

**Nonlocal quantum-field correlations and detection processes in quantum-field theory**Fabrizio Buscemi<sup>1,2,\*</sup> and Giuseppe Compagno<sup>3</sup><sup>1</sup>*ARCES, Alma Mater Studiorum, University of Bologna, Via Toffano 2/2, 40125 Bologna, Italy*<sup>2</sup>*S3 Research Center, CNR-INFN, Via Campi 213/A, I-Modena 41100, Italy*<sup>3</sup>*CNISM and Dipartimento di Scienze Fisiche ed Astronomiche, Università di Palermo, Via Archirafi 36, 90123 Palermo, Italy*

(Received 21 April 2009; published 19 August 2009)

Quantum detection processes in quantum field theory (QFT) must play a key role in the description of quantum-field correlations, such as the appearance of entanglement, and of causal effects. We consider the detection in the case of a simple QFT model with a suitable interaction to exact treatment, consisting of a quantum scalar field coupled linearly to a classical scalar source. We then evaluate the response function to the field quanta of two-level pointlike quantum model detectors, and analyze the effects of the approximation adopted in standard detection theory. We show that the use of the RWA, which characterizes the Glauber detection model, leads in the detector response to nonlocal terms corresponding to an instantaneously spreading of source effects over the whole space. Other detector models, obtained with nonstandard or no application of RWA, give instead local responses to field quanta, apart from source-independent vacuum contribution linked to preexisting correlations of zero-point field.

DOI: [10.1103/PhysRevA.80.022117](https://doi.org/10.1103/PhysRevA.80.022117)

PACS number(s): 03.65.Pm, 03.65.Ud, 03.65.Ta

**I. INTRODUCTION**

The appearance of nonlocal effects in quantum mechanics has received great attention beginning from the well-known Einstein-Podolsky-Rosen (EPR) paradox [1]. This indicates that the result of a measurement performed on one of a pair of correlated systems has a nonlocal effect on the correlated physical measurement on the partner distant system. Such a nonlocal behavior is related to the presence of quantum entanglement between the systems. Thus detection of quantum correlations between two separated systems plays a key role in establishing whether the systems are entangled or not. In particular models of measurements not causally connected are required to evaluate a genuine manifestation of the entanglement. In this case if correlations are detected they may violate Bell's inequality [2] and therefore the two systems can be considered as entangled. Instead some models of measurement, leading by their same nature to the instantaneous development of nonlocal effects over the whole space, could give rise in their interpretation to the appearance of entanglement even in the absence of real quantum correlations.

Another place where nonlocality may manifest itself is in the space-time evolution of single-particle wave function that gives place to nonzero contributions outside of the light cone [3–5]. This aspect of nonlocality in quantum mechanics, with the building up of probability on spacelike distances, appears instead to give rise to a violation of causality. It was however suggested that, since strict localization of a quantum system requires particle detectors sensitive to zero-point fluctuations that extend all over the space, this behavior shall not be in contrast with causal propagation [6,7]. It is one of the aims of this paper to investigate on this aspect. Nonlocal effects show up also in quantum-field theory (QFT) in the time evolution of initially localized quantum-field states both for free fields and for interacting matter-field models [3–5,8–12].

The use of appropriate model of the detection process for the interpretation of the measurements in the observation of nonlocal correlations plays a key role into evaluating the reality of these quantum correlations defining entanglement or even of effects that appear to not satisfy the causal propagation of signals. In this context it results thus to be important the adoption of suitable quantum detector models, and the appropriate detector model that must be adopted appears to be still questioned [13–16]. Different quantum detector models have been proposed in literature. Among them are the Glauber detector (GD) model, extensively used in photo-detection theory and quantum optics [17,18], and the Unruh-DeWitt detector (UDD) model utilized to describe accelerated detectors and their excitations as response to the inertial vacuum [19–22]. The GD model adopts the so-called rotating wave approximation (RWA) whose application in the solution of QFT systems seems, however, to lead to the appearance of nonlocal effects [23,24]. In particular, for the case of the interaction between atom and electromagnetic field within the dipole approximation, it has been shown that the use of the RWA leads to the atomic dipole being coupled to the field at points other than the position of the dipole [25]. From this point of view the GD model could result to be inappropriate in models of matter-field interaction in describing the experimental observation aimed to detect quantum entanglement. In fact by its nature this model gives rise to the appearance of quantum correlations over spacelike distances which do not represent a manifestation of a genuine entanglement. Thus the use of GD model also could lead to appearance of violation of the causal propagation of signals, even if the effective connection between RWA and causality in the Glauber detection theory is yet debated. In particular, it has been shown that the photocounting probabilities for short observation times appear to violate causality [15,16], and this has led some authors to suggest relevant modifications of the Glauber photodetection theory [15,16]. Other investigations seem instead to indicate that an appropriate use of the RWA in the GD model guarantees causality [13].

\*fabrizio.buscemi@unimore.it

Thus the observability and the measurement of quantum correlations in order to evidence entanglement and causal effects require the use of appropriate detector models and in particular the adoption of a suitable Hamiltonian that does not induce by itself nonlocality. The aim of this paper is to discuss the typical models adopted in describing quantum detection processes and their relation to the possible appearance of nonlocal effects in the context of QFT. To this purpose in the first part of the paper we will analyze the GD model and the role played by the RWA into the appearance of effects over spacelike distances. Then, in order to connect the measurement of quantum correlations to detection processes, we then shall analyze another suitable detector model and will obtain its response to the quantum field. To this end here we shall consider a system consisting of a quantum scalar field linearly interacting with a classical source localized in a finite space-time region [26–29]. Such a model, which can be exactly solved, appears to be of interest because it allows us to have a clear view of the role played by nonlocal effects in the quantum correlations in the system without the limitations linked to the perturbative calculations.

The paper is organized as follows. In Sec. II we illustrate the GD and UDD models, while in Sec. III a nonstandard application of the RWA to the quantum detection of fields generated by sources is analyzed. In Sec. IV we shall introduce the model of quantum scalar field coupled to a classical source and then evaluate the response function of the GD and UDD to the field for different situations. Finally in Sec. V we comment the results obtained.

## II. QUANTUM DETECTION MODELS OF SCALAR FIELD

The quantum theory of photodetection, with the construction of a model of detector, as developed by Glauber [17], has played a key role in quantum optics. Extensions of the GD model that take into account the back action of the detector on the field have also been considered [30]. However other kinds of detectors have also been used in QFT [19–22]. In both approaches the detectors are particle detectors and the detection process represents the quantum measurement to detect the quanta of the field. Here we shall utilize the GD and UDD models in the case of scalar field detection [20].

### A. Unruh-De Witt scalar detector

UDD model [19] is represented as an idealized particle of negligible spatial extension and with internal energy levels, coupled via a monopole interaction with a scalar field  $\Phi(x)$ . The latter may be expressed in terms of its positive and negative frequency part as

$$\Phi(x) = \Phi^+(x) + \Phi^-(x), \quad (1)$$

where, taking  $\hbar=1$  and  $c=1$ ,

$$\Phi^+(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\mathbf{k}}{2\omega} a(\mathbf{k}) e^{-ik \cdot x} \quad \text{and} \quad \Phi^-(x) = \Phi^{+\dagger}(x). \quad (2)$$

$\omega = \sqrt{|\mathbf{k}|^2 + m^2}$  and  $a(\mathbf{k})$  and  $a^\dagger(\mathbf{k})$  are, respectively, the usual annihilation and creation operators that satisfy the relativistic commutator rules,

$$[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = 2\omega \delta^3(\mathbf{k} - \mathbf{k}'). \quad (3)$$

The detector is characterized by two energy levels  $\omega_g$  and  $\omega_e$ , with eigenstates  $|g\rangle$  and  $|e\rangle$ , respectively. It moves along the line world line described by the function  $x(\tau)$ , with  $\tau$  the proper time. The UDD model in the case of scalar fields is defined by the following interaction Hamiltonian:

$$H_{\text{UDD}}^{\text{int}} = -c_1 \hat{m}(\tau) \Phi(x(\tau)), \quad (4)$$

with  $\hat{m}(\tau)$  the detector monopole moment and  $c_1$  the field-detector coupling constant. Notice that  $H_{\text{UDD}}^{\text{int}}$  contains both conserving and nonconserving energy terms. We shall take the interaction turned on only for a finite time interval  $\tau = \tau_f - \tau_i$ . The state of the detector-field system at initial time  $\tau_i$  is  $|i\rangle = |g\rangle |\psi_i\rangle = |g\rangle \otimes |\psi_i\rangle$ , where  $|g\rangle$  is the detector state ground and  $|\psi_i\rangle$  the field state.

Using the interaction picture the first-order transition amplitude from  $|g\rangle |\psi_i\rangle$  to  $|e\rangle |\psi_f\rangle$  is

$$\begin{aligned} A_{\text{UDD}|g\rangle|\psi_i\rangle \rightarrow |e\rangle|\psi_f\rangle} &= \langle e|\psi_f|U(t)|g\rangle|\psi_i\rangle \\ &= ic_1 m_{eg} \int_{\tau_i}^{\tau_f} e^{i\omega_{eg}\tau'} \langle \psi_f | \Phi(x(\tau')) | \psi_i \rangle d\tau', \end{aligned} \quad (5)$$

with  $m_{eg} = \langle e | \hat{m}(0) | g \rangle$  and  $\omega_{eg} = \omega_e - \omega_g$ . Using the positive and negative frequency parts of the field operator, the matrix elements appearing within the integral in Eq. (5) can be written as

$$\langle \psi_f | \Phi^+(x(\tau')) | \psi_i \rangle + \langle \psi_f | \Phi^-(x(\tau')) | \psi_i \rangle, \quad (6)$$

with the first term describing the absorption and the second term describing the emission of field quanta by the detector. In the UDD both the terms  $\langle \psi_f | \Phi^+(x(\tau')) | \psi_i \rangle$  and  $\langle \psi_f | \Phi^-(x(\tau')) | \psi_i \rangle$  contribute to the detector excitation amplitude and correspond, respectively, to detector excitation with absorption or emission of a field quantum. In particular the second term represents the response of the detector to the vacuum fluctuations. In order to have a better insight into the different kinds of processes occurring in the scalar field-UDD interaction, here we give the expression of the amplitude probability of excitation UDD in terms of annihilation and creation operators of scalar quanta. By inserting Eq. (2) in Eq. (5), we obtain

$$A_{\text{UDD}|g\rangle|\psi_i\rangle \rightarrow |e\rangle|\psi_f\rangle} = \frac{ic_1 m_{eg}}{(2\pi)^{3/2}} \int \frac{d^3\mathbf{k}}{2\omega} \int_{\tau_i}^{\tau_f} d\tau' [e^{ik \cdot x(\tau')} e^{i(\omega_{eg} - \omega)\tau'} \langle \psi_f | a(\mathbf{k}) | \psi_i \rangle + e^{-ik \cdot x(\tau')} e^{i(\omega_{eg} + \omega)\tau'} \langle \psi_f | a^\dagger(\mathbf{k}) | \psi_i \rangle], \quad (7)$$

where the emission of quanta of the field with energy  $\omega$  is given in the integrand by the factor  $e^{i(\omega_{eg}+\omega)\tau'}$ . The absorption process instead leads to the factor  $e^{i(\omega_{eg}-\omega)\tau'}$  in the integrand of the above expression.

The probability of detection of the UDD is thus obtained by taking the square modulus of Eq. (5) and summing over all the possible field final states,

$$P_{\text{UDD}}(\tau_f, \tau_i) = c_1^2 |m_{eg}|^2 \int_{\tau_i}^{\tau_f} \int_{\tau_i}^{\tau_f} d\tau' d\tau'' e^{i\omega_{eg}(\tau''-\tau')} \times \langle \psi_i | \Phi(x(\tau')) \Phi(x(\tau'')) | \psi_i \rangle. \quad (8)$$

From the above expression it comes out that the response of detector depends on the motion of the detector itself; the well-known Unruh effect is in fact related to this property. The response of a uniformly accelerated UDD with acceleration  $\alpha$  to the vacuum fluctuations is the same as that of an unaccelerated UDD immersed in a bath of thermal radiation at temperature  $T=1/(2\pi k\alpha)$  [19].

### B. Glauber scalar detector

The GD model [17], commonly adopted in quantum optics, is obtained by applying the RWA in the interaction term. The use of such an approximation, which permits to easily evaluate the photodetection probability, is valid as long as the measurement time and pulse length of detected field are long compared to a typical optical cycle.

The RWA can analogously be applied for the case of scalar detection in the field-detector interaction Hamiltonian of Eq. (4). It reduces to the Hamiltonian

$$H_{\text{GD}}^{\text{int}} = -c_1 [m_{ge}(\tau) |g\rangle \langle e| \Phi^-(x(\tau)) + m_{eg}(\tau) |e\rangle \langle g| \Phi^+(x(\tau))], \quad (9)$$

where the closure relation for detector eigenstates  $|g\rangle \langle g| + |e\rangle \langle e| = \mathbb{I}$  has been used and  $m_{e(g)g(e)}(\tau) = \langle e(g) | \hat{m}(\tau) | g(e) \rangle$ . Note that in the above expression, the antiresonant terms  $m_{eg}(\tau) |e\rangle \langle g| \Phi^-(x(\tau))$  and  $m_{ge}(\tau) |g\rangle \langle e| \Phi^+(x(\tau))$ , which describe the creation of scalar quanta with excitation of the detector and the annihilation of scalar quanta with the decay of the detector, respectively, do not appear. In fact the RWA implies the neglect of such counter-rotating terms.

According to this model the detection is only considered as an absorption process. As seen from Eq. (7), in the term describing the emission of quanta of the field the factor  $e^{i(\omega_{eg}+\omega)\tau'}$  appears, which is rapidly oscillating and gives a negligible contribution for  $(\tau_f - \tau_i) \gg 1/\omega_{eg}$ . In this sense such a process can be considered virtual since it can occur only for short time intervals  $(\tau_f - \tau_i)$  obeying  $\omega_{eg}(\tau_f - \tau_i) \leq 1$  and moreover does not conserve energy. Instead the absorption process is given in the integrand by the factor  $e^{i(\omega_{eg}-\omega)\tau'}$ . The adoption of RWA forbids the virtual transitions where the energy is not conserved. This implies that the only term, that now comes out in Eq. (5), is the matrix element  $m_{eg} \langle \psi_f | \Phi^+(x(\tau')) | \psi_i \rangle$ , thus only the positive frequency part of the field appears in the first-order amplitude transition from the initial state  $|g\rangle | \psi_i \rangle$  to  $|e\rangle | \psi_f \rangle$  for the GD model. This is given by

$$A_{\text{GD}}|g\psi_i\rangle \rightarrow |e\psi_f\rangle = ic_1 m_{eg} \int_{\tau_i}^{\tau_f} e^{i\omega_{eg}\tau'} \langle \psi_f | \Phi^+(x(\tau')) | \psi_i \rangle d\tau' \quad (10)$$

and then leads to the probability detection

$$P_{\text{GD}}(\tau_f, \tau_i) = c_1^2 |m_{eg}|^2 \int_{\tau_i}^{\tau_f} \int_{\tau_i}^{\tau_f} d\tau' d\tau'' e^{i\omega_{eg}(\tau''-\tau')} \times \langle \psi_i | \Phi^-(x(\tau')) \Phi^+(x(\tau'')) | \psi_i \rangle. \quad (11)$$

The response of the GD to the vacuum field state  $|0\rangle$  is

$$P_{\text{GD}}(\tau_f, \tau_i) = c_1^2 |m_{eg}|^2 \int_{\tau_i}^{\tau_f} \int_{\tau_i}^{\tau_f} d\tau' d\tau'' e^{i\omega_{eg}(\tau''-\tau')} \times \langle 0 | \Phi^-(x(\tau')) \Phi^+(x(\tau'')) | 0 \rangle = 0. \quad (12)$$

The GD, as a consequence of the RWA, therefore does not feel the zero-point vacuum fluctuations. From Eq. (12) it follows also that the response of the GD to the vacuum does not depend from the state of the motion of the same detector. The detection probability vanishes in particular for detectors traveling either along inertial or accelerated world lines. Therefore such a detector cannot show the well-known Unruh effect.

The adoption of the GD model, with its use of the RWA, to detect quantum correlations due to entanglement appears to be controversial in QFT systems [13,15,16]. By applying an approach already used in the photodetection of free electromagnetic fields [15,16], here we want to show the appearance of nonlocal effects in the Glauber scalar detection theory. To this aim we examine a free quantum scalar field  $|\Psi\rangle$ , expressed as

$$|\Psi\rangle = \exp \left\{ \int \frac{d^3k'}{2\omega} [\alpha(k') a^+(k') - \alpha^*(k') a(k')] \right\} |0\rangle, \quad (13)$$

and therefore  $|\Psi\rangle$  is the multimode coherent state satisfying  $a(\mathbf{k})|\Psi\rangle = \alpha(\mathbf{k})|\Psi\rangle$ . Taking into account Eqs. (1) and (13), the action of the operator  $\Phi^+(x)$  on  $|\Psi\rangle$  gives

$$\Phi^+(x)|\Psi\rangle = V(x)|\Psi\rangle, \quad (14)$$

where

$$V(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\mathbf{k}}{2\omega} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \alpha(\mathbf{k}). \quad (15)$$

It may easily be shown that  $V(x)$  satisfies the classical homogeneous Klein-Gordon equation

$$\left( \frac{\partial}{\partial t^2} - \frac{\partial}{\partial \mathbf{x}^2} + m^2 \right) V(x) = 0, \quad (16)$$

and can be therefore interpreted as a classical signal propagating freely. In Eq. (15) it appears only the factor  $e^{-i\omega t}$  with  $\omega > 0$ . Extending  $t$  to a complex variable, this corresponds in the complex plane  $t=t_1-it_2$ , to the appearance of the term  $e^{-\omega t_2}$ .  $V(x)$  is thus an analytical function in the lower com-

plex  $t$  half-plane and then its real and imaginary parts are therefore related by a Hilbert transformation,

$$\text{Im } V(\mathbf{x}, t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\text{Re } V(\mathbf{x}, t')}{t - t'} dt'. \quad (17)$$

Now we can easily evaluate the time evolution of quantities of interest in terms of  $V(x)$ . The first-order Glauber correlation function is defined as

$$G(x, x) = \langle \Phi^-(x) \Phi^+(x) \rangle, \quad (18)$$

and therefore is given by the expectation value of the operator  $\Phi^-(x) \Phi^+(x)$  on the quantum-field state. Such a function permits to estimate the rate of scalar quanta counting probability. By inserting Eq. (13) in Eq. (18), we obtain for it

$$G(x, x) = |V(x)|^2 = [\text{Re } V(x)]^2 + [\text{Im } V(x)]^2. \quad (19)$$

The mean value of the scalar field operator  $\Phi(x)$  has instead the form

$$\langle \Psi | \Phi(x) | \Psi \rangle = 2 \text{Re } V(x). \quad (20)$$

Let us consider a signal  $V(x)$  consisting of a plane wave with a sharp front moving in the positive  $\mathbf{z}$  axis direction whose real part is given by

$$\text{Re } V(x) = \Theta(t - \mathbf{z}) f(\mathbf{z} - t), \quad (21)$$

where  $\Theta$  is the Heaviside function and

$$f(\mathbf{z} - t) = \left. \begin{array}{l} f_0, \text{ for } t - \mathbf{z} \in [0, \Delta \bar{\mathbf{z}}] \\ 0, \text{ for } t - \mathbf{z} \notin [0, \Delta \bar{\mathbf{z}}] \end{array} \right\} \quad (22)$$

with  $\Delta \bar{\mathbf{z}}$  indicating the signal length here assumed to be small. We note from Eqs. (20) and (21) that  $\text{Re } V$  and therefore the mean value of the field reaches the detector at time  $t = \mathbf{z}$  and is equal to 0 for  $t < \mathbf{z}$ .  $\text{Im } V$ , related by Eq. (17) to  $\text{Re } V$ , will be given by

$$\text{Im } V(x) = \frac{f_0}{\pi} \ln \left| \frac{t - \mathbf{z}}{t - \mathbf{z} - \Delta \bar{\mathbf{z}}} \right|. \quad (23)$$

It results to differ from zero for all  $t$  even if  $\text{Re } V = 0$  for  $t < \mathbf{z}$ . Therefore the Glauber correlation function  $\langle \Phi^-(x) \Phi^+(x) \rangle$  written in Eq. (19) does not vanish before  $\langle \Phi(x) \rangle$  for  $t < \mathbf{z}$ .

The state consisting of the field vacuum and the atom in the ground state is the eigenstate of the GD Hamiltonian of Eq. (9) with eigenvalue 0. In this case the detector transition amplitude from the ground to the excited state is thus identically zero, meaning that each single measurement it performs when the field is in its vacuum state gives zero. Therefore the GD is not sensible to the field vacuum fluctuations of the field and non-null measurements must be attributed to the signal and not to preexisting vacuum correlations. Thus the behavior exhibited by the Glauber correlation function in relation to mean value of the field for the state  $|\Psi\rangle$  implies that the GD model, leading by its very nature to the development of effects over spacelike distances, is inappropriate to detect both the appearance of entanglement and causality in the time evolution of free fields [15,16].

### III. NONSTANDARD APPLICATION OF THE RWA TO QUANTUM DETECTION THEORY

In the previous section we have seen that already for a free scalar field, the Glauber detection correlation function presents a nonlocal behavior that cannot be accounted for by the pre-existing correlations of the zero-point field. This induces first to inquire if such a behavior may be observed when other kinds of detectors are used, and then to examine the detection processes in the case of field generated by quantum sources.

In particular, we shall here investigate the role played by a ‘‘nonstandard’’ application of the RWA to the quantum detection theory for the case of quantum scalar fields interacting with sources. Starting from a complete Hamiltonian  $H$ , which contains conserving and nonconserving energy terms, the detection probability rate for the UDD point like at rest and localized at  $\mathbf{x}$  with  $x(\tau) = x = (\mathbf{x}, t)$  is given by the time derivative of Eq. (8) and can be expressed as

$$\dot{P}_{\text{UDD}}(t) = 2c_1^2 |m_{eg}|^2 \text{Re} \int_0^t dt' \langle \psi_i | \Phi(\mathbf{x}, t) \Phi(\mathbf{x}, t') | \psi_i \rangle e^{i\omega_{eg}(t'-t)}, \quad (24)$$

where we have assumed that the field-detector interaction is turned from  $\tau_i = 0$  to  $\tau_f = t$ . We shall analyze the detection probability rate in the Heisenberg picture. It has again the form of Eq. (24), where  $\Phi'(\mathbf{x}, t)$  is now the Heisenberg operator satisfying the equation of the motion

$$\frac{\partial \Phi'(\mathbf{x}, t)}{\partial t} = i[H, \Phi'(\mathbf{x}, t)]. \quad (25)$$

Following the same approach previously adopted in quantum electrodynamics [13], a formal solution of Eq. (25) can be expressed by writing the Heisenberg operator  $\Phi'(\mathbf{x}, t)$  as

$$\Phi'(\mathbf{x}, t) = \Phi'_0(\mathbf{x}, t) + \Phi'_{RR}(\mathbf{x}, t) + \Phi'_s(\mathbf{x}, t) \quad (26)$$

with  $\Phi'_0(\mathbf{x}, t)$  the free field,  $\Phi'_{RR}(\mathbf{x}, t)$  the radiation reaction field of the detector on itself, while  $\Phi'_s(\mathbf{x}, t)$  indicates the field due to the source. The retarded source field can be expressed as  $\Phi'_s(\mathbf{x}, t) = F'(\mathbf{x}, t) \Theta(t - r)$ , where  $\Theta$  is the Heaviside function guaranteeing causality and therefore

$$\Phi'(\mathbf{x}, t) = \Phi'_0(\mathbf{x}, t) + \Phi'_{RR}(\mathbf{x}, t) + F'(\mathbf{x}, t) \Theta(t - r). \quad (27)$$

In the above expression we have assumed the external field source to be at distance  $r$  from the pointlike detector localized at  $\mathbf{x}$ .

Now let us define

$$\tilde{\Phi}'^{+(-)}(\mathbf{x}, t) = \Phi_0'^{+(-)}(\mathbf{x}, t) + \Phi_{RR}'^{+(-)}(\mathbf{x}, t) + \tilde{\Phi}_s'^{+(-)}(\mathbf{x}, t), \quad (28)$$

where  $\Phi_0'^{+(-)}$  and  $\Phi_{RR}'^{+(-)}$  are the positive (negative) frequency parts of the free and reaction radiation field, respectively, and  $\tilde{\Phi}_s'^{+(-)}$  is

$$\tilde{\Phi}'^{+(-)}_s(\mathbf{x}, t) = F'^{+(-)}(\mathbf{x}, t)\Theta(t-r) \quad (29)$$

with  $F'^{+(-)}$  indicating the positive (negative) frequency part of  $F'$ .

For time intervals larger than  $1/\omega_{eg}$ , we can adopt the approximation already used by Milonni *et al.* to treat the electromagnetic field case [13]. This consists in approximating Eq. (24) with the expression

$$\begin{aligned} \dot{P}_{\text{UDD}}(t) &\simeq \dot{P}_{\text{MD}}(t) \\ &= 2|m_{eg}|^2 \text{Re} \int_0^t dt' \langle \psi_i | \tilde{\Phi}'^-(\mathbf{x}, t) \tilde{\Phi}'^+(\mathbf{x}, t) \\ &\quad \times |\psi_i\rangle e^{i\omega_{eg}(t'-t)} \quad \text{for } t \gg 1/\omega_{eg}, \end{aligned} \quad (30)$$

which can be considered as the rate detection probability, evaluated in the Heisenberg picture, of a new scalar quantum detector model, which is the ‘‘Milonni detector’’ (MD). In Eq. (30), instead of the field operator  $\Phi(\mathbf{x}, t)$  which appears in Eq. (24) and thus contains also terms including positive and negative frequency parts, such as  $\Phi'^+\Phi'^-$ , only the combination  $\tilde{\Phi}'^-\tilde{\Phi}'^+$  is present.

We stress that the approximation used in Eq. (30) has been applied only after calculating the fields based on full Hamiltonian including conserving and nonconserving energy terms. Now we will show that this way of using such an approximation represents a nonstandard application of the RWA as originally performed in Glauber formulation and as a matter of fact a different one. In fact from Eq. (29) we observe that while  $\tilde{\Phi}'^{+(-)}$  gives the retarded positive (negative) frequency part of the external source field, it does not coincide with positive (negative) frequency part of the retarded operator  $\Phi'_s$ , which should be inserted according to the standard application of the RWA. Indeed the  $\Theta$  function by itself consists of positive and negative frequency parts as

$$\begin{aligned} \Theta(\tau) &= \Theta^-(\tau) + \Theta^+(\tau) \\ &= \lim_{\epsilon \rightarrow 0} \frac{-1}{2\pi i} \left\{ \int_{-\infty}^0 \frac{d\omega e^{-i\omega\tau}}{\omega + i\epsilon} + \int_0^{\infty} \frac{d\omega e^{-i\omega\tau}}{\omega + i\epsilon} \right\}. \end{aligned} \quad (31)$$

Therefore the causally retarded source field  $\tilde{\Phi}'^{+(-)}$  contains both positive and negative frequency components. The approach described here that replaces in the detection probability rate of Eq. (30) the full retarded fields with the retarded positive (negative) frequency part of the field is different from the standard form of the RWA that is performed at the beginning in the Glauber detection theory in the Hamiltonian of the system. This gives place to the detection rate probability of the defined MD, whose use prevents the development of quantum correlations over spacelike distances as we now will show.

By taking in the rate detection probability of MD, given by Eq. (30), as the initial field state, the vacuum state  $|\psi_i\rangle = |0\rangle$ , and then inserting Eqs. (28) and (29) we obtain

$$\begin{aligned} \dot{P}_{\text{MD}}(t) &= 2|m_{eg}|^2 \text{Re} \int_0^t dt' [\langle \Phi'_{RR}{}^-(\mathbf{x}, t) \Phi'_{RR}{}^+(\mathbf{x}, t') \rangle \\ &\quad + \Theta(t' - r) \langle \Phi'_{RR}{}^-(\mathbf{x}, t) F'^+(\mathbf{x}, t') \rangle \\ &\quad + \Theta(t - r) \langle F'^-(\mathbf{x}, t) \Phi'_{RR}{}^+(\mathbf{x}, t') \rangle \\ &\quad + \Theta(t - r) \Theta(t' - r) \langle F'_-(\mathbf{x}, t) F'^+(\mathbf{x}, t') \rangle] e^{i\omega_{eg}(t'-t)}. \end{aligned} \quad (32)$$

Under the assumption that the monopole detector atom is only weakly perturbed, the above expression becomes, similarly to the electromagnetic field case [13],

$$\begin{aligned} \dot{P}_{\text{MD}}(t) &\simeq 2|m_{eg}|^2 \Theta(t-r) \text{Re} \int_r^t dt' \langle F'^-(\mathbf{x}, t) F'^+(\mathbf{x}, t') \rangle e^{i\omega_{eg}(t'-t)}. \end{aligned} \quad (33)$$

The presence of the function  $\Theta(t-r)$  in the rate probability expression guarantees that the influence of the source field is not vanishing only inside the light cone centered on the external field source. Thus the adoption of the MD for models of matter-field interaction being the sources quantum, like the in Fermi model [9,31], or classical like in other models [26–28], does not lead to quantum correlations spreading in the whole space and is moreover causal, even if such a behavior appears to be ‘‘forced’’ by the approximation used in rate detection probability.

#### IV. QUANTUM DETECTION IN A SCALAR QFT MODEL

An analysis of the measurement and the possible observability of quantum correlations must use suitable detectors and can be strictly accomplished within exactly solvable physical models. In this spirit a simple QFT system, consisting of a quantum scalar field coupled to a classical source, has been recently investigated with none of the limitations related to perturbative calculations. So it appears of interest to study for this system the response of the various detectors to the field generated by localized sources.

##### A. Model

We consider a QFT model of a quantum scalar field  $\Phi(x)$  linearly interacting with a classical scalar source  $j(x)$ , assumed to be localized in a finite space-time region and turned for a finite time [26–28]. The Hamiltonian term describing the interaction is given by

$$\begin{aligned} H_{int}(t) &= g \int_{-\infty}^{+\infty} d^3\mathbf{x} [\Phi^+(\mathbf{x}, t) + \Phi^-(\mathbf{x}, t)] j(\mathbf{x}, t) \\ &= H_{int}^+(t) + H_{int}^-(t), \end{aligned} \quad (34)$$

where  $g$  is the source-field coupling constant.

Initially ( $t=0$ ) the field is taken in its vacuum state  $|0\rangle$ . The state  $|t\rangle$ , describing the system at time  $t$ , will be

$$|t\rangle = U(t)|0\rangle, \quad (35)$$

where  $U(t)$  is the interaction picture time evolution operator. Solving the equation of motion that derives from Eq. (34) we get for  $U(t)$  a formal expression valid at all orders in  $g$  as

$$U(t) = \exp \left[ -i \int_0^t dt' H_{int}^-(t') \right] \times \exp \left[ -i \int_0^t dt' H_{int}^+(t') \right] e^{-\xi(t)} e^{\alpha(t)}. \quad (36)$$

In Eq. (36) the coefficients  $\alpha(t)$  and  $\xi(t)$  depend explicitly on the source as

$$\begin{aligned} \alpha(t) &= \frac{ig^2}{2} \int_0^t dt_1 \int_0^t dt_2 \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 j(\mathbf{x}_1, t_1) \\ &\quad \times \Delta_-(\mathbf{x}_1 - \mathbf{x}_2, t_1 - t_2) j(\mathbf{x}_2, t_2), \\ \xi(t) &= \frac{ig^2}{2} \int_0^t dt_1 \int_0^t dt_2 \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 \\ &\quad \times \Delta(\mathbf{x}_1 - \mathbf{x}_2, t_1 - t_2) j(\mathbf{x}_1, t_1) j(\mathbf{x}_2, t_2) \Theta(t_1 - t_2), \end{aligned} \quad (37)$$

where  $\Delta$  is the two-point function, given by the field commutator as  $[\Phi(x), \Phi(y)] = i\Delta(x-y)$ , and  $\Delta_-$  is its negative frequency part [32,33].

It has been previously shown that the dynamics of any local observable  $\hat{O}[\Phi(x), \partial_\mu \Phi(x)]$ , satisfying the microcausality principle and represented by an analytical function of the field operator and its space and time derivatives, depends causally on the source [28,29]. With this model the presence of nonlocality has also been investigated by analyzing the localization properties of average values of local operators in connection to Hegerfeldt's theorem [3–5], which seems to imply causality violation for the time evolution of the wave functions, and one-point positive localization observables. In the same spirit and in the connection to the relevance of the detection theory for relating the results of measurements with the form of quantum correlation functions, here we will evaluate the expectation values, on the quantum state  $|t\rangle$  describing the system, of the Glauber and Newton-Wigner operators, which have recently been used in QFT models both of free fields and of matter-field interaction [8,9].

The Glauber operator for the scalar field is defined as  $\hat{\rho}_G(x) = \Phi^-(x)\Phi^+(x)$  and its expectation value on the state  $|t\rangle$  is

$$\langle t | \hat{\rho}_G(x) | t \rangle = g^2 \tilde{\Delta}_+(x-y) \tilde{\Delta}_-(x-y), \quad (38)$$

where the function  $\tilde{\Delta}_\pm(\tilde{\Delta}_-)$ , defined as

$$\tilde{\Delta}_\pm(x-y) \equiv \int_0^t dt' \int d^3\mathbf{x}' \Delta_\pm(\mathbf{x} - \mathbf{x}', t - t') j(\mathbf{x}', t'), \quad (39)$$

is not zero outside the light cone containing the source. Therefore the expectation value of  $\hat{\rho}_G$  given by Eq. (38) does not show a causal behavior.

The Newton-Wigner operator for scalar field has instead the form [9,34,35]

$$\rho_{NW}(x) = a_{NW}^\dagger(x) a_{NW}(x), \quad (40)$$

where  $a_{NW}^\dagger(x)$  and  $a_{NW}(x)$  may be expressed in terms of the negative (positive) frequency part of the field operator  $\Phi(x)$  as

$$\begin{aligned} a_{NW}^\dagger(x) &= R(\mathbf{x}) \Phi^-(x), \\ a_{NW}(x) &= R(\mathbf{x}) \Phi^+(x), \end{aligned} \quad (41)$$

where

$$R(\mathbf{x}) = \sqrt{2} \left[ m^2 - \left( \frac{\partial}{\partial \mathbf{x}} \right)^2 \right]^{1/4}. \quad (42)$$

$R(\mathbf{x})$  is a nonlocal operator that may be shown to correspond to a nonlocal integral transformation [35]. The expectation value of  $\rho_{NW}(x)$  on  $|t\rangle$  is

$$\langle t | \rho_{NW}(x) | t \rangle = g^2 R(\mathbf{x}) \tilde{\Delta}_+(x-y) R(\mathbf{x}) \tilde{\Delta}_-(x-y). \quad (43)$$

The expectation value of the Newton-Wigner operator  $\rho_{NW}$  on  $|t\rangle$  immediately shows a local behavior. In fact it contains the action of the nonlocal operator  $R(\mathbf{x})$  on the functions  $\tilde{\Delta}_+(x-y)$  and  $\tilde{\Delta}_-(x-y)$ , which already present contributions outside the light cone centered on the source. Nonlocal effects shown by both the Glauber and Newton-Wigner operators are, however, attributable to the fact that these operators do not satisfy the microcausality principle [28]. This implies that the measurement on one space-time point has influence another point at a spacelike distance.

## B. Response of UD detector

The appearance in the scalar model of nonlocal effects seems to be at variance with the results found in previous works that use local operator functions of the field and of its time and space derivatives [28,29]. However they are connected to the use of localization operators that do not satisfy the microcausality principle. All of this stresses once more the key role played by a proper detection theory in the question concerning nonlocality and measurement of quantum correlations due to a genuine entanglement and of causal effects. Here we will calculate explicitly the response of the pointlike UD detector to the field in our QFT scalar model.

In order to keep the problem simple we will assume the detector at rest at space point  $\mathbf{x}$ , so that the function describing its world line becomes  $x(\tau) = x = (\mathbf{x}, t)$ . Therefore the effects that depend from the motion of the detector, as the Unruh ones, will not appear in the detection probability. Moreover we shall assume that the source coupled to the quantum field is classical and localized within a sufficiently small space-time region around the space-time point  $y = (\mathbf{y}, y_0)$ . This source is thus effectively pointlike and we shall assume that it is turned on and off for an infinitesimal time interval. In this case from Eq. (36), the quantum-field state describing the evolving system at time  $t$  takes the form

$$|t\rangle = \exp[-ig\Theta(t-y_0)\Phi^-(y)] |0\rangle e^{\alpha_0(t)} \quad (44)$$

with

$$\alpha_0(t) = \frac{ig^2}{2} \Theta^2(t-y_0) \lim_{x \rightarrow 0} \Delta_-(x). \quad (45)$$

The above expression for  $\alpha_0(t)$  is formally divergent. Therefore one should regularize the space-time integrals by using a cutoff  $\lambda$ , which makes the source localized in a small, but not exactly pointlike, space-time region and we shall consider the limit  $\lambda \rightarrow \infty$  in those matrix elements where  $\lambda$  appears. However we will see that in our case the matrix elements we are interested in do not depend from the regularization of the integrals.

Using Eqs. (44) and (45) in Eq. (8) the detection probabilities to the field generated by the source can then be evaluated with no kind of approximation and becomes

$$P_{\text{UDD}} = c_1^2 |m_{eg}|^2 \int_{t_i}^{t_f} \int_{t_i}^{t_f} dt' dt'' e^{i\omega_{eg}(t''-t')} \langle t_i | \Phi(\mathbf{x}, t') \Phi(\mathbf{x}, t'') | t_i \rangle, \quad (46)$$

where we have assumed that the UDD-field interaction occurs in the interval time  $[t_f, t_i]$ . Three different physical situations can occur for:  $y_0 < t_i$ ,  $t_i < y_0 < t_f$ , and  $t_f < y_0$ .

(i)  $\mathbf{y}_0 < \mathbf{t}_i$ . The classical pointlike source is turned on at  $y_0 < t_i$ . In this case the response of the UDD takes the form

$$\begin{aligned} P_{\text{UDD}} &= P_1(t_f, t_i) + g^2 |P_2(s_f, s_i)|^2 \\ &= c_1^2 |m_{eg}|^2 \int_{t_i}^{t_f} \int_{t_i}^{t_f} dt' dt'' e^{i\omega_{eg}(t''-t')} \langle 0 | \Phi(\mathbf{x}, t') \Phi(\mathbf{x}, t'') | 0 \rangle \\ &\quad + g^2 c_1^2 |m_{eg}|^2 \Theta^2(t_i - y_0) \\ &\quad \times \left| \int_{t_i}^{t_f} dt'' e^{i\omega_{eg} t''} \Delta(\mathbf{x} - \mathbf{y}, t'' - y_0) \right|^2, \end{aligned} \quad (47)$$

where  $\Delta$  is the propagator function coming from the field commutator  $[\Phi(x), \Phi(y)] = i\Delta(x-y)$  and is vanishing when its argument is spacelike. Therefore to the last time integral of Eq. (47) contribute only the values of  $\Delta(x)$  such that  $x$  is inside the light cone centered on the space-time point  $y$ , where the classical source is localized. The detection probability for UDD can be seen to be made of two terms. The first, representing the vacuum contribution to the detector response function, is source independent and presents non-zero contributions outside the light cone centered on the source. The second is source dependent and, as shown in Appendix A,  $P_2(s_f, s_i)$  can be put in the form

$$\begin{aligned} P_2(s_f, s_i) &= 2c_1 |m_{eg}| \Theta(t_i - y_0) \Theta(s_f^2) \left[ \Theta(-s_i^2) \left( F_1(0, s_f^2) \right. \right. \\ &\quad \left. \left. - \frac{e^{i\omega_{eg}(r+y_0)}}{8\pi r} \right) + \Theta(s_i^2) F_1(s_i^2, s_f^2) \right], \end{aligned} \quad (48)$$

where  $F_1(u^2, v^2)$  is defined in Eq. (A3) and  $s_{f(i)}^2 = (t_{f(i)} - y_0)^2 - r^2$  with  $r = |\mathbf{x} - \mathbf{y}|$ . Because  $\Theta(s_f^2)$  appears in expression (48), the source dependent contribution of the UDD detector response turns out to be automatically causally retarded

(ii)  $\mathbf{t}_i < \mathbf{y}_0 < \mathbf{t}_f$ . The field-classical source coupling is turned in the time interval  $[t_f, t_i]$ . We can analyze this situation assuming that the coupling of the detector with the field

is turned on from  $t_i$  to  $y_0 - \epsilon$  and from  $y_0 + \epsilon$  to  $t_f$ , while the source-field interaction is effective in the interval time from  $y_0 - \epsilon$  to  $y_0 + \epsilon$ . Then we will take the limit  $\epsilon \rightarrow 0$  in the expressions obtained. Following the same procedure used to calculate the response of detector in the previous situation, we obtain for the UDD probability detection

$$P_{\text{UDD}} = P_1(t_f, t_i) + g^2 |P_2(s_f, 0)|^2. \quad (49)$$

Again  $P_1(t_f, t_i)$  represents the vacuum response contribution and coincides with the one of Eq. (47), while  $P_2(s_f, 0)$  is linked to the variation in the source and is given in this case by

$$P_2(s_f, 0) = 2c_1 m_{eg} \Theta(s_f^2) \left( F_1(0, s_f^2) - \frac{e^{i\omega_{eg}(r+y_0)}}{8\pi r} \right), \quad (50)$$

where we have assumed  $\lim_{x \rightarrow 0} \Theta(x) = 1$ . Also in this case we observe that the source dependent part of the UDD response is vanishing outside the light cone centered on the source.

(iii)  $\mathbf{t}_f < \mathbf{y}_0$ . The classical pointlike source is turned on at  $y_0 > t_f$ . Such a physical situation is not of interest for evaluating the detector response of the field generated by the source. In fact in this case the quantum-field state describing our system in the time interval  $[t_i, t_f]$  is the vacuum state  $|0\rangle$ , and therefore the response of the UDD is simply given by vacuum contribution with  $P_{\text{UDD}} = P_1(t_f, t_i)$ .

The results presented in this section show that the use UDD to detect field generated by a classical pointlike source does not give rise to the instantaneous appearance of quantum correlation over the whole space, and therefore the UDD response is causally retarded. This causal behavior comes out naturally from the detector models and is not put in a sense by ‘‘hand.’’

### C. Response of Glauber detector

In relation to the causal response of the UDD to the field generated by the classical source, localized in space and time, in our scalar QFT model it appears also of interest to evaluate here the response of the GD model in order to see how realistic its use in determining the structures of quantum correlations function. As seen in Sec. II, its use for free fields leads by its very nature to the development of effects developing over spacelike distances and at variance with Einstein’s causality.

In a manner analogous to the calculation of UDD response, here we again consider a source localized in an infinitesimal space-time region around the space-time point  $y$ . The detection probability for the Glauber detector  $P_{\text{GD}}$ , assumed to be at rest and located in  $\mathbf{x}$ , may thus be obtained by inserting Eqs. (44) and (45) in Eq. (11),

$$P_{\text{GD}} = c_1^2 |m_{eg}|^2 \int_{t_i}^{t_f} \int_{t_i}^{t_f} dt' dt'' e^{i\omega_{eg}(t''-t')} \langle t_i | \Phi^-(\mathbf{x}, t') \Phi^+(\mathbf{x}, t'') | t_i \rangle. \quad (51)$$

Again three different configurations may be considered.

(i)  $\mathbf{y}_0 < \mathbf{t}_i$ . In this case the response of the GD, as shown in the explicit calculation of Appendix, may be expressed as

$$P_{\text{GD}} = g^2 \left| \frac{1}{2} P_2(s_f, s_i) + P_3(s_f, s_i) \right|^2 \quad (52)$$

with  $P_2(s_f, s_i)$  and  $P_3(s_f, s_i)$  given with respect by Eq. (48) and

$$P_3(s_f, s_i) = c_1 |m_{eg}| \Theta(t_i - y_0) \{ \Theta(s_f^2) [ \Theta(s_i^2) F_2(s_i^2, s_f^2) + \Theta(-s_i^2) \times [F_2(0, s_f^2) + F_3(s_i^2, 0)] ] + \Theta(-s_f^2) F_3(s_i^2, s_f^2) \}, \quad (53)$$

where  $F_2(u^2, v^2)$  and  $F_3(u^2, v^2)$  are defined in the Eqs. (A9) and (A10). We observe in Eq. (52) that all terms are source dependent. In particular from expression (53) we also note the Heaviside function  $\Theta(-s^2)$  appears in  $P_3(s_f, s_i)$ . This implies that GD may instantaneously respond to the variation in the source, giving rise to nonlocality in our model in agreement with what seen in Sec II.

(ii)  $\mathbf{t}_i < \mathbf{y}_0 < \mathbf{t}_f$ . The GD detection probability is

$$P_{\text{GD}} = g^2 \left| \frac{1}{2} P_2(s_f, 0) + P_3(s_f, r) \right|^2, \quad (54)$$

where  $P_2(s_f, 0)$  is given in Eq. (50) and  $P_3(s_f, r)$  is defined as

$$P_3(s_f, r) = c_1 m_{eg} \{ \Theta(s_f^2) [F_2(0, s_f^2) + F_3(-r^2, 0)] + \Theta(-s_f^2) F_3(-r^2, s_f^2) \}.$$

We note that again in the detection probability of Eq. (54), consisting of all source dependent contributions, noncausal terms appear.

(iii)  $\mathbf{t}_f < \mathbf{y}_0$ . In this case the response of the GD vanishes, because this detector model is not sensitive to the vacuum fluctuations.

Finally we point that the appearance of noncausal terms in the response function of GD, given in Eqs. (52) and (54), cannot be related to the zero-point vacuum fluctuations, different from what happens for UDD detector. Thus noncausal behavior must be ascribed to the fact that the quantity  $\Phi^- \Phi^+$  which does not satisfy the microcausality principle, appears in the probability of detection for GD, given by Eq. (11), as a consequence of the standard application of the RWA. This is again in agreement with the previous results showing that the use of the RWA leads to the development, over spacelike distances, of quantum correlations not describing genuine entanglement and at variance with causal propagation of the signals [23,25,36].

## V. CONCLUSIONS

The evaluation of nonlocal quantum correlations, such as the entanglement between two or more systems consisting of separated quanta field, can be obtained by interpreting measurements performed with suitable quantum detector models not inducing by themselves nonlocally. In fact nonlocal effects due to the use of inappropriate model detectors could lead to the development of correlations over the whole space mimicking a not physical entanglement and even violating Einstein's causality. Thus the theory of detection is of importance in the interpretation of measurement and observability of quantum nonlocal effects.

Mainly two detector models are commonly used, that is, GD and UDD models [17,19–22]. The difference relevant,

for our purpose, between these kinds of detector models is that in the Hamiltonian describing the GD model the RWA is adopted and it thus responds only to the positive frequency of the field it detects, while in the UDD model the Hamiltonian maintains the counter-rotating terms in field-detector interaction and thus responds both to positive and negative field frequencies.

Because a rigorous analysis of the measurement and observability of correlations through quantum detection processes in QFT requires the use of suitable models that can be solved exactly [31], we have used a QFT system, formed by a quantum scalar field coupled linearly to a classical scalar source localized in a finite space-time region, which when presented by these characteristics, can be considered a good model [29].

The use of the GD to interpret the appearance of quantum correlations in QFT models has been questioned [13,15,16]. In the first part of this paper we have shown that by taking a coherent state of a quantum scalar field, whose expectation value of the field operator given by a wave plane with a sharp front, the scalar quanta counting probability, evaluated according to the GD model, comes out different from zero before the signal reaches the detector. Such a result, which is in agreement with what already obtained in the case of free electromagnetic field [15,16], raises also the question relative to the role played by RWA in the models of quantum detection theory and its relation with the development of spurious quantum correlations at spacelike distances. In the same spirit here we have extended our analysis by also examining the detection process in the case of a quantum scalar field generated by the sources. We have then adopted a procedure, previously used for the electromagnetic fields case [13], that makes use of the Heisenberg picture and then applies RWA to the formal solution of the detector-field interaction. The fields are thus obtained from the complete Hamiltonian, which describes the quantum field interacting both with the source and the detector, where also the nonconserving energy terms are kept. In such models it is possible to calculate the detection probability rate. We have shown that by first obtaining the fields with the complete Hamiltonian, including both conserving and nonconserving energy terms, then separating the full retarded field in a retarded positive and negative frequency parts, and finally applying the RWA on the field themselves, a causal rate of detection probability is obtained. This approach, due originally to Milonni, is really different from the standard application of the RWA in the Hamiltonian. It does not in fact give rise to the appearance of nonlocal effects in the evolution of the positive and negative frequency parts of the field. This deep difference in the final results must be associated to the fact that the spectral decomposition of the retarded positive (negative) frequency part of the field does contain both positive and negative frequencies coming from the  $\Theta$ -like retarded terms. This is therefore different from the standard procedure adopted in the Glauber theory of detection where only positive or negative frequencies of the complete field are kept. Thus such an application of the RWA leads to an effective new detector model, which differs from the GD one and does not give rise to quantum correlations at spacelike distances, but the causal behavior results to be put by “hand” in the solution of complete detector-field interaction.



We have shown that in our scalar model local operator function of the field develop causally from the source, nevertheless, nonlocal effects appear in the expectation values of one-point positive localization observables, such as the Glauber and Newton-Wigner operators. The reason of this result is however that these operators do not satisfy the microcausality principle and therefore induce, by their very definition, effects over spacelike distances [28]. Thus they must not be used, for example by calculating their correlations, to furnish indications of the presence of nonlocal effects.

A valid interpretation of the appearance of quantum correlations in our model requires analyzing realistic models that describe the detection of the scalar field generated by the source. To this end we have explicitly evaluated the response function of two detector models, that is, the UDD and GD models, at rest in our reference frame in order to avoid the appearance of Unruh-like effects. We have then shown that the UDD detection probability causally responds to the field generated by the source and is not characterized by nonlocal effects, apart from the source-independent vacuum contribution related to the UDD sensitiveness to zero-point fluctuations. Anyway this term must not be considered to describe the appearance of nonlocality due to the variations in the source and can in principle be taken into account to interpret the results [23,24,36]. On the other hand the response function of the GD model gives source dependent terms in the detection probability rate that corresponds to an instantaneous spreading of source effects over the whole space. If taken at face value this would seem to imply a violation of causal propagation of signals in our QFT system, and therefore our results confirm previous ones, indicating that the

standard use of RWA does lead to development of nonlocal effects with time [23,25,36].

In conclusion to measure either the quantum correlations or those causal effects linked to the time varying sources in our QFT model, the adoption of the GD model, with the RWA in the Hamiltonian turns out to be inappropriate inducing by its very definition nonlocality. Another detector model, the MD, obtained by a nonstandard application of the RWA in the Heisenberg picture field solution, although it guarantees causality, presents the characteristic that its behavior is somehow imposed in the detection theory from the outside, and one may then not be sure whether source relevant terms may also be thrown out. Instead the UDD model must be preferred to describe quantum correlation for quantized fields because the appearance of nonlocal effects in its response to field quanta is only due the zero-point vacuum fluctuations and does not depend from the source. Moreover this behavior comes out naturally from the detection model itself. It would then be also of interest to analyze the behavior of such a quantum detector model when it or the source is in arbitrary motion in order to study the relation between the appearance of Unruh effects and nonlocal quantum effects.

#### APPENDIX A: PHOTODETECTION PROBABILITIES

Here we shall give the explicit calculation of the detection probability of the UDD and GD in our QFT system when  $y_0 < t_i$ .

For the UDD the source dependent contribution of response to the field is given by the second term of the right side of expression (47). The integral appearing in it, after inserting the explicit form of  $\Delta$  function, can be expressed as

$$\int_{t_i}^{t_f} dt'' e^{i\omega_{eg}t''} \Delta(\mathbf{x} - \mathbf{y}, t'' - y_0) = 2e^{i\omega_{eg}y_0} \int_{y_0}^t dt'' e^{i\omega_{eg}(t''-y_0)} \left[ -\frac{1}{4\pi} \delta(s''^2) + \frac{m\Theta(s''^2)}{8\pi\sqrt{s''^2}} J_1(m\sqrt{s''^2}) \right], \quad (\text{A1})$$

with  $s''^2 = (t'' - y_0)^2 - r^2$  and  $J_1$  indicating the Bessel function of first order [37]. Performing the change  $t'' \rightarrow s''^2$  in integration variable [ $s^2(t)$  is monotone in the integration variable for  $t''$ ] Eq. (A1) may be put in the form

$$\int_{t_i}^{t_f} dt'' e^{i\omega_{eg}t''} \Delta(\mathbf{x} - \mathbf{y}, t'' - y_0) = \Theta(s_f^2) \left\{ \Theta(-s_i^2) \left[ -\frac{e^{i\omega_{eg}(r+y_0)}}{8\pi r} + e^{i\omega_{eg}y_0} \int_0^{s_f^2} \frac{ds''^2}{2\sqrt{s''^2+r^2}} e^{i\omega_{eg}\sqrt{s''^2+r^2}} \frac{m}{8\pi\sqrt{s''^2}} J_1(m\sqrt{s''^2}) \right] \right. \\ \left. + \Theta(s_i^2) \int_{s_i^2}^{s_f^2} \frac{ds''^2}{2\sqrt{s''^2+r^2}} e^{i\omega_{eg}\sqrt{s''^2+r^2}} \frac{m}{8\pi\sqrt{s''^2}} J_1(m\sqrt{s''^2}) \right\}. \quad (\text{A2})$$

By using the last equation and defining the function  $F_1(u^2, v^2)$  as

$$F_1(u^2, v^2) = 2e^{i\omega_{eg}y_0} \left\{ e^{i\omega_{eg}r} \left( -\frac{1}{8\pi r} \right) + \int_{v^2}^{u^2} \frac{ds''^2}{2\sqrt{s''^2+r^2}} e^{i\omega_{eg}\sqrt{s''^2+r^2}} \frac{m}{8\pi\sqrt{s''^2}} J_1(m\sqrt{s''^2}) \right\}, \quad (\text{A3})$$

we obtain expression (48), whose square modulus gives the source dependent term of UDD detection probability in our QFT model.

Now we evaluate explicitly the response function of the GD when  $y_0 < t_i$ . To this purpose we insert Eq. (44) in Eq. (51) and we obtain

$$P_G(t_f, t_i) = c_1^2 |m_{eg}|^2 g^2 \Theta^2(t - y_0) \left| \int_{t_i}^{t_f} dt'' e^{i\omega_{eg} t''} \Delta_+(\mathbf{x} - \mathbf{y}, t'' - y_0) \right|^2. \quad (\text{A4})$$

The square modulus in the above expression can be written after decomposing  $\Delta_+$  in its real and imaginary parts as

$$\left| \int_{t_i}^{t_f} dt'' e^{i\omega_{eg} t''} \Delta_+(\mathbf{x} - \mathbf{y}, t'' - y_0) \right|^2 = \left| \int_{t_i}^{t_f} dt'' e^{i\omega_{eg} t''} \text{Re } \Delta_+ + i \int_{t_i}^{t_f} dt'' e^{i\omega_{eg} t''} \text{Im } \Delta_+ \right|^2. \quad (\text{A5})$$

Because  $\text{Re } \Delta_+ = \frac{1}{2} \Delta$  it can be easily shown that the first term in Eq. (A5) gives a contribution proportional to expression (A1),

$$\int_{t_i}^{t_f} dt'' e^{i\omega_{eg} t''} \text{Re } \Delta_+(\mathbf{x} - \mathbf{y}, t'' - y_0) = \frac{1}{2} \int_{t_i}^{t_f} dt'' e^{i\omega_{eg} t''} \Delta(\mathbf{x} - \mathbf{y}, t'' - y_0) \quad (\text{A6})$$

Instead using the explicit forms of  $\text{Im } \Delta_+(x)$  for  $x_0 > 0$  [37]

$$\text{Im } \Delta_+(x) = -i \left[ \frac{m\Theta(x^2)}{8\pi\sqrt{x^2}} N_1(m\sqrt{x^2}) + \frac{2m\Theta(-x^2)}{8\pi^2\sqrt{-x^2}} K_1(m\sqrt{-x^2}) \right] \quad (\text{A7})$$

with  $N_1(z)$  and  $K_1(z)$ , respectively, the first-order Neumann and Mac Donald functions and performing the change in the integration variable  $t'' \rightarrow s''^2$ , we can evaluate the second integral in the right side of Eq. (A5),

$$\begin{aligned} i \int_{t_i}^{t_f} dt'' e^{i\omega_{eg} t''} \text{Im } \Delta_+(\mathbf{x} - \mathbf{y}, t'' - y_0) &= \Theta(s_f^2) \left[ \Theta(s_i^2) e^{i\omega_{eg} y_0} \int_{s_i^2}^{s_f^2} \frac{ds''^2}{2\sqrt{s''^2 + r^2}} e^{i\omega_{eg} \sqrt{s''^2 + r^2}} \frac{m}{8\pi\sqrt{s''^2}} N_1(m\sqrt{s''^2}) + \Theta(-s_i^2) \right. \\ &\quad \times \left( e^{i\omega_{eg} y_0} \int_0^{s_f^2} \frac{ds''^2}{2\sqrt{s''^2 + r^2}} e^{i\omega_{eg} \sqrt{s''^2 + r^2}} \frac{m}{8\pi\sqrt{s''^2}} N_1(m\sqrt{s''^2}) \right. \\ &\quad \left. \left. + e^{i\omega_{eg} y_0} \int_{s_i^2}^0 \frac{ds''^2}{2\sqrt{s''^2 + r^2}} e^{i\omega_{eg} \sqrt{s''^2 + r^2}} \frac{2m}{8\pi^2\sqrt{-s''^2}} K_1(m\sqrt{-s''^2}) \right) \right] \\ &\quad + \Theta(-s_f^2) e^{i\omega_{eg} y_0} \int_{s_i^2}^{s_f^2} \frac{ds''^2}{2\sqrt{s''^2 + r^2}} e^{i\omega_{eg} \sqrt{s''^2 + r^2}} \frac{2m}{8\pi^2\sqrt{-s''^2}} K_1(m\sqrt{-s''^2}). \end{aligned} \quad (\text{A8})$$

Now inserting the Eqs. (A6) and (A8) in Eq. (A4) and defining the functions  $F_2(u^2, v^2)$  and  $F_3(u^2, v^2)$  as

$$F_2(u^2, v^2) = e^{i\omega_{eg} y_0} \int_{v^2}^{u^2} \frac{ds''^2}{2\sqrt{s''^2 + r^2}} e^{i\omega_{eg} \sqrt{s''^2 + r^2}} \frac{m}{8\pi\sqrt{s''^2}} N_1(m\sqrt{s''^2}), \quad (\text{A9})$$

$$F_3(u^2, v^2) = e^{i\omega_{eg} y_0} \int_{v^2}^{u^2} \frac{ds''^2}{2\sqrt{s''^2 + r^2}} e^{i\omega_{eg} \sqrt{s''^2 + r^2}} \frac{2m}{8\pi^2\sqrt{-s''^2}} K_1(m\sqrt{-s''^2}), \quad (\text{A10})$$

with respect, we obtain for the GD response expression (52).

- 
- [1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).  
 [2] J. S. Bell, Physics **1**, 195 (1964).  
 [3] G. C. Hegerfeldt, Phys. Rev. Lett. **54**, 2395 (1985).  
 [4] G. C. Hegerfeldt, Phys. Rev. D **10**, 3320 (1974).  
 [5] G. C. Hegerfeldt and S. N. M. Ruijsenaars, Phys. Rev. D **22**, 377 (1980).  
 [6] G. C. Hegerfeldt, Phys. Rev. Lett. **72**, 596 (1994).  
 [7] G. C. Hegerfeldt, *Irreversibility and Causality*, edited by A. Bohm, H. D. Doebner, and P. Kielanowski, Lecture Notes in Physics Vol. 504 (Springer, Berlin, 1998), pp. 238–245.  
 [8] T. Petrosky, G. Ordonez, and I. Prigogine, Phys. Rev. A **62**, 042106 (2000).  
 [9] E. Karpov, G. Ordonez, T. Petrosky, I. Prigogine, and G. Pronko, Phys. Rev. A **62**, 012103 (2000).  
 [10] F. Buscemi and G. Compagno, Open Sys. Inf. Dyn. **13**, 301 (2006).  
 [11] M. H. Rubin, Phys. Rev. D **35**, 3836 (1987).  
 [12] D. Buchholz and J. Yngvason, Phys. Rev. Lett. **73**, 613 (1994).  
 [13] P. W. Milonni, D. F. V. James, and H. Fearn, Phys. Rev. A **52**, 1525 (1995).  
 [14] M. Fleischhauer, J. Phys. A **31**, 453 (1998).  
 [15] V. P. Bykov and V. I. Tatarskii, Phys. Lett. A **136**, 77 (1989).  
 [16] V. I. Tatarskii, Phys. Lett. A **144**, 491 (1990).  
 [17] R. J. Glauber, Phys. Rev. **130**, 2529 (1963); **131**, 2766 (1963).

- [18] P. L. Kelley and W. H. Kleiner, *Phys. Rev.* **136**, A316 (1964).  
[19] W. G. Unruh, *Phys. Rev. D* **14**, 870 (1976).  
[20] B. F. Svaiter and N. F. Svaiter, *Phys. Rev. D* **46**, 5267 (1992).  
[21] S. Schlicht, *Class. Quantum Grav.* **21**, 4647 (2004).  
[22] P. Langlois, *Ann. Phys.* **321**, 2027 (2006).  
[23] G. Compagno, R. Passante, and F. Persico, *J. Mod. Opt.* **37**, 1337 (1990).  
[24] G. Compagno, G. M. Palma, R. Passante, and F. Persico, *Chem. Phys.* **198**, 19 (1995).  
[25] A. A. Clerk and J. E. Sipe, *Found. Phys.* **28**, 639 (1998).  
[26] I. Antoniou, E. Karpov, and G. Pronko, *Found. Phys.* **31**, 1641 (2001).  
[27] L. Maiani and M. Testa, *Phys. Lett. B* **356**, 319 (1995).  
[28] F. Buscemi and G. Compagno, *Phys. Lett. A* **334**, 357 (2005).  
[29] F. Buscemi and G. Compagno, *J. Phys. B* **39**, S695 (2006).  
[30] M. D. Srinivas and E. B. Davies, *Opt. Acta* **28**, 981 (1981);  
**29**, 235 (1982).  
[31] E. A. Power and T. Thirunamachandran, *Phys. Rev. A* **56**, 3395 (1997).  
[32] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).  
[33] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Westview, New York, 1995).  
[34] T. D. Newton and E. P. Wigner, *Rev. Mod. Phys.* **21**, 400 (1949).  
[35] G. Fleming and J. Butterfield, *Physics to Philosophy: Essays in Honor of Micheal Redhead* (Cambridge University Press, London, 1998).  
[36] A. K. Biswas, G. Compagno, G. M. Palma, R. Passante, and F. Persico, *Phys. Rev. A* **42**, 4291 (1990).  
[37] K. Gottfried and V. F. Weisskopf, *Concepts of Particle Physics* (Oxford University Press, London, 1986), Vol. 1.