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Enhanced sensitivity of a passive optical cavity by an intracavity dispersive medium

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The pushing of the modes of a Fabry-Perot cavity by an intracavity rubidium cell is measured. The scale factor of the modes is increased by the anomalous dispersion and is inversely proportional to the sum of the effective group index and an additional cavity delay factor that arises from the variation of the Rb absorption over a free spectral range. This additional positive feedback further increases the effect of the anomalous dispersion and goes to zero at the lasing threshold. The mode width does not grow as fast as the scale factor as the intracavity absorption is increased resulting in enhanced measurement sensitivities. For absorptions larger than the scale factor pole, the atom-cavity response is multivalued and mode splitting occurs.

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I. INTRODUCTION

The use of strongly dispersive materials whose resonant features can speed up, slow down, stop, or even reverse the propagation of pulses of light as a result of their substantially modified group velocities [1–8] has been proposed for the enhancement of interferometers [9–12], the autostabilization of lasers [12–16], and the enhancement of optical gyroscopes [16–22]. In fact, it has long been recognized that the dispersion associated with the gain (or absorption) of an intracavity medium results in a slight frequency pulling (or pushing) and linewidth narrowing (or broadening) of the cold cavity modes [23–29]. The dispersion simply introduces an additional phase shift, whose size and sign is dependent on the original phase shift to be measured, in a manner that is analogous to the feedback of a regenerative amplifier. The question of particular interest is whether, and under what conditions, such frequency pushing effects result in enhanced measurement sensitivities.

Previously we demonstrated that the sensitivity of a laser gyro (or of the modes of a laser to some external perturbation) is inversely proportional to the group index of the intracavity medium [16]. This result applies only to the case where the gain medium is located within the cavity in question. An alternative manifestation of the "laser" gyro involves the use of a passive cavity that is maintained on resonance with an externally injected laser using feedback. In this geometry, the cavity operates far from the threshold condition. As a result, the dependence of the cavity sensitivity on the group index is substantially different from the case of a laser. In this Rapid Communication we derive the correct expressions for the cavity sensitivity as a function of the group index and confirm these derivations by measuring the amount of mode pushing due to an atomic vapor cell located within such a passive cavity. We demonstrate that the sensitivity, defined as the scale factor divided by the mode width, may be enhanced for such a system.

The transmittance of the coupled atom-cavity system can be written in terms of the detuning $\Delta=\nu-\nu_0$ from the atomic resonance frequency ν_0 as [30]

$$T(\Delta,\delta_q)=\frac{(1-r)^2/r}{[1-g(\Delta)]^2/g(\Delta)+4\sin^2\left[\dfrac{\phi(\Delta)+\Phi(\Delta)}{2}\right]},\tag{1}$$

where $\phi(\Delta)=2\pi\tau_c(\Delta-\delta_q)=2\pi\tau_c(\nu-\nu_q)$ is the cold cavity round-trip phase shift and $\delta_q=\nu_q-\nu_0$ is the detuning of the qth cold cavity mode from the medium resonance. The net electric field gain (or loss) per round trip is $g(\Delta)=ra\tau(\Delta)=g_0\tau(\Delta)/\tau_0$, where g_0 and τ_0 are line center values, r is the round-trip reflectivity of the mirrors, and a accounts for other frequency-independent losses in the cavity. The complex round-trip transmittivity of the medium is $\tilde{\tau}(\Delta)=\tau(\Delta)\exp i\Phi(\Delta)$, where $\tau(\Delta)=\exp[-\hat{\alpha}(\Delta)L/2]$ and $\Phi(\Delta)=[\hat{n}(\Delta)-1]2\pi\nu L/c$ are the real-valued transmittivity and phase shift, respectively, L is the round-trip length of the cavity, and $\hat{\alpha}(\Delta)$ and $\hat{n}(\Delta)$ are the effective absorption coefficient and refractive index of the medium, respectively. The relations between these effective parameters and their more conventional phenomenological counterparts are $\hat{\alpha}(\Delta)=\alpha(\Delta)\ell/L$ and $\hat{n}(\Delta)=1+[n(\Delta)-1]\ell/L$, where ℓ is the round-trip path length of the medium.

The specific detunings of the transmittance maxima and minima are determined from the derivative of Eq. (1) with respect to Δ, i.e., from the solution of the transcendental equation

$$\Delta_p=\delta_q-\frac{\Phi(\Delta_p)+F(\Delta_p,g)+\pi p}{2\pi\tau_c},\tag{2}$$

where p is twice the pushed mode number, $\tau_c=L/c$ is the round-trip time, and

$$F(\Delta_p,g)=-\sin^{-1}\left[\frac{1-g(\Delta_p)^2}{2g(\Delta_p)}\frac{1}{2\pi\tau_c\hat{n}_g(\Delta_p)}\frac{d\ln\tau(\Delta_p)}{d\Delta}\right]\tag{3}$$

is an additional cavity feedback that accounts for the contribution of the absorption to the mode pushing. By defining an effective propagation constant $\hat{k}(\nu)=2\pi\nu/c+\Phi(\nu)/L$, we have written Eq. (3) in terms of the effective group index

$$\hat{n}_g(\Delta_p) = \frac{c}{2\pi}\frac{d\hat{k}(\Delta_p)}{d\Delta} = 1 + \frac{1}{2\pi\tau_c}\frac{d\Phi(\Delta_p)}{d\Delta}, \tag{4}$$

where $T_d=(1/2\pi\tau_c)d\Phi/d\Delta$ corresponds to the delay time of a long monochromatic pulse as a result of the presence of the medium. Again, $\hat{n}_g=1+(n_g-1)\ell/L$, where n_g is the actual group index of the medium. The scale factor sensitivity of these modes to an external perturbation is then given by the derivative of Eq. (2) with respect to δ, i.e.,

$$S(\Delta_p,g) = \frac{d\Delta_p}{d\delta} = [\hat{n}_g(\Delta_p) + T_{cav}(\Delta_p,g)]^{-1}, \tag{5}$$

where $T_{cav}(\Delta_p,g)\equiv(1/2\pi\tau_c)dF(\Delta_p,g)/d\Delta_p$ is an additional dimensionless time delay resulting from the interaction with the cavity.

As a consequence of the strong frequency dependence of the absorption in comparison with the free spectral range, the function $F(\Delta,g)$ must be added to the round-trip phase shift $\phi+\Phi(\Delta)$ to obtain the detuning with respect to the pushed mode frequencies, i.e., $\phi+\Phi(\Delta)+F(\Delta,g)=2\pi\tau_c(\Delta-\Delta_p)$, but while both $\Phi(\Delta)$ and $F(\Delta,g)$ contribute to the feedback, only $F(\Delta,g)$ depends explicitly on the net round-trip gain. For a laser the net gain at the free-running oscillation frequency is clamped at the threshold value, i.e., $g(\Delta_p)=1$, equivalent to a pole in Eq. (1), such that $F(\Delta,g)$ and $T_{cav}(\Delta_p,g)$ become zero, and we recover the result in [16,21], specifically that the scale factor enhancement for a laser is inversely proportional to the group index $S^{(L)}(\Delta_p)=1/\hat{n}_g(\Delta_p)$. Below threshold, $g(\Delta_p)<1$, the factor $F(\Delta,g)$ has the same sign as $\Phi(\Delta)$ and thus reinforces the feedback, whereas above the lasing threshold $g(\Delta_p)>1$, the factor $F(\Delta,g)$ has the opposite sign as $\Phi(\Delta)$, thereby diminishing the feedback. Therefore, given the same level of absorption, larger dispersive feedbacks are obtained for a passive cavity than for a laser.

The mode widths, on the other hand, are larger for a passive cavity. The detunings of the half maxima of the transmittance for the pth mode are given by the solution of

$$\Delta_\pm = \delta_q - \frac{\Phi(\Delta_\pm) \mp 2\sin^{-1}[\sqrt{z(\Delta_p,\Delta_{p\pm1}) - y(\Delta_\pm)}]}{2\pi\tau_c}, \tag{6}$$

where $z(\Delta_p,\Delta_{p\pm1})=(1-r)^2/4rT_{avg}(\Delta_p,\Delta_{p\pm1})$, $y(\Delta_\pm)=[1-g(\Delta_\pm)]^2/4g(\Delta_\pm)$, and $T_{avg}(\Delta_p,\Delta_{p\pm1})=[T(\Delta_p)+T(\Delta_{p\pm1})]/2$ is the average of the maximum and minimum transmittance on either side of the pth mode. The full width at half maximum of the cavity mode is then given by $W=\Delta_+-\Delta_-$.

The mode linewidth and scale factor depend differently on the group index. The physical explanation for this is that the group index is not constant over the mode width. Note that the scale factor becomes infinite when the denominator of Eq. (5) goes to zero, i.e., when $\hat{n}_g(\Delta_p)+T_{cav}(\Delta_p,g)=0$, but the linewidth remains finite. If we define the sensitivity of the cavity as the scale factor divided by the linewidth, i.e., $S(\Delta_p,g)/W$ then the sensitivity becomes "infinite" at two values of the net gain $g(\Delta_p)$: (i) for some value of $g(\Delta_p)<1$ corresponding to a pole in Eq. (5) owing to the very strong dispersive feedback, and (ii) at the lasing threshold $g(\Delta_p)=1$ as a result of the zero in the mode width as determined from Eq. (6).

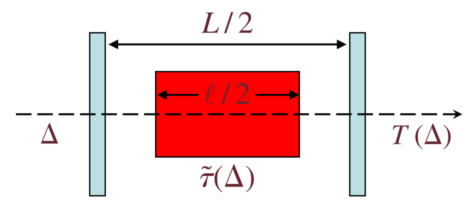

FIG. 1. (Color online) A Fabry-Perot cavity with an intracavity dispersive medium.

II. EXPERIMENT

An external-cavity diode laser having a linewidth of ≤ 1 MHz at 780 nm was used to scan over the modes of an $L/2=15$ cm, 1 GHz Fabry-Perot cavity containing an isotopically enriched ^{87}Rb cell as shown in Fig. 1. A second ^{87}Rb cell in a counterpropagating pump-probe saturated absorption spectrometer was used to provide an absolute frequency reference. The center frequency of the laser was adjusted to coincide with the Doppler broadened $F=2$ to F' transitions of the ^{87}Rb D2 line by looking for the fluorescence from the saturated absorption spectrometer, and the laser frequency was subsequently scanned via external tuning over its full range of almost 5 GHz, corresponding to several times the free spectral range of the cavity. Additionally, a Michelson interferometer was used to correct for nonlinearity in the frequency scan. The detuning between the cavity mode and the atomic resonance, δ, was varied by adjusting the voltage of the piezo on the Fabry-Perot cavity mirror and spectra were recorded for a variety of detunings.

Experiments were performed on Rb cells of two different lengths: 3 and 8 cm. The Fabry-Perot cavity mirrors were Au thin films evaporated onto $\lambda/10$ substrates having a wedge angle of greater than 30°. The measured finesse was 5.10 for the 8 cm cell and 4.35 for the 3 cm cell, and the exact free spectral range (FSR) was 985 MHz. To obtain the complex transmittivity of the medium $\tilde{\tau}(\Delta)$, the transmittance spectra of the Rb cells were measured at an intensity well below saturation and fit by assuming a single Gaussian absorption profile whose center frequency roughly coincides with the $F=2$ to $F'=3$ transition. A slightly better fit that accounts for the Rb line asymmetry can be obtained by incorporating additional Gaussian functions into the model of the Doppler broadened line to account for the less prominent hyperfine transitions, i.e., the $F=2$ to $F'=1$ and $F'=2$ transitions, but this does not significantly change the results. The phase shift $\Phi(\Delta)$ was subsequently determined by the Kramers-Krönig relations. The round-trip frequency dependent and independent loss parameters were determined to be $\tau_0=0.62$ (3 cm cell), $\tau_0=0.29$ (8 cm cell), and $a=0.83$, respectively, by measuring the off-resonance cell transmission, as well as the normal incidence absorption of the Au mirrors. The round-trip cavity reflectivity was determined to be $r=0.55$ by fitting the finesse of the atom-cavity transmittance $T(\Delta,\delta)$ to that of the

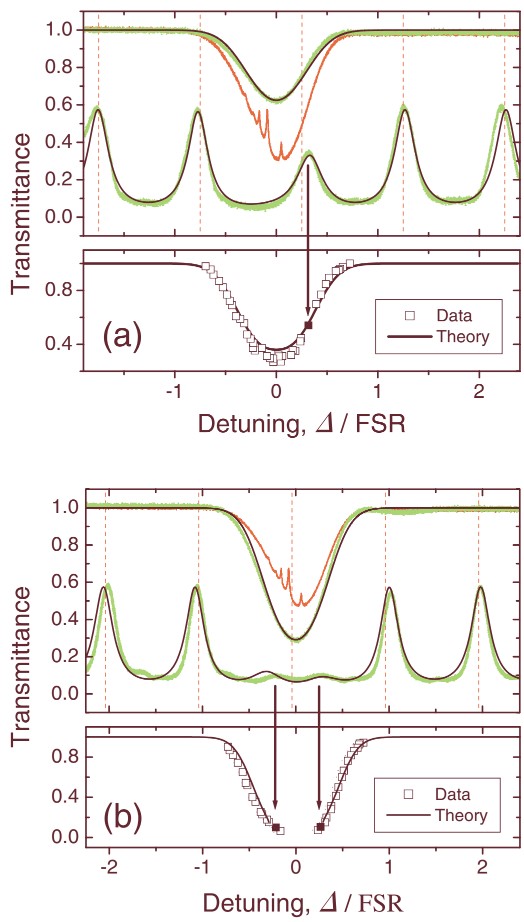

FIG. 2. (Color online) Mode pushing of a Fabry-Perot cavity mode by the $F=2$ to F' transitions of the ^{87}Rb D2 line for (a) a 3 cm cell at $\delta=0.25$ and (b) an 8 cm cell at $\delta=-0.04$. In (a) the mode is pushed to the right and in (b) the pushing results in mode splitting. The Rb saturated and unsaturated transmittance spectra are shown for comparison. The bottom panes show the relative transmittance of the peak of the mode as it is scanned across the resonance. The vertical dashed lines represent the unpushed mode detunings.

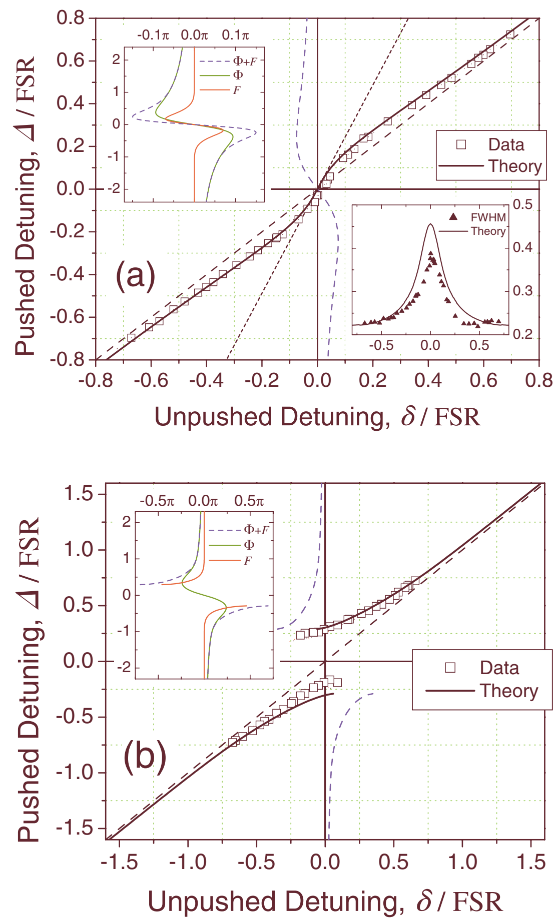

FIG. 3. (Color online) Cavity scale factor, i.e., pushed vs unpushed mode detuning. Near the Rb resonance, the scale factor is (a) increased for a 3 cm cell by anomalous dispersion and (b) decreased for an 8 cm cell as a result of mode splitting. The dashed curves represent the dispersive and nondispersive contributions to the mode frequency. The dispersive part is further separated into phase and cavity feedback functions (upper left inset). The dotted line in (a) is a linear phase, constant group index, approximation. The FWHM of the mode is also shown (lower right inset).

data. This value did not correspond to the maximum value obtained by measurement of the mirror reflectivity ($r=0.93$). However, the measured reflectivity varied strongly with spot position, therefore, the decrease in cavity finesse was likely a result of phase and amplitude variations across the diameter of the Gaussian beam. Moreover, the diameter of the laser beam was kept less than 1 mm, resulting in some divergence from plane-wave conditions. The incident and maximum transmitted intensities were $I_{\rm in}=72\ \mathrm{mW/cm^2}$ and $I_{\rm out}=0.1\ \mathrm{mW/cm^2}$, respectively. From the maximum output intensity, we infer that the intensity inside the cavity was always much lower than the saturation intensity for the $F=2$ to $F'=3$ transition with π-polarized light ($3\ \mathrm{mW/cm^2}$) [31].

In Fig. 2(b) the effect of mode pushing on the cavity mode closest to the Rb resonance is shown for the 8 cm cell. All frequencies are in units of the FSR. As the detuning gets smaller, the mode is pushed away from the resonance by an amount that depends on the detuning. For small enough detunings the mode splits as the left and right parts of the mode are pushed in opposite directions. This mode splitting does not occur in the case of the 3 cm cell [Fig. 2(a)] because the absorption and hence the dispersion are considerably smaller, however, the mode width does increase as a result of the mode pushing.

In Fig. 3 the frequency detuning of the mode closest to the Rb resonance is plotted against its unpushed detuning (the detuning in the absence of the dispersive medium) resulting in a plot of the cavity scale factor. Near the resonance, the scale factor is increased by a factor of 2.4 for the 3 cm cell [Fig. 3(a)], whereas the linewidth is increased by only a factor of 1.7 (inset), a result of the variation in the group index over the width of the mode. In the case of the 8 cm cell [Fig. 3(b)], the dispersion is larger than that required for infinite scale factor, such that the cavity feedback factor F becomes undefined and the solution to Eq. (2) becomes multivalued, resulting in a mode splitting. In this case the slope of Eq. (5) at $\Delta_p=0$ becomes negative. Notably, the on-resonance group indices were $\hat{n}_g(0)=0.77$ and $\hat{n}_g(0)=0.32$ for the 3 cm and 8 cm cells, respectively; i.e., the scale factor pole occurs at an effective group index greater than zero in contrast to the case of a laser cavity where the pole occurs at zero effective group index. Therefore, for the case of a passive cavity, less dispersion is required to obtain the same enhancement in scale factor.

In contrast with previous findings [22] that the sensitivity of the modes of a passive cavity, defined as the scale factor divided by the normalized mode width, cannot grow above unity, we have demonstrated that the sensitivity can be increased by anomalous dispersion. It is apparent by inspection of the two cases presented here that as the cell absorption increases such that the scale factor passes through infinite slope, the linewidth always remains finite as a result of the group velocity dispersion. The deviation of the data from the theoretical prediction in each of the figures is most likely a result of optical pumping effects involving the hyperfine resonances on the low frequency side of the Doppler broadened line. The sensitivity should be controllable by increasing the number of atoms in the $F=2$ level via optical repumping or by introducing an additional element into the cavity, i.e., by increasing the frequency dependent or independent losses, respectively.

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