# Derivation of the density matrix of a single photon produced in parametric down-conversion

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We discuss an effective numerical method of density matrix determination of fiber coupled single photon generated in process of spontaneous parametric down conversion in type I noncollinear configuration. The presented theory has been successfully applied in case of source utilized to demonstrate the experimental characterization of spectral state of single photon, what was reported in Wasilewski, Kolenderski, and Frankowski [Phys. Rev. Lett. **99**, 123601 (2007)].

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## I. INTRODUCTION

The source of single photons is a prerequisite for implementation of linear optical quantum information processing schemes [1–4], quantum teleportation [5], and quantum cryptography protocols [6]. Although deterministic single photon sources have been recently developed [7–10], most of the fundamental experiments still utilize the process of spontaneous parametric down conversion (SPDC). In these phenomena pairs of daughter photons are produced in a pumped nonlinear crystal and are typically coupled into single mode fibers (SMFs). SPDC provides unique ease of shaping spectral and spatial properties of generated nonclassical light. Detection of one photon, conventionally called signal, heralds the presence of the other, called idler [11]. Its statistical properties are satisfactory for postselection type experiments as long as the pump power is low enough to make multiple pair generation events negligible [12]. However, the idler photon is typically prepared in a mixed state [13] with respect to the temporal degree of freedom. Only by careful spectral filtering or precise adjustment of collecting setup one can produce pure wave packets [14–17]. Furthermore, coupling into SMFs decreases significantly the total number of useful heralded photons. In a typical experimental scenario coupling only one photon out of the pair is significantly more probable than coupling both photons. This comes at an expense of loosing sub-Poissonian statistics of the counts but is beneficial for a certain test experiments [18] or measuring vital characteristics of the source.

We report here an effective numerical method of determination of the state of single photon generated in process of type I SPDC in configuration in which idler photon is coupled into SMF and the signal is disregarded. This scheme has been previously used to demonstrate the method of characterization of a single photon state in Ref. [18]. The theoretical method, adapted form the results of Ref. [17], is based on an observation that optical fiber define a relatively narrow range of directions that need to be included in the calculations. This justifies the paraxial approximation, which makes a substantial portion of the problem tractable analytically. In consequence we are able to derive the expression for the spectral density matrix of idler photon $\rho_i(\omega_i,\omega_i')$ propagating in SMF. Finally we present numerical simulations of $\rho_i(\omega_i,\omega_i')$ for typical experimental settings. The developed efficient method of calculating spectral density matrix $\rho_i(\omega_i,\omega_i')$ can provide a valuable insight in a future experiments with single photons obtained from SPDC.

## II. THEORETICAL MODEL

Below we will develop a set of effective approximations for calculating the reduced density matrix $\rho_i(\omega_i,\omega_i')$ in an idler arm of typical fiber coupled single-pass parametric down-conversion source. We start by writing down the full biphoton wave function in the perturbative approximation. We describe the coupling of idler photon into a SMF and we trace over the degrees of freedom of the signal photon. This way we obtain an involved, multidimensional integral for the reduced density matrix $\rho_i(\omega_i,\omega_i')$. At this stage we point out that by applying a paraxial approximation and judicious reordering of the integrals this expression can be largely simplified.

Let us begin by describing the typical source depicted in Fig. 1. It comprises the nonlinear crystal of length $L$, pumped by a beam of ultrashort pulses centered around frequency

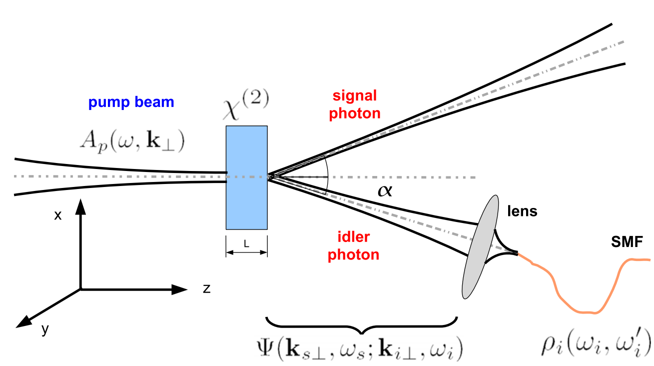


FIG. 1. (Color online) The setup is comprised of $\chi^{(2)}$ nonlinear crystal of length $L$, pumped by a Gaussian beam of $w_p$ comprised of pulses described by a spectral envelope function $A_{sp}$. The idler photon is collected to a single mode fiber SMF, provided it is emitted into a spatial mode of characteristic width $w_f$ at the crystal.

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$2\omega_0$. The elementary process taking place in nonlinear crystal is a decay of a pump photon into signal and idler photons. We chose to parameterize the interacting waves by their frequencies $\omega_p$, $\omega_s$, $\omega_i$ and transverse components of their wave vectors $\mathbf{k}_{p\perp}$, $\mathbf{k}_{s\perp}$, $\mathbf{k}_{i\perp}$. The energy conservation principle is expressed in this parametrization by an equation $\omega_p=\omega_s+\omega_i$, while the momentum conservation requires that the transverse components of wave vectors match $\mathbf{k}_{p\perp}=\mathbf{k}_{s\perp}+\mathbf{k}_{i\perp}$. Note that for each of the waves once $\mathbf{k}_\perp$ and $\omega$ are given $k_z$ is fixed. Therefore an ideal match of the longitudinal wave vectors $k_{pz}$, $k_{sz}$, and $k_{iz}$ is in general impossible. We choose a perfect matching for pump photon of frequency $\omega_p=2\omega_0$ propagating along the $z$ axis ($\mathbf{k}_{p\perp}=0$) and signal and idler photons of equal frequencies $\omega_s=\omega_i=\omega_0$ propagating symmetrically at the angle $\alpha$ with respect to $z$ axis. The propagation directions of both generated photons correspond to a transverse wave vectors $|\mathbf{k}_{s0\perp}|=|\mathbf{k}_{i0\perp}|=\omega_0/c \sin\alpha$ pointing in opposite directions. Thus a phase matching, which we consider here, corresponds to fulfilling the following criterion:

$$\Delta k_z(\mathbf{k}_{s0\perp},\omega_0;\mathbf{k}_{i0\perp},\omega_0) = k_{pz}(\mathbf{k}_{s0\perp}+\mathbf{k}_{i0\perp},2\omega_0) - k_{sz}(\mathbf{k}_{s0\perp},\omega_0) - k_{iz}(\mathbf{k}_{i0\perp},\omega_0) = 0. \quad (1)$$

Let us write down an expression for the probability amplitude of generation of two photons of frequencies $\omega_s$, $\omega_i$ and transverse wave vectors $\mathbf{k}_{s\perp}$, $\mathbf{k}_{i\perp}$ [17,19],

$$\Psi(\mathbf{k}_{s\perp},\omega_s;\mathbf{k}_{i\perp},\omega_i) = \mathcal{N}\int_{-L/2}^{L/2} dz A_p(\mathbf{k}_{s\perp}+\mathbf{k}_{i\perp},\omega_s+\omega_i) \times e^{i\Delta k_z(\mathbf{k}_{s\perp},\omega_s;\mathbf{k}_{i\perp},\omega_i)z}, \quad (2)$$

where $\mathcal{N}$ is normalization constant. The above formula expresses a fact that the probability amplitude of generating pair of photons characterized by $\mathbf{k}_{s\perp},\omega_s$, and $\mathbf{k}_{i\perp},\omega_i$ is proportional to the pump amplitude $A_p(\mathbf{k}_{s\perp}+\mathbf{k}_{i\perp},\omega_s+\omega_i)$. In turn the integration sums the contributions form the slices of the crystal perpendicular to $z$ axis with exponential term representing propagation phases. Indeed, $\Delta k_z(\mathbf{k}_{s\perp},\omega_s;\mathbf{k}_{i\perp},\omega_i)z$ is the phase acquired by the pump while propagating to the slice of interaction and by the photon pair while propagating toward the end of the crystal.

Now we must take a certain spatial shape of the pump and the fiber spatial mode, preferably leading to analytically integrable expressions. We assume there is no spatiotemporal correlation in the pump pulse and its amplitude is a product of spectral $A_{\mathrm{sp}}(\omega)$ and spatial $u_p(\mathbf{k}_\perp)$ parts,

$$A_p(\omega,\mathbf{k}_\perp) = A_{\mathrm{sp}}(\omega)u_p(\mathbf{k}_\perp). \quad (3)$$

We impose a spatial profile to be Gaussian, corresponding to a beam of waist $w_p$,

$$u_p(\mathbf{k}_\perp) = \frac{w_p}{\sqrt{\pi}}\exp\left(-\frac{w_p^2}{2}\mathbf{k}_\perp^2\right).$$

However, we still provide freedom of choice of the pump spectral amplitude $A_{\mathrm{sp}}(\omega)$.

Next we find the wave function with respect to the fiber for an idler photon and with respect to the free space for the signal photon. This can be done by projecting $\psi(\mathbf{k}_{s\perp},\omega_s;\mathbf{k}_{i\perp},\omega_i)$ on the fiber mode profile. We approximate the spatial mode coupled in SMF $u_i(\mathbf{k}_{s\perp})$ by a Gaussian function centered around the phase matching direction $\mathbf{k}_{i0\perp}$ corresponding to a beam of width $w_f$ [20],

$$u_i(\mathbf{k}_{i\perp},\omega_i) = \frac{w_f}{\sqrt{\pi}}\exp\left(-\frac{w_f^2}{2}(\mathbf{k}_{i\perp}-\mathbf{k}_{i0\perp})^2\right). \quad (4)$$

Without loss of generality we assume the collecting optics and the fibers are in the $xz$ plane. Therefore the transverse wave vector of the idler photon as a function of its frequency $\omega_i$ and the angle of observation $\alpha$ is given by $\mathbf{k}_{i0\perp}=-\omega_i\sin(\alpha)\hat{x}/c$. Here $\hat{x}$ is a unit vector in the direction of $x$ axis. Thus the biphoton wave function of idler photon inside SMF and signal propagating in free space $\Psi_i(\mathbf{k}_{s\perp},\omega_s,\omega_i)$ is a partial overlap of the free space wave function $\Psi(\mathbf{k}_{s\perp},\omega_s;\mathbf{k}_{i\perp},\omega_i)$ and the fiber mode profile $u_i(\mathbf{k}_{s\perp})$,

$$\Psi_i(\mathbf{k}_{s\perp},\omega_s,\omega_i) = \mathcal{N}A_{\mathrm{sp}}(\omega_s+\omega_i)\int_{-L/2}^{L/2}\mathrm{d}z\int \mathrm{d}^2\mathbf{k}_{i\perp}u_i^*(\mathbf{k}_{i\perp},\omega_i) \times u_p(\mathbf{k}_{s\perp}+\mathbf{k}_{i\perp})e^{i\Delta k_z(\mathbf{k}_{s\perp},\omega_s;\mathbf{k}_{i\perp},\omega_i)z}. \quad (5)$$

In the next step let us write an expression for the density matrix of the idler photon in the fiber $\rho_i(\omega_i,\omega_i')$. It is obtained by taking a trace of the photon pair density matrix $\Psi^*(\mathbf{k}_{s\perp},\omega_s;\mathbf{k}_{i\perp},\omega_i)\Psi(\mathbf{k}'_{s\perp},\omega'_s;\mathbf{k}'_{i\perp},\omega'_i)$ over the degrees of freedom of the signal photon $\omega_s$ and $\mathbf{k}_{s\perp}$,

$$\rho_i(\omega_i,\omega_i') = \int \mathrm{d}\omega_s\mathrm{d}\mathbf{k}_{s\perp}\Psi_i^*(\mathbf{k}_{s\perp},\omega_s,\omega_i)\Psi_i(\mathbf{k}_{s\perp},\omega_s,\omega_i'). \quad (6)$$

Hence a general structure of the expression for spectral density matrix $\rho_i(\omega_i,\omega_i')$ after substitution of Eq. (5) into Eq. (6) is given by

$$\rho_i(\omega_i,\omega_i') \propto \int \mathrm{d}z\mathrm{d}z'\mathrm{d}\omega_s A_{\mathrm{sp}}(\omega_s+\omega_i)A_{\mathrm{sp}}(\omega_s+\omega_i') \times \int \mathrm{d}\mathbf{k}_{s\perp}\mathrm{d}\mathbf{k}_{i\perp}\mathrm{d}\mathbf{k}'_{i\perp}u_p^*u_p'u_i^*u_i' \times\exp(i\Delta k_z'z'-i\Delta k_z z), \quad (7)$$

where the functions $u_p$, $u_i$, and $\Delta k_z$ are taken for $\mathbf{k}_{s\perp}$, $\mathbf{k}_{i\perp}$, $\omega_s$, $\omega_i$, and $u_p'$, $u_i'$ and $\Delta k_z'$ refer to the respective quantities taken at $\mathbf{k}_{s\perp}$, $\mathbf{k}'_{i\perp}$, $\omega_s$, $\omega_i'$. A classical approach to evaluate this integrals would be first to integrate analytically over the length of the crystal $z$ and $z'$ and then evaluate remaining seven integrals numerically. However this would require tremendous computational effort. Therefore we adopted another approach in order to alleviate the numerical cost. First, we will apply the paraxial approximation to the phase mismatch exponent $i\Delta k_z'z'-i\Delta k_z z$. Then the integrals in second line of Eq. (7) can be evaluated analytically. Finally we will end up with the integrals over $z$, $z'$, and $\omega_s$ for numerically computation.

Let us first justify our approach. The paraxial approximation of the phase mismatch exponent $i\Delta k_z'z'-i\Delta k_z z$ is accurate as long as waists of the beam involved are much bigger than a wavelength. Mathematically this is equivalent to observing that in the integral Eq. (7) the range of $\mathbf{k}_{i\perp}$ is in fact restricted by the aperture of the collection optics described by $u_i(\mathbf{k}_{s\perp})$ to $\mathbf{k}_{i\perp} \simeq \mathbf{k}_{i0\perp} \pm 1/w_f$, as seen in Eq. (4). Then the range of perpendicular component of the signal wave vector $\mathbf{k}_{s\perp}$ is also restricted because of the finite wave vector spread of the pump. Direct calculation reveals the range of approximately [21]: $\mathbf{k}_{s\perp} \simeq \mathbf{k}_{s0\perp} \pm \sqrt{1/w_f^2 + 1/w_p^2}$, where $\mathbf{k}_{s0\perp} = \omega_s \sin(\alpha)\hat{x}/c$. Those two facts enable us to approximate the phase mismatch $\Delta k_z(\mathbf{k}_{s\perp},\omega_s;\mathbf{k}_{i\perp},\omega_i)$ around the center of both ranges up to second order in $\mathbf{k}_{s\perp}$ and $\mathbf{k}_{i\perp}$,

$$\begin{aligned}\Delta k_z(\mathbf{k}_{s\perp},\omega_s;\mathbf{k}_{i\perp},\omega_i) &\coloneqq k_{pz}(\mathbf{k}_{s\perp}+\mathbf{k}_{i\perp},\omega_s+\omega_i) - k_{sz}(\mathbf{k}_{s\perp},\omega_s)\\ &\quad - k_{iz}(\mathbf{k}_{i\perp},\omega_i)\\ &\approx \Delta k_z(\mathbf{k}_{s0\perp},\omega_s;\mathbf{k}_{i0\perp},\omega_i) + \mathbf{D}_1^T(\omega_s,\omega_i)\boldsymbol{\kappa}\\ &\quad + \boldsymbol{\kappa}^T\mathbf{D}_2(\omega_s,\omega_i)\boldsymbol{\kappa}, \end{aligned} \tag{8}$$

where for sake of brevity we introduced $\boldsymbol{\kappa} = (\mathbf{k}_{s\perp}-\mathbf{k}_{s0\perp},\mathbf{k}_{i\perp}-\mathbf{k}_{i0\perp})^T$ to denote the four component vector of deviations form the directions of phase matching. Furthermore $\Delta k_z(\mathbf{k}_{s0\perp},\omega_s;\mathbf{k}_{i0\perp},\omega_i)$ is spatially constant term while

$$\mathbf{D}_1(\omega_s,\omega_i) = \begin{pmatrix}\mathbf{d}_s(\omega_s,\omega_i)\\ \mathbf{d}_i(\omega_s,\omega_i)\end{pmatrix} \tag{9}$$

and

$$\mathbf{D}_2(\omega_s,\omega_i) = \frac{1}{2}\begin{pmatrix}\mathbf{d}_{ss}(\omega_s,\omega_i), & \mathbf{d}_{si}(\omega_s,\omega_i)\\ \mathbf{d}_{si}(\omega_s,\omega_i), & \mathbf{d}_{ii}(\omega_s,\omega_i)\end{pmatrix}, \tag{10}$$

where we denote the blocks as

$$\mathbf{d}_\mu(\omega_s,\omega_i) = \left(\frac{\partial \Delta k_z}{\partial k_{\mu x}}, \frac{\partial \Delta k_z}{\partial k_{\mu y}}\right), \tag{11}$$

and

$$\mathbf{d}_{\mu\nu}(\omega_s,\omega_i) = \begin{pmatrix}\dfrac{\partial^2 \Delta k_z}{\partial k_{\mu x}\,\partial k_{\nu x}}, & \dfrac{\partial^2 \Delta k_z}{\partial k_{\mu x}\,\partial k_{\nu y}}\\ \dfrac{\partial^2 \Delta k_z}{\partial k_{\mu y}\,\partial k_{\nu x}}, & \dfrac{\partial^2 \Delta k_z}{\partial k_{\mu y}\,\partial k_{\nu y}}\end{pmatrix}, \tag{12}$$

In the above indices $\mu,\nu = s,i$ refers to signal and idler photons. Note that for each pair of frequencies $\omega_s$ and $\omega_i$, the expansion coefficients is different, which is a signature of the fact that we keep the exact dispersion relations. This is motivated by the observation that the spectrum of the single photons can easily span 100 nm or more, which makes the expansion of the $k_z$ component of wave vector as a function of frequency inaccurate.

With Taylor expansion given by Eq. (8) at hand, we can perform inner integrals in the expression for density matrix Eq. (7). Let us rewrite the integrand of Eq. (7) using a vector of deviations $\tilde{\boldsymbol{\kappa}} = (\mathbf{k}_{s\perp}-\mathbf{k}_{s0\perp},\mathbf{k}_{i\perp}-\mathbf{k}_{i0\perp},\mathbf{k}'_{i\perp}-\mathbf{k}'_{i0\perp})$ to shorten the notation,

$$u_p^* u_p' u_i^* u_i' \exp(i\Delta k_z' z' - i\Delta k_z z) = \exp(-\tilde{\boldsymbol{\kappa}}\mathbf{M}_2\tilde{\boldsymbol{\kappa}} + \mathbf{M}_1\tilde{\boldsymbol{\kappa}} + \mathbf{M}_0), \tag{13}$$

where $\mathbf{M}_0$, $\mathbf{M}_1$, and $\mathbf{M}_2$ depend on $\omega_s$, $\omega_i$, $\omega_i'$, $z$, $z'$ and are given by

$$\begin{aligned}\mathbf{M}_2 = \frac{w_p^2}{2}&\begin{pmatrix}2\mathbb{1}_2 & \mathbb{1}_2 & \mathbb{1}_2\\ \mathbb{1}_2 & \left(1+\dfrac{w_f^2}{w_p^2}\right)\mathbb{1}_2 & \mathbb{0}_2\\ \mathbb{1}_2 & \mathbb{0}_2 & \left(1+\dfrac{w_f^2}{w_p^2}\right)\mathbb{1}_2\end{pmatrix} +\\ &- \frac{1}{2}\begin{pmatrix} iz'\mathbf{d}'_{ss} - iz\mathbf{d}_{ss} & -iz\mathbf{d}_{si} & iz'\mathbf{d}'_{si}\\ -iz\mathbf{d}_{is} & -iz\mathbf{d}_{ii} & \mathbb{0}_2\\ iz'\mathbf{d}'_{is} & \mathbb{0}_2 & iz'\mathbf{d}'_{ii}\end{pmatrix},\end{aligned} \tag{14}$$

$$\mathbf{M}_1 = \begin{pmatrix} -(\boldsymbol{\Delta_0}+\boldsymbol{\Delta}'_{\mathbf{0}})w_p^2 - iz\mathbf{d}_s + iz'\mathbf{d}'_s\\ -\boldsymbol{\Delta_0}w_p^2 - iz\mathbf{d}_i\\ -\boldsymbol{\Delta}'_{\mathbf{0}}w_p^2 + iz'\mathbf{d}'_i\end{pmatrix}, \tag{15}$$

$$\begin{aligned}\mathbf{M}_0 = -\frac{w_p^2}{2}(\boldsymbol{\Delta_0}^2 + \boldsymbol{\Delta_0}'^2) - iz\Delta k_z(\mathbf{k}_{s0\perp},\omega_s;\mathbf{k}_{i0\perp},\omega_i)\\ + iz'\Delta k_z(\mathbf{k}_{s0\perp},\omega_s;\mathbf{k}'_{i0\perp},\omega'_i).\end{aligned} \tag{16}$$

We use $\mathbb{1}_2$ and $\mathbb{0}_2$ to denote two-dimensional identity and zero matrices, furthermore $\boldsymbol{\Delta_0} = \mathbf{k}_{s0\perp} + \mathbf{k}_{i0\perp}$, $\boldsymbol{\Delta}'_{\mathbf{0}} = \mathbf{k}_{s0\perp} + \mathbf{k}'_{i0\perp}$ and all primed Taylor series coefficients components $\mathbf{d}'_{\mu\nu}$ and $\mathbf{d}'_\mu$ are taken for the primed idler frequency $\omega'_i$.

Now the integral in Eq. (7) can be partially evaluated and the density matrix $\rho_i(\omega_i,\omega'_i)$ is found to be

$$\begin{aligned}\rho_i(\omega_i,\omega'_i) &= |\mathcal{N}|^2\int \mathrm{d}\omega_s A^*_{\mathrm{sp}}(\omega_s+\omega_i)A_{\mathrm{sp}}(\omega_s+\omega'_i)\\ &\times \int_{-L/2}^{L/2}\mathrm{d}z\int_{-L/2}^{L/2}\mathrm{d}z'\,\frac{\exp\mathbf{M}_0}{\sqrt{\det\mathbf{M}_2}}\\ &\times\exp\left(-\frac{1}{4}\mathbf{M}_1^T\mathbf{M}_2^{-1}\mathbf{M}_1\right).\end{aligned} \tag{17}$$

## III. NUMERICAL RESULTS

In order to proceed we fix the spectral shape of pump pulse $A_{\mathrm{sp}}(\omega)$. We assume it to be a Gaussian of duration $\tau_p$

$$A_{\mathrm{sp}}(\omega) = \frac{\sqrt{\tau_p}}{\sqrt[4]{\pi}}\exp\left(-\frac{\tau_p^2}{2}(\omega-2\omega_0)^2\right). \tag{18}$$

In typical experimental scenario photons are spectrally filtered in order to ameliorate its characteristics or reduce a noise. We can model spectral filtering effects multiplying the density matrix by the amplitude transition functions $\Lambda(\omega')$,

$$\rho_i(\omega,\omega') \to \Lambda(\omega)\Lambda(\omega')\rho_i(\omega,\omega'). \tag{19}$$

We approximate amplitude transmission of spectral filter by a Gaussian function defined in the following way:

$$\Lambda(\omega) = \exp\left(-2\ln 2\frac{(\omega-\omega_0)^2}{\sigma^2}\right). \tag{20}$$

We have simulated here the spectral density matrix of a photon generated in a typical experimental scenario, where

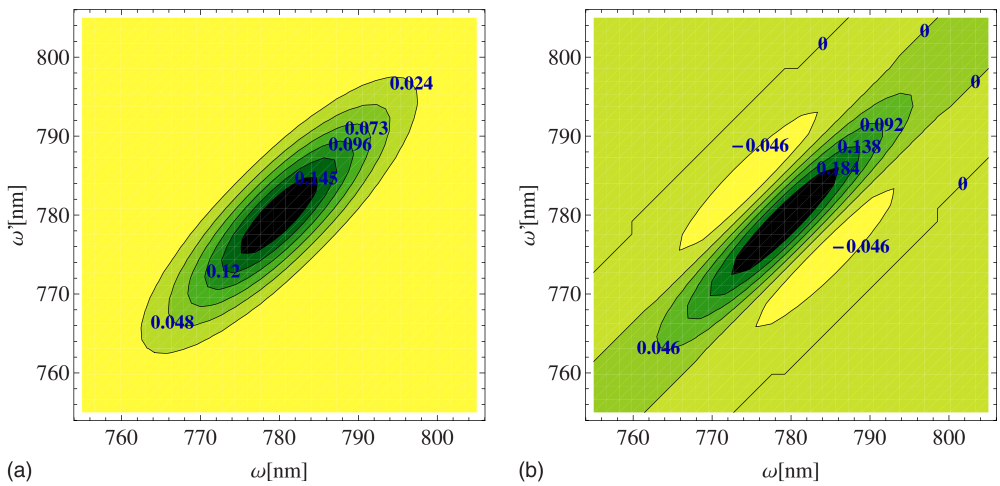


FIG. 2. (Color online) The spectral density matrix $\rho(\omega,\omega')$ of photons transmitted through 20 nm spectral filter. The plots were generated for $L=1$ mm long BBO crystal cut for $\theta_c=30°$, pumped using Gaussian beam of $w_p=100$ $\mu$m waist. The pulse duration was set to $\tau_p=100$ fs. The collecting optics and the single mode fiber was set to couple the Gaussian mode of $w_f=100$ $\mu$m. The observation angle was set to (a) $\alpha=2.2°$ (direction of perfect phase matching) and (b) $\alpha=3.2°$.

the nonlinear crystal is pumped by a Gaussian pulses Eq. (18). Note that for real Gaussian pulses density matrix is real. Figure 2 shows the spectral density matrices $\rho_i(\omega_i,\omega_i')$ of photons generated in $L=1$ mm long beta barium borate (BBO) crystal cut for $\theta_c=30°$, pumped using Gaussian pulsed beam of $w_p=100$ $\mu$m waist and duration $\tau_p=100$ fs. The collecting optics and the single mode fiber were set to couple the Gaussian mode of waist equal $w_f=200$ $\mu$m. The spectral filters of $\sigma=20$ nm were applied. Panel (a) shows density matrix when the observation direction coincides with the phase matching direction $\alpha=2.2°$. In panel (b) the observation direction is set to $\alpha=3.2°$.

Let us point out that in case of collection optics set far from the direction of perfect phase matching, the spectral density matrix $\rho_i(\omega_i,\omega_i')$ often assumes negative off diagonal values, see for example Fig. 2(b). In such cases generated photons acquire complicated temporal mode structure. This effect is beyond the scope of a Gaussian approximation of the phase matching function [22,23].

## IV. CONCLUSIONS

We have derived a simple method of predicting the spectral density matrix $\rho_i(\omega_i,\omega_i')$ of a photon generated in one arm of pulsed SPDC. The model allows to reduce dramatically the computational effort while retaining the accuracy of results. Those results, by the virtue of their simplicity, may become very useful when engineering photon sources to meet a certain needs for the spectral characteristics.

The presented model has been successfully used to predict the measurement outcome in Ref. [18]. Note that the model Eq. (17) may be used to predict the spectral density matrix in case of any type of spectral pumping.

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