Goos-Hänchen shifts in frustrated total internal reflection studied with wave-packet propagation

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We have investigated the Goos-Hänchen (GH) shifts in frustrated total internal reflection (FTIR) studied with wave-packet propagation. In the first-order approximation of the transmission coefficient, the GH shift is exactly the expression given by a stationary phase method, thus saturating an asymptotic constant in two different ways depending on the angle of incidence. Taking the second-order approximation into account, the GH shift does not saturate with increasing gap width when the small beam size is used. The GH shift increases by decreasing the beamwidth at the small incidence angles, while for the large incidence angles it reveals a strong decrease by decreasing the beamwidth. These phenomena offer the better understanding of the GH shift and tunneling delay time in FTIR.

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It is well known that a light beam totally reflected from an interface between two dielectric media undergoes lateral shift from the position predicted by geometrical optics [1]. This phenomenon was referred to as Goos-Hänchen (GH) effect [2] and was theoretically explained by Artmann's stationary phase method [3] and Renard's energy flux method [4]. Because of the potentials applications in integrated optics [2], optical waveguide switch [5], and optical sensors [6,7], the GH shifts including other three nonspecular effects such as angular deflection, focal shift, and waist-width modification have been extensively investigated in partial reflection [8–13], attenuated total reflection [14,15], and frustrated total internal reflection (FTIR) [16–20].

From a somewhat different perspective, the optical tunneling phenomenon in FTIR has attracted much attention in the last two decades [21–25] because of the analogy between FITR and quantum tunneling. Theoretical [21,23] and experimental [22,24] investigations have demonstrated that the GH shifts in FTIR play an important role in the superluminal tunneling time and the well-known "Hartman effect" [26], which describes that the group delay for quantum particles tunneling though a potential barrier saturates to a constant for an opaque barrier. Recently, Martinez and Polatdemir [27] studied the effect of width of the beam on the GH shift (which is proportional to tunneling time) to offer the complementary insights into the origin of Hartman effect in FTIR. In addition, Haibel *et al.* [19] once carried out a comprehensive study of the GH shift in FTIR as a function of the polarization, beamwidth, and incidence angle in the microwave experiment, which challenges its theoretical descriptions. In fact, the expressions of the GH shifts given by stationary phase method and energy flux method are independent of the beamwidth. Thus, an exact theory for GH shift is still not available [28].

The main purpose of this Brief Report is to investigate that the GH shifts in FTIR by wave-packet propagation. It is shown that the GH shift in the first-order approximation of the transmission coefficient is exactly the expression of the GH shift given by stationary phase method. The GH shift in this case approaches the saturation value in two different ways depending on the incidence angle. Considering the second-order approximation, the GH shift does not saturate with increasing gap width when the beam size is very small. The GH shift will become strongly dependent on the beamwidth.

For simplicity, we consider TE polarized beam incident into the double-prism structure with the angular frequency ω and incidence angle θ_0, as shown in Fig. 1, where a is the width of air gap. The permittivity, permeability, and refractive index of the prism are denoted by ε, μ, and n, respectively. For a well-collimated beam, the electric field of the incident beam can be expressed by

$$\Psi_{in}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k_y) \exp[i(k_x x + k_y y)] dk_y, \qquad (1)$$

where $k_x = nk \cos\theta$, $k_y = nk \sin\theta$, $k = \omega/c$, $n = \sqrt{\epsilon\mu}$, c is the speed of light in vacuum, θ is the incident angle of the plane-wave component under consideration, and time dependence $\exp(-i\omega t)$ is implied and suppressed. For a Gaussian-shaped incident beam whose peak is assumed to be located at $x=0$,

$$\Psi_i(x=0,y) = \exp\left(-\frac{y^2}{2w_y^2}\right)\exp(ik_{y0}y), \qquad (2)$$

its angular spectral distribution is also a Gaussian function, $A(k_y) = w_y \exp[-(w_y^2/2)(k_y - k_{y0})^2]$, around its central $k_{y0} = k \sin\theta_0$, $w_y = w_0/\cos\theta_0$, w_0 is the width of the beam at waist. According to Maxwell equations and boundary conditions, the field of the transmitted beam is found to be

$$\Psi_t(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} TA(k_y) \exp\{i[k_x(x-a) + k_y y]\} dk_y, \qquad (3)$$

with the transmitted coefficient $T = \exp(i\phi)/f$ is given by

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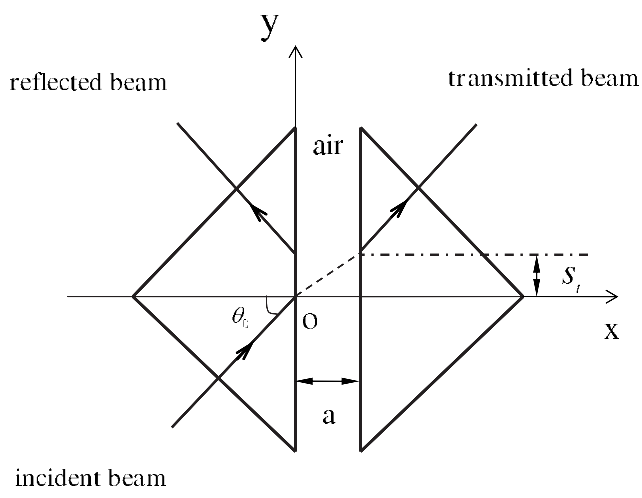

FIG. 1. Schematic diagram of the GH shift s_t in FTIR, where the width of air gap is a.

$$f\exp(i\phi)=\cosh\kappa a+\frac{i}{2}\left(\frac{k_x}{\kappa}-\frac{\kappa}{k_x}\right)\sinh\kappa a, \tag{4}$$

where $\kappa=(k_y^2-k^2)^{1/2}$.

First, we look at the GH shift in the first-order approximation of the transmission coefficient. Expanding the exponent in Taylor series at k_{y0} and retaining up to the first-order term, then we will obtain

$$T(k_y)=\exp[\ln T(k_y)]\approx T_0\exp\left[\frac{1}{T_0}\frac{dT}{dk_{y0}}(k_y-k_{y0})\right], \tag{5}$$

where $T_0=T(k_{y0})$ and d/dk_{y0} denotes the derivative with respect to k_y evaluated at $k_y=k_{y0}$. Introduce two real parameters L_t' and L_t'' defined as

$$L=L_t'+iL_t''=\frac{i}{T_0}\frac{dT}{dk_{y0}}, \tag{6}$$

then, in terms of the phase and magnitude of T, we will have

$$L_t'=-\frac{d\phi}{dk_{y0}} \quad \text{and} \quad L_t''=\frac{d}{dk_{y0}}\ln|T(k_y)|.$$

Substituting Eq. (5) into Eq. (3) and employing the paraxial approximation condition,

$$k_x\approx k_{x0}-(k_y-k_{y0})\tan\theta_0-\frac{(k_y-k_{y0})^2}{2k\cos^2\theta_0}, \tag{7}$$

we obtain the transmitted beam at $x=a$,

$$\Psi_t(a,y)\approx T_0\exp\left[-\frac{(y-L_t')^2}{2w_y^2}\right]\exp\left[i\left(k_{y0}+\frac{L_t''}{w_y^2}\right)y\right], \tag{8}$$

where $L_t'=-d\phi/dk_{y0}$ is exactly the GH shift obtained by the stationary phase method [3] and is thus given by

$$s_t^p=\frac{s_c}{2f_0^2}\left[\left(\frac{\kappa_0}{k_{x0}}-\frac{k_{x0}}{\kappa_0}\right)+\left(\frac{k_{x0}}{\kappa_0}+\frac{\kappa_0}{k_{x0}}\right)\left(1+\frac{\kappa_0^2}{k_{x0}^2}\right)\frac{\sinh 2\kappa_0 a}{2\kappa_0 a}\right], \tag{9}$$

with $s_c=ak_{y0}/\kappa_0$. When the width of air gap is large enough, that is, $a\gg 1/\kappa$, the GH shift tends to a constant,

$$s_{asymp}^p\equiv\lim_{\kappa_0 a\to\infty}s_t^p=\frac{2k_{y0}}{\kappa_0 k_{x0}}. \tag{10}$$

With increasing the air gap, the GH shift reaches a asymptotic constant, which is in agreement with the experimental results [19,22] and is also closely related to the counterintuitive Hartman effect of the tunneling delay time in the limit of an opaque barrier [22,24].

More interestingly, what we emphasize here is that the GH shift approaches the saturation value in two different ways depending on the angle of incidence. The GH shift [Eq. (9)] can be expressed by the following form:

$$s_t^p=\frac{g_0}{1+g_0^2}\left[\left(1+\frac{\kappa_0^2}{k_{x0}^2}\right)\left(1+\frac{k_{x0}^2}{\kappa_0^2}\right)\frac{k_{y0}}{k_{x0}^2-\kappa_0^2}-\frac{2ak_{y0}}{\kappa_0\sinh 2\kappa_0 a}\right], \tag{11}$$

where $g_0=(k_{x0}^2-\kappa_0^2)\tanh\kappa_0 a/2k_{x0}\kappa_0$. Keeping the next term to leading term for large a shows the approach to asymptotic value by

$$s_t^p\simeq s_{asymp}^p+8a\left(\frac{k_{y0}}{k_{x0}}\right)\left[\frac{k_{x0}^2(\kappa_0^2-k_{x0}^2)}{(\kappa_0^2+k_{x0}^2)^2}\right]e^{-2\kappa_0 a}. \tag{12}$$

It is clearly evident from the above expression that for $\kappa_0^2-k_{x0}^2<0$ the GH shift increases monotonically to reach the saturation value, while for $\kappa_0^2-k_{x0}^2>0$ it reaches the saturation value from above, that is, there is a hump before it attains saturation. Therefore, when the necessary condition for incident angle,

$$\theta_0>\theta_p\equiv\sin^{-1}\sqrt{\frac{1+n^2}{2n^2}}, \tag{13}$$

is satisfied, the GH shift can approach the saturation limit with negative slope and can be larger than the saturation value for the intermediate values of the air gap. The position of the maximum of the hump (a_0) is given by

$$a_0=\frac{1}{2\kappa_0}\frac{\kappa_0^4+3k_{x0}^2\kappa_0^2}{(k_{x0}^2+\kappa_0^2)^2}. \tag{14}$$

Figure 2 shows that for the large a the GH shift s_t^p is independent of the width a of the air gap hence it saturates a asymptotic constant, where $\lambda=32.8$ mm and $n=1.605$ (corresponding to critical angle $\theta_c=38.5°$ for total reflection and $\theta_p=56.4°$) [19]. Furthermore, the GH shift approaches the asymptotic limit from above for $\theta_0>\theta_p$ and from below for $\theta_0<\theta_p$. This phenomenon does result from the interference between the incident and reflected beams and is not due to the interference time [27]. Of course, it can also been seen from the relationship between the GH shift and group delay time discussed in Ref. [29] that the group delay time in FTIR also saturates to a constant from above for $\theta_0>\theta_p$ [30] in the

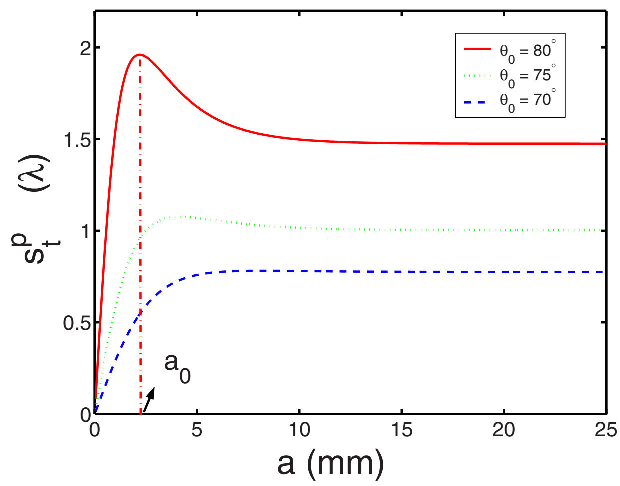

FIG. 2. (Color online) Dependence of the GH shifts s_t^p(in unit of λ) on the air gap width a in FTIR, where $\lambda=32.8$ mm and $n=1.605$.

same way as that in the quantum tunneling for $E<V_0/2$ [31,32] since the self-interference delay time that comes from the overlap of incident and reflected waves in front of barrier is of great importance [33].

Next, in what follows we will show the influence of the beam waist width on the GH shift. To this end, we consider the exponent of the transmission coefficient is approximated to the second-order term,

$$T(k_y) \approx T_0 \exp\left[\frac{1}{T_0}\frac{dT}{dk_{y0}}(k_y-k_{y0}) + \frac{1}{2}\frac{d}{dk_{y0}}\left(\frac{1}{T}\frac{dT}{dk_y}\right)(k_y-k_{y0})^2\right]. \quad (15)$$

Introducing two new real parameters F_t' and F_t'' defined as

$$F_t = F_t' + iF_t'' = -i\frac{d}{dk_{y0}}\left(\frac{1}{T}\frac{dT}{dk_y}\right), \quad (16)$$

then, with the phase and magnitude of $T(k_y)$, we have

$$F_t' = \frac{d^2\phi}{dk_{y0}^2}$$

and

$$F_t'' = -\frac{d^2}{dk_{y0}^2}\ln|T|.$$

Substituting expression (15) into Eq. (3) using paraxial condition (7) and neglecting some unimportant factors, we finally obtain the following field of the transmitted beam at $x=a$,

$$\Psi_t(a,y) \approx T_0 \exp\left[-\frac{1}{2w_{tf}^2}\left(y-L_t'+\frac{\eta_t F_t'}{w_{ty}}\right)^2\right] \times \exp\left[i\left(k_{y0}+\frac{\eta_t}{w_{ty}}\right)y\right], \quad (17)$$

where $\eta_t=L_t''/w_{ty}$, $w_{tf}=(w_{ty}^2-iF_t')^{1/2}$, and $w_{ty}^2=w_y^2+F_t''$ correspond to the angular deflection, focal shift, and waist-width modification, respectively [13]. The GH shift in this case can be expressed by

$$s_t = L_t' - \frac{\eta_t F_t'}{w_{ty}}. \quad (18)$$

Obviously, the second term on the right-handed side of Eq. (18) is a second-order correction, which leads to the dependence of the GH shift on the beamwidth. In addition, it also results in its dependence on the width of air gap in the opaque barrier limit.

Figure 3 demonstrates that the GH shift in the second-order approximation depends on the beamwidth, where (a) $\theta_0=45°$, (b) $\theta_0=75°$, and other parameters are the same as in Fig. 2. Compared with Fig. 2 discussed above, the GH shift becomes dependent on the width a in the limit of an opaque barrier due to the second correction. When the beamwidth is large, that is, the divergence angle becomes small, the correction to GH shift can be neglected, thus for a well-collimated beam the GH shift is in agreement with that given by the stationary phase method. More importantly, Fig. 3 shows that the GH shift increases by decreasing the beamwidth at $\theta_0=45°$, while the GH shift for $\theta_0=75°$ shows a strong decrease by decreasing the beamwidth. As shown in Fig. 3(a), the GH shift becomes dependent linearly on the width of air gap because the Fourier components of the incident beam above the critical angle are strongly depressed so that the plane-wave components just below the critical angle start to dominate. That is to say, when the incidence angle is larger than but close to the critical angle, the wave vector filter is more pronounced for a larger beamwidth, the

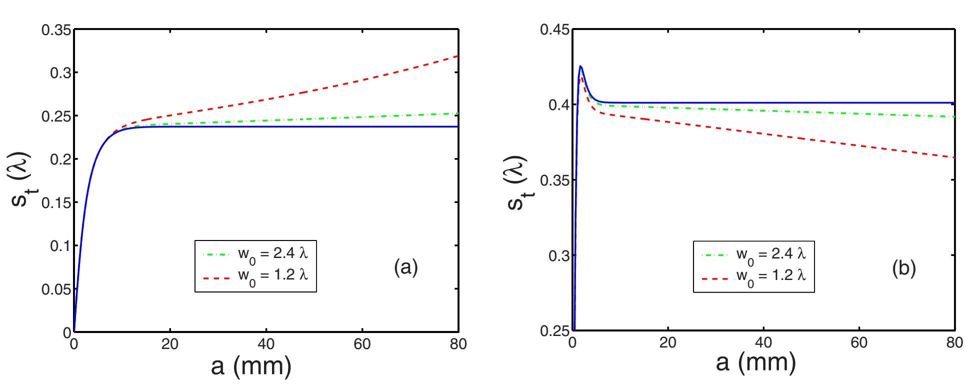

FIG. 3. (Color online) Dependence of GH shift s_t (in unit of λ) on width of air gap, where (a) $\theta_0=45°$, (b) $\theta_0=75°$, and other parameters are the same as in Fig. 2. The solid corresponds to GH shift in the first-order approximation, the dashed and dotted curves correspond to the GH shifts in the second-order approximation.

transmission is essentially not tunneling at all, thus the GH shift increases with increasing the width a, as one expects classically. This also implies the violation of Hartman effect for the quantum tunneling in time domain [34].

Finally, we have a brief look at the microwave experiment on GH shifts [19]. The experimental results show that the beamwidth is an important parameter for the GH shift in FTIR. Martinez *et al.* [27] once explained the influence of beamwidth on GH shift by the factor η due to the curvature effect. However, they cannot explain further that the GH shifts do not saturate to constant when the beamwidth is very small. To explain further the results of Figs. 4 and 5 in Ref. [19], we plot Fig. 3 to demonstrate that the GH shift increases by deceasing beam dimension corresponding to the beam waist width, while it has the strong decreases in GH shift when the incidence angle is far away from the critical angle. As a matter of fact, the modified GH shift given by Martinez *et al.* is quite different from the modified GH shift discussed here. Obviously, the former modification is related to the radius of curvature of the wave fronts comprising the incident beam. Otherwise, the latter one does result from second-order correction of transmission coefficient during the wave-packet propagation. It is worthwhile to point out that the beam waist of the incident beam indicated by Eq. (2) is assumed to be at the interface of $x=0$ so that the curvature effect of the wave fronts can be neglected. In a word, the formula of GH shift for an actual incident beam, whose beam waist is not located at the interface $x=0$ as measured in the microwave experiment [19], can be further modified by the curvature effect and second-order correction presented here simultaneously, especially when the beamwidth is small.

To summary, we have investigated the GH shifts in FTIR by wave-packet propagation. It is found that the GH shift in the first-order approximation of the transmission coefficient, which is exactly the expression of the GH shift obtained by stationary phase method, approaches the saturation value in two different ways depending on the angle of incidence. The explicit expression of the GH shift in the second-order approximation shows the strong dependence on the beamwidth. It is further shown that the GH shift with the second-order correction increases by decreasing the beamwidth at the small incident angles, while for the large incident angles the GH shift reveals a decrease by decreasing the beamwidth. In this case, the GH shift does not trend to a constant with increasing the air gap. All these phenomena can be applicable to give better understanding of the GH shift and tunneling delay time in FTIR.

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[1] F. Goos and H. Hänchen, Ann. Phys. **436**, 333(1947); **440**, 251 (1949).
[2] H. K. V. Lotsch, Optik (Stuttgart) **32**, 116 (1970); **32**, 189 (1970); **32**, 299 (1971); **32**, 553 (1971).
[3] K. V. Artmann, Ann. Phys. (Leipzig) **2**, 87 (1948).
[4] R. H. Renard, J. Opt. Soc. Am. **54**, 1190 (1964).
[5] T. Sakata *et al.*, Appl. Phys. Lett. **76**, 2841 (2000).
[6] X. B. Yin and L. Hesselink, Appl. Phys. Lett. **89**, 261108 (2006).
[7] T.-Y. Yu *et al.*, Opt. Lett. **33**, 1001 (2008).
[8] C. W. Hsue and T. Tamir, J. Opt. Soc. Am. A **2**, 978 (1985).
[9] R. P. Riesz and R. Simon, J. Opt. Soc. Am. A **2**, 1809 (1985).
[10] T. Tamir, J. Opt. Soc. Am. A **3**, 558 (1986).
[11] C.-F. Li, Phys. Rev. Lett. **91**, 133903 (2003).
[12] D. Müller *et al.*, Europhys. Lett. **73**, 526 (2006).
[13] C.-F. Li *et al.*, Chin. Phys. B **17**, 1758 (2008).
[14] X. Yin *et al.*, Appl. Phys. Lett. **85**, 372 (2004).
[15] F. Pillon *et al.*, J. Opt. Soc. Am. B **22**, 1290 (2005).
[16] J. J. Cowan and B. Anicin, J. Opt. Soc. Am. **67**, 1307 (1977).
[17] J. L. Agudin and A. M. Platzeck, J. Opt. (Paris) **9**, 187 (1987).
[18] A. K. Ghatak *et al.*, Opt. Commun. **56**, 313 (1986).
[19] A. Haibel *et al.*, Phys. Rev. E **63**, 047601 (2001).
[20] J. Broe and O. Keller, J. Opt. Soc. Am. A **19**, 1212 (2002).
[21] A. M. Steinberg and R. Y. Chiao, Phys. Rev. A **49**, 3283 (1994).
[22] Ph. Balcou and L. Dutriaux, Phys. Rev. Lett. **78**, 851 (1997).
[23] A. A. Stahlhofen, Phys. Rev. A **62**, 012112 (2000).
[24] A. Haibel and G. Nimtz, Ann. Phys. (Leipzig) **10**, 707 (2001).
[25] I. R. Hooper *et al.*, Phys. Rev. Lett. **97**, 053902 (2006).
[26] T. E. Hartman, J. Appl. Phys. **33**, 3427 (1962).
[27] J. C. Martinez and E. Polatdemir, Phys. Lett. A **351**, 31 (2006).
[28] G. Nimtz, Nat. Photonics **3**, 319 (2009).
[29] H. G. Winful and C. Zhang, Phys. Rev. A **79**, 023826 (2009).
[30] The necessary condition can also be derived from $\tau_i > \tau_d$ given in Ref. [29] in the limit of thin barrier.
[31] A. Paul *et al.*, Eur. Phys. J. D **42**, 495 (2007).
[32] D. R. Solli *et al.*, J. Opt A, Pure Appl. Opt. **10**, 075204 (2008).
[33] H. G. Winful, Phys. Rev. Lett. **91**, 260401 (2003).
[34] X. Chen and C.-F. Li, EPL **82**, 30009 (2008).