1}{\pi}\int d^2z\chi(z)e^{-|z|^2/2-uz+vz^*+uv}\right]\Bigg|_{u,v=0}.\end{aligned} \quad \text{(B5)}$$

Substituting the characteristic function of a given state into Eq. (B5), then elements of the density matrix can be one by one obtained by completing the integral in the brackets of Eq. (B5) and the differentiation with respect to auxiliary parameters u and v.

For completeness, we list the characteristic functions of the single-mode Gaussian state and the corresponding teleported states for entangled resources (1)–(5), which are involved in the calculation of Bures fidelity (22). The characteristic function of the input Gaussian state with the covariance matrix Eq. (21) is

$$\chi_{in}(z) = \exp\left[-\left(n+\frac{1}{2}\right)|z|^2 - \frac{1}{2}m^*z^2 - \frac{1}{2}mz^{*2}\right]. \quad \text{(B6)}$$

Note that the second-order moments n and m used here are different from the indexes in $\langle n|\rho|m\rangle$ used previously in this appendix. Then the characteristic function of the teleported state using the two-mode squeezed state as the entangled resource is

$$\chi(z) = \exp\left[-\left(\frac{1-\lambda}{1+\lambda}+n+\frac{1}{2}\right)|z|^2 - \frac{1}{2}m^*z^2 - \frac{1}{2}mz^{*2}\right]. \quad \text{(B7)}$$

The characteristic function of the teleported state using the photon-subtracted state as the entangled resource is of the form

$$\chi_s(z) = (1 + c_1|z|^2 + c_2|z|^4)\exp\left[-\left(\frac{1-\lambda}{1+\lambda} + n + \frac{1}{2}\right)|z|^2 - \frac{1}{2}m^* z^2 - \frac{1}{2}m z^{*2}\right], \tag{B8}$$

where the coefficients c_is $(i=1,2)$ are only dependent of λ. The characteristic function of the teleported state using the photon-added state as the entangled resource is of the same form but with different c_is.

The characteristic function of the teleported state using the photon-added-then-subtracted state as the entangled resource is of the form

$$\chi_{sa}(z) = (1 + c_1|z|^2 + c_2|z|^4 + c_3|z|^6 + c_4|z|^8) \times\exp\left[-\left(\frac{1-\lambda}{1+\lambda} + n + \frac{1}{2}\right)|z|^2 - \frac{1}{2}m^* z^2 - \frac{1}{2}m z^{*2}\right], \tag{B9}$$

where the coefficients c_is $(i=1,2,3,4)$ are only dependent of λ. The characteristic function of the teleported state using the photon-subtracted-then-added state as the entangled resource is of the same form but with different c_is.

[1] S. D. Bartlett, B. C. Sanders, S. L. Braunstein, and K. Nemoto, Phys. Rev. Lett. **88**, 097904 (2002).

[2] S. D. Bartlett and B. C. Sanders, Phys. Rev. Lett. **89**, 207903 (2002).

[3] J. Eisert, S. Scheel, and M. B. Plenio, Phys. Rev. Lett. **89**, 137903 (2002).

[4] J. Fiurášek, Phys. Rev. Lett. **89**, 137904 (2002).

[5] G. Giedke and J. I. Cirac, Phys. Rev. A **66**, 032316 (2002).

[6] J. Niset, J. Fiurášek, and N. J. Cerf, Phys. Rev. Lett. **102**, 120501 (2009).

[7] J. Wenger, R. Tualle-Brouri, and P. Grangier, Phys. Rev. Lett. **92**, 153601 (2004).

[8] A. Zavatta, S. Viciani, and M. Bellini, Science **306**, 660 (2004).

[9] A. Zavatta, S. Viciani, and M. Bellini, Phys. Rev. A **72**, 023820 (2005).

[10] A. Zavatta, V. Parigi, and M. Bellini, Phys. Rev. A **75**, 052106 (2007).

[11] V. Parigi, A. Zavatta, M. Kim, and M. Bellini, Science **317**, 1890 (2007).

[12] M. S. Kim, H. Jeong, A. Zavatta, V. Parigi, and M. Bellini, Phys. Rev. Lett. **101**, 260401 (2008).

[13] P. Marek, H. Jeong, and M. S. Kim, Phys. Rev. A **78**, 063811 (2008).

[14] S. S. Mizrahi and V. V. Dodonov, J. Phys. A **35**, 8847 (2002).

[15] M. S. Kim, E. Park, P. L. Knight, and H. Jeong, Phys. Rev. A **71**, 043805 (2005).

[16] A. R. Usha Devi, R. Prabhu, and M. S. Uma, Eur. Phys. J. D **40**, 133 (2006).

[17] A. Biswas and G. S. Agarwal, Phys. Rev. A **75**, 032104 (2007).

[18] Y. Yang and F.-L. Li, J. Opt. Soc. Am. B **26**, 830 (2009).

[19] T. Opatrný, G. Kurizki, and D.-G. Welsch, Phys. Rev. A **61**, 032302 (2000).

[20] P. T. Cochrane, T. C. Ralph, and G. J. Milburn, Phys. Rev. A **65**, 062306 (2002).

[21] S. Olivares, M. G. A. Paris, and R. Bonifacio, Phys. Rev. A **67**, 032314 (2003).

[22] S. Olivares and M. G. A. Paris, Phys. Rev. A **70**, 032112 (2004).

[23] C. Invernizzi, S. Olivares, M. G. A. Paris, and K. Banaszek, Phys. Rev. A **72**, 042105 (2005).

[24] H. Jeong, Phys. Rev. A **78**, 042101 (2008).

[25] A. Kitagawa, M. Takeoka, M. Sasaki, and A. Chefles, Phys. Rev. A **73**, 042310 (2006).

[26] A. Ourjoumtsev, A. Dantan, R. Tualle-Brouri, and P. Grangier, Phys. Rev. Lett. **98**, 030502 (2007).

[27] F. Dell'Anno, S. De Siena, L. Albano, and F. Illuminati, Phys. Rev. A **76**, 022301 (2007).

[28] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).

[29] S. L. Braunstein and H. J. Kimble, Phys. Rev. A **61**, 042302 (2000).

[30] R. E. S. Polkinghorne and T. C. Ralph, Phys. Rev. Lett. **83**, 2095 (1999).

[31] S. M. Tan, Phys. Rev. A **60**, 2752 (1999).

[32] P. van Loock and S. L. Braunstein, Phys. Rev. A **61**, 010302(R) (1999).

[33] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994).

[34] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).

[35] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[36] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).

[37] L. Vaidman, Phys. Rev. A **49**, 1473 (1994).

[38] A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282**, 706 (1998).

[39] T. Ide, H. F. Hofmann, A. Furusawa, and T. Kobayashi, Phys. Rev. A **65**, 062303 (2002).

[40] A. V. Chizhov, L. Knöll, and D.-G. Welsch, Phys. Rev. A **65**, 022310 (2002).

[41] P. Marian and T. A. Marian, Phys. Rev. A **74**, 042306 (2006).

[42] B. G. Englert and K. Wódkiewicz, Int. J. Quantum Inf. **1**, 153 (2003).

[43] A. Ferraro, S. Olivares, and M. G. A. Paris, e-print arXiv:quant-ph/0503237.

[44] H.-R. Li, F.-L. Li, and Y. Yang, Chin. Phys. **15**, 2947 (2006).

[45] R. Jozsa, J. Mod. Opt. **41**, 2315 (1994).

[46] Gh.-S. Paraoanu and H. Scutaru, Phys. Rev. A **61**, 022306 (2000).