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Dynamics of a one-dimensional spinor Bose liquid: A phenomenological approach

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The ground state of a spinor Bose liquid is ferromagnetic, while the softest excitation above the ground state is the magnon mode. The dispersion relation of the magnon in a one-dimensional liquid is periodic in the wave number q with the period of $2\pi n$, determined by the density n of the liquid. Dynamic correlation functions, such as, e.g., spin-spin correlation function, exhibit power-law singularities at the magnon spectrum, $\omega \to \omega_m(q,n)$. Without using any specific model of the interparticle interactions, we relate the corresponding exponents to independently measurable quantities $\partial\omega_m/\partial q$ and $\partial\omega_m/\partial n$.

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Bosons with an internal degree of freedom, "spin," exhibit a ferromagnetic ground state [1]. The presence of the internal states yields an excitation, magnon, in addition to conventional waves of the mass density [2,3]. Because of the ferromagnetic ground-state order, the dispersion relation $\omega_m(q)$ of magnons at small wave vectors q is quadratic. In the case of spin-isotropic liquid, this is the softest excitation of the system, since $\omega_m(0)=0$, while the density waves in the $q\to 0$ limit propagate with a finite sound velocity v. Away from the isotropic limit $\omega_m(0)>0$, still the magnon is the lowest-energy *spin excitation*. One may think of a magnon as of a "quantum impurity" with the spin opposite to the majority direction, moving in a spin-polarized host liquid.

A spin flip in a compressible system may excite a wave of density in it, which affects the spin-spin correlation function $A_m(q,\omega)$. This back action of the medium is the strongest in a one-dimensional (1D) system. In a 1D Heisenberg ferromagnet on a rigid lattice, $A_m(q,\omega)\propto f(q)\delta(\omega-\omega_m(q))$ with $f(q)$ being periodic in the reciprocal lattice. A finite compressibility of liquid transforms the δ-function response into a power-law edge singularity, $A_m(q,\omega)\propto[\omega-\omega_m(q)]^{\mu_m}\theta(\omega-\omega_m(q))$, at the magnon spectrum. While the existence of that singularity can be argued on the basis of the scale invariance [4], the exponent $\mu_m(q)$ has been evaluated thus far only for an SU(2) symmetric system in the limit of strong repulsion between the bosons [4,5] and for an integrable Yang-Gaudin model [6].

The singularities in $A_m(q,\omega)$, along with the previously studied singularities in the dynamic responses of single-component quantum fluids [7,8], and the spectral function singularity in the quantum impurity problem share a common root and can be related [9–11] to the physics of Fermi edge singularity. The exponent $\mu_m(q)>-1$ originates in the interaction of the quantum impurity with low-energy density waves of the majority-spin polarization.

Here we express the exponent $\mu_m(q)$ through the dispersion of the magnon mode $\omega_m=\omega_m(q,n)$ and its derivatives over the wave number q and the density of the majority-spin component n. For this purpose, we find the constants of the quantum impurity Hamiltonian which describes the dynamics of the magnon excitation and its interaction with fluctuations of density. That, in turn, reduces the problem of finding the critical behavior of a dynamic response to a much simpler problem of evaluation of an excitation energy as a function of n and q. Our results for $\mu_m(q)$ are not limited to small momenta and independent of the interaction strength and the presence of the SU(2) symmetry. In addition to $A_m(q,\omega)$, we consider also the single-particle spectral function $A_d(q,\omega)$ of a boson with the spin opposite to the majority polarization added to the fully polarized liquid. It also displays a power-law threshold behavior, and we find the corresponding wave-number-dependent exponent $\mu_d(q)$.

To find the exponents $\mu_m(q)$ and $\mu_d(q)$, we start with introducing a representation suitable for the description of the low-energy spin dynamics. For definiteness we consider spin-1/2 particles and assume that all spins point in the positive-z direction in the state with a maximal spin polarization. Addressing the spin-density operator

$$\mathbf{s}(x)=\frac{1}{2}\sum_j \sigma^{(j)}\delta(x-x_j), \tag{1}$$

we concentrate first on its z component. Within the set of states of maximal spin polarization, spin-density fluctuations are proportional to the fluctuations of the number density. Thus we represent the long-wavelength part of density $s_z(x)$ in terms of the boson field ϕ as $s_z(x)=\partial_x\phi/(2\pi)$, where the field ϕ has the meaning of the displacement field of spin-up particles. The long-wavelength dynamics of the displacements is described by the Luttinger liquid Hamiltonian [12]

$$H_0=\frac{v}{2\pi}\int dx\left[K(\partial_x\theta)^2+\frac{1}{K}(\partial_x\phi)^2\right], \tag{2}$$

where θ is the conjugate to ϕ variable, $[\phi(x),\partial_x\theta(y)]=i\pi\delta(x-y)$. For Galilean invariant system the Luttinger parameter is $K=\pi n/(m_\uparrow v)$ with $m_\uparrow$ being the bare mass of spin-up bosons.

Considering the operator $s_-(x)$ in the subspace of excitations with energies close to the magnon energy $\omega_m(q)$, we replace the general form (1) with an operator

$$s_-(x)\propto d^\dagger(x)e^{iqx}e^{i\theta(x)}. \tag{3}$$

Acting on a fully polarized state, this operator "extracts" a particle with spin up and replaces it (by acting with the creation operator $d^\dagger$ on vacuum) with a spin-down particle at the same point x. Representation (3) of $s_-(x)$ operator is adequate for $|q|\le\pi n$ and requires [12] a generalization (considered later in this Rapid Communication) for larger momenta.

The Hamiltonian describing the magnon dynamics should include the dynamics of the d quasiparticle and its interaction with the fields ϕ and θ representing the density fluctuations in the spin-up state. Similar to Refs. [8,13,14], we present the effective quantum impurity Hamiltonian in the form

$$H = H_0 + \int dx\, d^\dagger[\omega_m(q,n) - v_m(q)(-i\partial_x)]d + \int dx[V_\phi(\partial_x\phi) + V_\theta(\partial_x\theta)]d^\dagger d, \qquad (4)$$

where $v_m(q)=\partial_q\omega_m(q,n)$. The first of the two added parts to H_0 establishes $\omega_m(q,n)$ as the lower edge for the spectral function [15]

$$A_m(q,\omega) = \sum_\nu \delta(\omega - E_\nu(q))|\langle\nu,q|s_-(q)|0\rangle|^2,$$

where $|\nu,q\rangle$ is a many-body eigenstate of the system with momentum q and energy $E_\nu(q)$. The gradient expansion used in that part is sufficient for finding the behavior of $A_m(q,\omega)$ in the vicinity of the edge. The last term in the right-hand side of Eq. (4) describes the interactions of the quantum impurity with the density waves. The strengths of such interactions, V_ϕ and V_θ, can be expressed through independently measurable characteristics of the system.

To determine V_θ, we note that the corresponding term in the Hamiltonian (4) is nothing but a modification of the energy-momentum relation, $\omega_m^u(q,n)=\omega_m(q)+V_\theta m_\uparrow u$, for the d quasiparticle (a mobile "impurity") in the presence of finite velocity $u=\partial_x\theta/m_\uparrow$ of the fluid formed by the spin-up particles. That allows [16] one to use the Galilean invariance to find V_θ. Indeed, in the presence of a flow of the liquid with velocity u, the magnon energy remains unchanged in the comoving frame. In the laboratory frame, the magnon momentum is $q+m_\downarrow u$, while its energy is $\omega_m^u(q+m_\downarrow u,n)=\omega_m(q,n)+qu+m_\downarrow u^2/2$. Changing the momentum in the last formula, $q+m_\downarrow u\to q$, and comparing the above two expressions for $\omega_m^u(q,n)$, we find in the limit of small u

$$V_\theta(q) = \frac{q}{m_\uparrow} - \frac{m_\downarrow}{m_\uparrow}\partial_q\omega_m(q,n). \qquad (5)$$

Hereinafter we assume that the mass of bosons is spin independent, $m_\uparrow=m_\downarrow=m$, thus reducing Eq. (5) to

$$V_\theta(q) = \frac{q}{m} - v_m(q). \qquad (6)$$

At $|q|\ll\pi n$, the magnon spectrum is quadratic in q, i.e., $\omega(q)\approx\omega_m(0)+q^2/(2m^*)$ with some effective mass m^*; in this limit, $V_\theta=[(m^*-m)/m]v_m$.

To determine V_ϕ, we may consider the effect of a long-wavelength density variation, $\delta\rho=(1/\pi)\nabla\phi$, on the energy of the system. According to Eq. (4), it adds a term $V_\phi\pi\delta\rho$ to the energy density. This variation should be equal to the corresponding value, $(\partial\omega_m/\partial n)+(\partial\mu/\partial n)$, defined phenomenologically (here μ is the chemical potential of the spin-up bosons). Expressing $\partial\mu/\partial n$ in terms of K, one finds [17]

$$V_\phi(q) = \frac{1}{\pi}\frac{\partial\omega_m(q,n)}{\partial n} + \frac{v}{K}. \qquad (7)$$

As follows from the time-reversal symmetry of the problem, $V_\theta(q)$ and $V_\phi(q)$ are correspondingly odd and even functions of the momentum q.

It is instructive to consider separately the $q=0$ limit. An introduction of a static impurity (spin-down boson with $q=0$) creates a displacement $-(\pi K V_\phi/v)\mathrm{sgn}(y-x)$ in the field ϕ, while the operator $\exp[i\theta(x)]$ in the definition of the spin density (3) creates a shift $\pi\,\mathrm{sgn}(y-x)$. The sum of the two is related to the dilatation δ_l caused by the spin flip, $-\pi K V_\phi/v+\pi=\pi\delta_l$, yielding $V_\phi=(v/K)(1-\delta_l)$. Comparing that with Eq. (7) at $q=0$, we find

$$\delta_l = -\frac{K}{\pi v}\frac{\partial\omega_m(0,n)}{\partial n}.$$

Derived from considering a single spin flip, this relation under some assumptions can also be derived from the thermodynamics of a system at a constant pressure. One must assume that magnons do not form bound states, so $\omega_m(0,n)=\partial\mathcal{E}/\partial S$ with $\mathcal{E}$ being the ground-state energy and S being the total spin. In the presence of SU(2) invariance, $\partial\mathcal{E}/\partial S=0$, thus flipping a spin with $q=0$ does not cause dilatation, $\delta_l=0$.

Once the interaction constants V_θ and V_ϕ are established, we may proceed in full analogy with Ref. [17]: (i) rescale variables, $\phi=\tilde{\phi}\sqrt{K}$, $\theta=\tilde{\theta}/\sqrt{K}$ thus giving Eq. (2) the appearance of a Hamiltonian of free "particles;" (ii) eliminate the linear in ϕ,θ part of the full Hamiltonian, Eqs. (2) and (4), by a unitary transformation with proper values of δ_+,δ_-,

$$U^\dagger = \exp\left(-i\int dx\left\{\frac{\delta_+(q)}{2\pi}[\tilde{\theta}(x)-\tilde{\phi}(x)] - \frac{\delta_-(x)}{2\pi}[\tilde{\theta}(x)+\tilde{\phi}(x)]\right\}d(x)d^\dagger(x)\right), \qquad (8)$$

(iii) express the exponents of the sought correlation functions in terms of the phase shifts δ_+ and δ_- which quantum impurity ($d^\dagger d=1$) causes for comoving (+) and counterpropagating (−) particles. The values of the phase shifts can be written in a relatively compact form as

$$\frac{\delta_\pm(q)}{\pi} = \frac{1}{v_m(q)\mp v}\left(\sqrt{K}V_\phi \pm \frac{1}{\sqrt{K}}V_\theta\right). \qquad (9)$$

We are now in the position to evaluate the correlation functions of interest. Using Eq. (3) we may represent $A_m(x,t)$ as

$$A_m(q,\omega) = \mathrm{Im}\int dxdt\, e^{iqx-i\omega t}\langle 0|d(0,0)e^{-i\theta(0,0)}e^{i\theta(x,t)}d^\dagger(x,t)|0\rangle. \qquad (10)$$

In the case of spinor Bose liquid, one may envision tunneling of a spin-up or a spin-down particle into otherwise fully spin-up polarized system. The spectral function for a spin-up particle is identical to the one evaluated for the liquid of spinless bosons [7,8]; the corresponding tunneling threshold spectrum is $\omega = vq$. For a spin-down particle, the tunneling threshold is determined by the magnon spectrum, and the tunneling probability is proportional to the spectral function of the d quasiparticle,

$$A_d(q,\omega) = \mathrm{Im}\int dxdt\, e^{iqx-i\omega t}\langle 0|d(x,t)d^\dagger(0,0)|0\rangle. \tag{11}$$

Functions $A_m(q,\omega)$ and $A_d(q,\omega)$ exhibit a power-law behavior above the threshold,

$$A_{m,d}(q,\omega) \propto \Theta(\omega-\omega_m(q))(\omega-\omega_m(q))^{\mu_{m,d}(q)}.$$

Using transformation (8) and the standard methods of bosonization, one finds for $\mu_d(q)$ in the region $|q|\leq \pi n$

$$\mu_d(q) = -1 + \left(\frac{\delta_+(q)}{2\pi}\right)^2 + \left(\frac{\delta_-(q)}{2\pi}\right)^2, \tag{12}$$

like in Ref. [17]. Similarly, the exponent $\mu_m(q)$ at $|q|\leq \pi n$ reads as

$$\mu_m(q) = -1 + \left(\frac{1}{2\sqrt{K}} + \frac{\delta_+(q)}{2\pi}\right)^2 + \left(\frac{1}{2\sqrt{K}} - \frac{\delta_-(q)}{2\pi}\right)^2. \tag{13}$$

Equations (12) and (13) together with Eq. (9) relate the exponents of the correlation functions to the properties of the magnon branch of excitation spectrum $\omega_m(q,n)$ for the principal interval of momenta $|q|\leq \pi n$.

Due to the peculiarity of 1D systems, the lowest-energy excitations corresponding to a single flipped spin or to an added spin-down particle at a given momentum q are periodic functions of the momentum, $\omega_m(q,n) = \omega(q-2\pi nl,n)$, for any integer l. To extend the above results for $\mu_{m,d}(q)$ beyond the principal interval of momenta, we notice that introducing a d quasiparticle with the lowest energy amounts to a momentum boost of $2\pi nl$ of the spin-up liquid and exciting a magnon with the residual momentum belonging to the principal interval $-\pi n < q-2\pi nl \leq \pi n$. The boost accompanying the creation of a d quasiparticle corresponds to the modified definition of the spin-down particle creation operator,

$$d^\dagger(x) \rightarrow e^{-2il\phi(x)}d^\dagger(x), \quad d(x)\rightarrow d(x)e^{2il\phi(x)}. \tag{14}$$

Performing these replacements in Eq. (11) and repeating the steps which have lead to Eq. (12), we find now

$$\mu_d(q) = -1 + \left(\frac{\delta_+(q^*)}{2\pi} - l\sqrt{K}\right)^2 + \left(\frac{\delta_-(q^*)}{2\pi} - l\sqrt{K}\right)^2, \tag{15}$$

where $q^* = q - 2\pi nl$ with the integer l chosen to have $|q^*|\leq \pi n$. A similar procedure for $A_m(q,\omega)$ yields

$$\mu_m(q) = -1 + \left(\frac{1}{2\sqrt{K}} + \frac{\delta_+(q^*)}{2\pi} - l\sqrt{K}\right)^2 + \left(\frac{1}{2\sqrt{K}} - \frac{\delta_-(q^*)}{2\pi} + l\sqrt{K}\right)^2. \tag{16}$$

Equations (15) and (16) along with Eq. (9) provide the values of the edge exponents for an arbitrary momentum q.

Transferring momentum to the liquid as a whole allows tunneling of a particle at low energy, $\omega \rightarrow \omega_m(q^*,n)$, even at high momentum $|q| > \pi n$. The price for that, however, is a reduced tunneling probability reflected by the presence of integer l in Eq. (15): while the spectrum $\omega_m(q)$ is periodic, the exponent $\mu_d(q)$ is increasing with moving from one period to another, with larger $|l|$. The suppressed tunneling probability is a manifestation of the orthogonality catastrophe. Similarly the exponent μ_m, describing the probability of the spin-flip photon absorption near the edge, is increased due to the orthogonality catastrophe. Indeed, for $|l|>0$ the final state includes the spin-flipped particle along with the moving fluid with the momentum $2\pi nl$, which has the progressively smaller overlap with the initial state of the fluid at rest.

The periodic dispersion relation reaches its maxima at $q = \pi n(2l-1)$. Depending on the microscopic interaction strength between the bosons, the magnon velocity $v_m = \partial_q \omega_m(q,n)$ may have jumps at $q = \pi n(2l-1)$ or be a continuous function. In the latter case, obviously, $v_m(\pi n(2l-1)) = 0$. The transition between the two types of behavior upon the increase in the interaction strength is equivalent to the "quantum phase transition" in the Kondo problem controlled by tuning the exchange constant through zero [11]. The $v_m(\pi n(2l-1)) = 0$ regime corresponds to the strong-coupling side of the transition. The above developed Luttinger liquid representation is applicable on either side of the transition, similar to the scattering phase description of the low-energy physics of the Kondo problem. The region of applicability in $\omega - \omega_m(q,n)$, of course, gets narrow close to the transition point, as the corresponding Kondo energy scale becomes small. In the strong-coupling regime, v_m and $\partial_n \omega_m(q,n)$ have no discontinuities at $q = \pi n(2l-1)$. It is not clear *a priori* that the same is true for the exponents $\mu_{m,d}(q)$: after all, the definition of the response functions involves different operators [see Eq. (14)] at subsequent intervals of momenta. It is quite striking to see directly from Eqs. (15), (16), and (9) the continuity of $\mu_d(q)$ and $\mu_m(q)$. Indeed, the substitution of $v_m(q^* = \pm \pi n) = 0$ and the use of relations $K = \pi n/(mv)$ and $\partial_n \omega_m(q^*,n)|_{q^*=\pi n} = \partial_n \omega_m(q^*,n)|_{q^*=-\pi n}$ in Eq. (9) yield

$$\frac{\delta_\pm(\pi n)}{2\pi} = \frac{\delta_\pm(-\pi n)}{2\pi} - \sqrt{K}. \tag{17}$$

With the help of Eqs. (15) and (16), this immediately implies that for $q \rightarrow \pi n(2l-1) \pm 0$ both $\mu_d(q)$ and $\mu_m(q)$ are continuous functions *and* their first derivatives over q are continuous as well. At the "weak-coupling" side of the transition, the exponents, together with $v_m(q)$, are discontinuous at $q = \pi n(2l-1)$.

Around the local minima, $q = 2\pi nl$, i.e., at $q^* = 0$, one finds

$$\mu_d(2\pi nl) = -1 + \frac{(1-\delta_l)^2}{2K} + 2l^2K \tag{18}$$

and

$$\mu_m(2\pi nl) = -1 + \frac{\delta_l^2}{2K} + 2l^2K. \qquad (19)$$

The fact that $\mu_d(q=0) \neq -1$ is due to the orthogonality catastrophe [18]: the tunneled spin-down boson shakes up the liquid of spin-up particles. The same mechanism has led [8] to nontrivial tunneling exponents $\overline{\mu_\pm}$ for spinless bosons in the Lieb-Liniger [19] model, once the exponents are evaluated beyond the Luttinger liquid approximation. The direct comparison of Eq. (18) with the corresponding result (22) of Ref. [8] is possible only at $K \to 1$, when the shakeup becomes independent of the impurity velocity relative to $\pm v$; the two equations agree with each other, yielding $\mu_d = \overline{\mu_\pm} = 1/2$. The similar physics is at work for $A_m(q,\omega)$ correlation function away from the SU(2) symmetric point, i.e., when $\delta_l \neq 0$. The amplitude of a spin-flip process for a finite dilatation δ_l is suppressed by the orthogonality, resulting in $\mu_m(0) \neq -1$.

In the SU(2) invariant case, where $\partial\omega_m(0,n)/\partial n \propto \delta_l = 0$, one may also deduce universal results for the momentum dependence of $\mu_{m,d}(q)$ in the region of small momenta $|q| \ll m^* v$,

$$\mu_d(q) = -1 + \frac{1}{2K} + \frac{Kq^2}{2(\pi n)^2}\left(1 + \frac{m}{m^*}(2+2\sigma)\right), \qquad (20)$$

where $\sigma = -(n/2m^*)\,\partial m^*/\partial n$. Similarly,

$$\mu_m(q) = -1 + \frac{Kq^2}{2(\pi n)^2}\left[1 + \left(\frac{q}{m^* v}\right)^2 (3 + 4\sigma + \sigma^2)\right]. \qquad (21)$$

The q^2 term of the $\mu_m(q)$ exponent depends only on the parameters of the Luttinger liquid and for the strong-coupling limit that was derived in Refs. [4,5]. It is interesting that the q^4 term here may be expressed through the effective mass of the magnon mode m^*. The latter was evaluated in various limits for the integrable contact-interaction model [20], which allows us finding σ in Eqs. (20) and (21). In the strong-coupling limit $m^* = 3\gamma m/(2\pi^2)$, where $\gamma = mg/n \gg 1$ and thus $\sigma = 1/2$; here, g is the interaction strength. In the weak-coupling limit $m^* = m(1 + 2\sqrt{\gamma}/3\pi)$ and thus $\sigma = \sqrt{\gamma}/6\pi \ll 1$. These considerations are also applicable for the q^2 term in $\mu_d(q)$.

In conclusion, the dynamic response functions (10) and (11) of a homogeneous ferromagnetic one-dimensional Bose liquid exhibit power-law asymptotes at the threshold defined by the spectrum of the magnon $\omega_m(q)$. Independent of any model assumptions, the corresponding exponents $\mu_m(q)$ and $\mu_d(q)$ at any wave vector can be expressed in terms of a few independently measurable parameters: the sound velocity v, the corresponding Luttinger liquid parameter K, and the derivatives of ω_m with respect to q and liquid density n. Further simplification of $\mu_m(q)$ and $\mu_d(q)$ is possible in the vicinities $q = 2\pi nl$ of the minima of the magnon spectrum.

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[1] E. Eisenberg and E. H. Lieb, Phys. Rev. Lett. **89**, 220403 (2002).
[2] E. P. Bashkin, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 11 (1981) [JETP Lett. **33**, 8 (1981)]; Zh. Eksp. Teor. Fiz. **87**, 1948 (1984) [Sov. Phys. JETP **60**, 1122 (1985)].
[3] C. Lhuillier and F. Laloe, J. Phys. (Paris) **43**, 197 (1982); **43**, 225 (1982).
[4] M. B. Zvonarev, V. V. Cheianov, and T. Giamarchi, Phys. Rev. Lett. **99**, 240404 (2007).
[5] K. A. Matveev and A. Furusaki, Phys. Rev. Lett. **101**, 170403 (2008).
[6] M. Zvonarev, V. Cheianov, and T. Giamarchi, e-print arXiv:0905.0598.
[7] M. Khodas, M. Pustilnik, A. Kamenev, and L. I. Glazman, Phys. Rev. Lett. **99**, 110405 (2007).
[8] A. Imambekov and L. I. Glazman, Phys. Rev. Lett. **100**, 206805 (2008).
[9] A. Imambekov and L. Glazman, Science **323**, 228 (2009).
[10] M. Pustilnik, M. Khodas, A. Kamenev, and L. I. Glazman, Phys. Rev. Lett. **96**, 196405 (2006).
[11] A. Lamacraft, Phys. Rev. B **79**, 241105(R) (2009).
[12] V. N. Popov, Theor. Math. Phys. **11**, 565 (1972); K. B. Efetov and I. A. Larkin, Zh. Eksp. Teor. Fiz. **69**, 764 (1975) [Sov. Phys. JETP **42**, 390 (1975)]; F. D. M. Haldane, Phys. Rev. Lett. **47**, 1840 (1981).
[13] S. Sorella and A. Parola, Phys. Rev. Lett. **76**, 4604 (1996).
[14] R. G. Pereira, S. R. White, and I. Affleck, Phys. Rev. Lett. **100**, 027206 (2008).
[15] The correlation functions of various spin components form a tensor. For the ferromagnetic ground state, $\langle s_+(x,t)s_-(0,0)\rangle$ is its only nontrivial component.
[16] G. Baym and C. Ebner, Phys. Rev. **164**, 235 (1967).
[17] A. Imambekov and L. I. Glazman, Phys. Rev. Lett. **102**, 126405 (2009).
[18] P. W. Anderson, Phys. Rev. Lett. **18**, 1049 (1967).
[19] E. H. Lieb and W. Liniger, Phys. Rev. **130**, 1605 (1963); E. H. Lieb, *ibid.* **130**, 1616 (1963).
[20] J. N. Fuchs, D. M. Gangardt, T. Keilmann, and G. V. Shlyapnikov, Phys. Rev. Lett. **95**, 150402 (2005).