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Elastic Scattering of Fast Electrons by Helium*

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The analysis of electron-helium elastic scattering at small and intermediate angles given in a previous paper is extended to include the case of large-angle scattering. Agreement with absolute large-angle measurements is excellent.

In a recent paper[1] (to be referred to hereafter as I), we proposed a new approach—the eikonal-Born-series (EBS) method—to analyze elastic electron-atom collisions at intermediate energies. We applied this method to give a comprehensive treatment of electron-hydrogen and electron-helium elastic scattering at incident electron energies above 100 eV and for small- and intermediate-angle scattering. Excellent agreement was found with the experiment data.[2-6] In this paper we wish to complete the picture by considering the angular distributions at larger angles. Restricting our attention to electron-helium scattering, for which absolute experimental data are available,[4-6] we want to show that large angle scattering can be understood very simply in terms of elastic scattering by the static potential V_S of the target.[7] We shall also comment briefly on the validity of the EBS method and of the Glauber approximation at large momentum transfers.

Using the notation of I, we begin by remarking that the second Born terms, $\mathrm{Re}\bar{f}_{B2}$ and $\mathrm{Im}\bar{f}_{B2}$, are dominated at large momentum transfers by the contribution arising from the ground state acting as an intermediate state. A detailed analysis of higher terms of the Born and Glauber series using asymptotic techniques[8] yields similar conclusions, although the ground-state dominance becomes

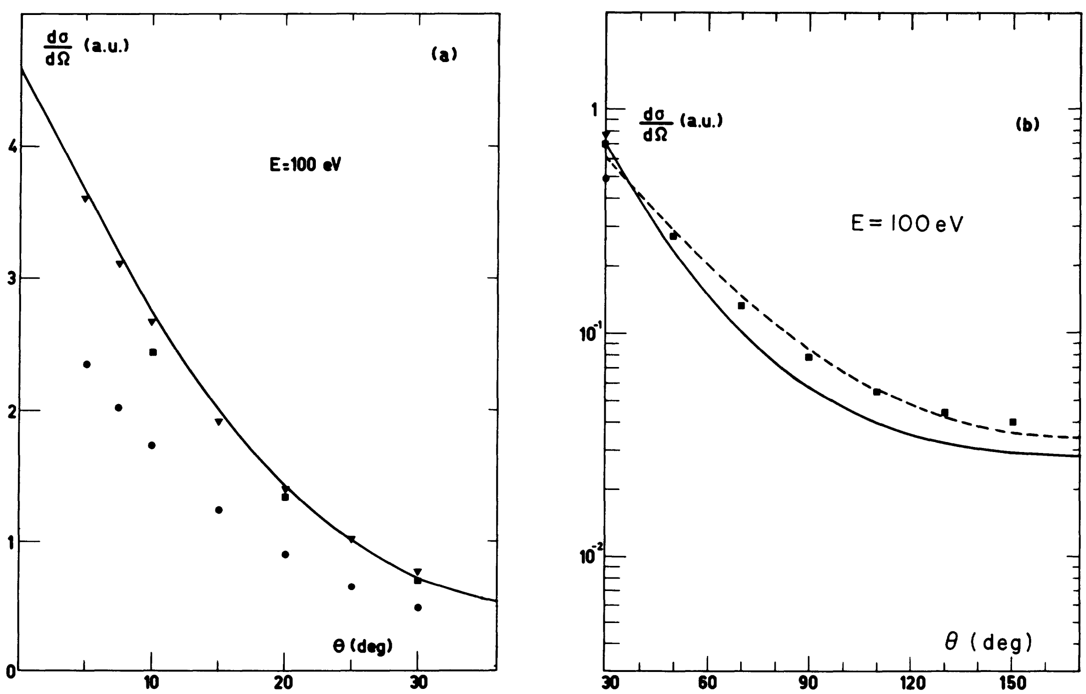

FIG. 1. (a) Small-angle elastic scattering of electrons by helium at 100 eV. Solid curve represents the eikonal-Born-series theoretical results. Triangles show the experimental data of Ref. 3; circles show the experimental data of Ref. 3 as renormalized by Ref. 5; and squares show the experimental data of Ref. 6. (b) Large-angle elastic scattering of electrons by helium at 100 eV. Solid curve represents the eikonal-Born-series theoretical results, while the dashed curve represents the "static-plus-exchange" calculation. Squares show the experimental results of Ref. 6.

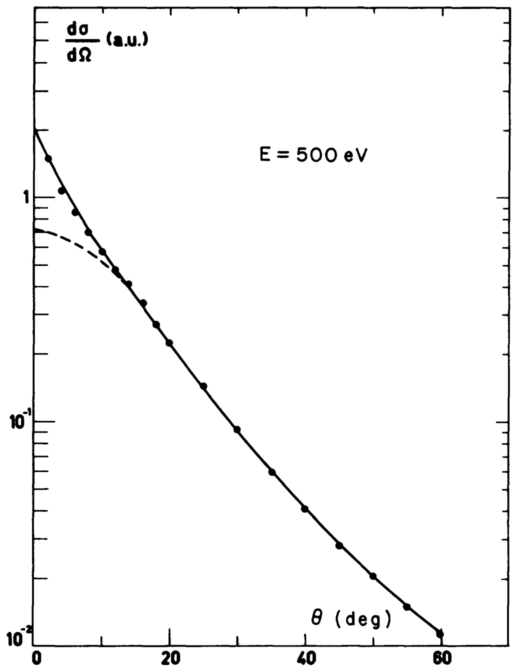

FIG. 2. Elastic scattering of electrons by helium at 500 eV. Solid curve represents the eikonal-Born-series theoretical results, while the dashed curve represents the "static-plus-exchange" calculation. Circles show the experimental results of Ref. 4.

weaker as one goes to progressively higher terms in perturbation theory. The important role played by the ground state in multiple scattering confirms one's intuitive expectation that large-angle scattering is mainly governed by the static potential V_S.

We have therefore computed the differential scattering cross section arising from the static helium potential V_S, obtained by using a Hartree-Fock wave function,[9] to represent the helium ground state. This may be done in a straightforward way by solving numerically the partial-wave radial equations to obtain the direct static scattering amplitude, f_d^S. Exchange effects, which are rather small at large angles, were taken into account by keeping the leading (Ochkur) contribution,[10] as discussed in I.

Calling $f_{SE} = f_d^S - g_{Och}$ the "static-plus-exchange" amplitude, we display in Figs. 1 and 2 the corresponding differential cross sections $d\sigma_{SE}/d\Omega = |f_{SE}|^2$ for electron energies of 100 and 500 eV (dashed lines). We see that the SE results give a good account of the scattering at *not-too-small angles*. Also shown in these figures are the results of the EBS method of I (solid lines). Outside the small-angle region, both methods (SE and EBS) agree fairly well with each other (within 30%) at 100 eV and extremely well at 500 eV. Moreover, we note from Figs. 1a and 2 that at *small angles* the EBS results agree very well with the data of Vriens *et al.*[3] at 100 eV and with the data of Bromberg[4] at 500 eV. The reasons for the reliability of the EBS method at these angles and energies have been discussed in detail in I. They may be summarized by recalling that the EBS calculations have been performed through order k_i^{-2} (where k_i is the incident wave number) and therefore require a fairly rapidly convergent perturbation series. At larger angles and relatively low energies (such as 100 eV) the Born series is not rapidly convergent, and the accuracy of the EBS results is reduced significantly [see Fig. 1b)]. The convergence at large angles is adversely affected relative to the convergence at small angles by the presence of powers of $\ln K$, where K is the momentum transfer. As the energy increases, the EBS results become much more accurate, even at large angles, as is shown in

TABLE I. A comparison of various theoretical and experimental differential cross sections for elastic electron-helium scattering at an incident electron energy of 200 eV. All results are in units of a_0^2/sr.

θ (degrees)	Theoretical values: Born	Glauber	EBS	Static + Exch.	Experimental values: Vriens *et al.*	Chamberlain *et al.*	Crooks and Rudd
0	6.27(−1)	∞	3.16	8.54(−1)	⋯	⋯	⋯
5	6.05(−1)	1.15	2.12	8.22(−1)	2.04	1.64	⋯
10	5.44(−1)	6.97(−1)	1.34	7.37(−1)	1.28	1.04	1.93
20	3.73(−1)	3.30(−1)	5.83(−1)	5.01(−1)	5.61(−1)	4.58(−1)	7.13
30	2.25(−1)	1.70(−1)	2.88(−1)	2.99(−1)	2.75(−1)	2.24(−1)	3.25(−1)
50	7.80(−2)	5.07(−2)	8.76(−2)	1.03(−1)	⋯	⋯	1.03(−1)
70	3.15(−2)	1.88(−2)	3.61(−2)	4.23(−2)	⋯	⋯	4.23(−2)
90	1.56(−2)	9.48(−3)	1.97(−2)	2.17(−2)	⋯	⋯	2.33(−2)
110	9.18(−3)	5.66(−3)	1.32(−2)	1.34(−2)	⋯	⋯	1.41(−2)
130	6.31(−3)	4.00(−3)	1.01(−2)	9.64(−3)	⋯	⋯	1.05(−2)
150	4.97(−3)	2.88(−3)	8.60(−3)	7.81(−3)	⋯	⋯	8.43(−3)
180	4.36(−3)	2.59(−3)	7.88(−3)	6.97(−3)	⋯	⋯	⋯

TABLE II. Same as Table I, but for an incident electron energy of 400 eV.

θ (degrees)	Theoretical values Born	Glauber	EBS	Static + Exch.	Experimental values Vriens *et al.*	Chamberlain *et al.*	Crooks and Rudd
0	6.27(−1)	∞	2.24	7.39(−1)	...	...	...
5	5.83(−1)	7.63(−1)	1.18	6.86(−1)	1.15	1.04	...
10	4.75(−1)	4.91(−1)	6.89(−1)	5.57(−1)	6.88(−1)	6.22(−1)	7.61(−1)
20	2.45(−1)	2.12(−1)	2.85(−1)	2.85(−1)	2.62(−1)	2.37(−1)	3.17(−1)
30	1.12(−1)	9.07(−2)	1.23(−1)	1.30(−1)	1.05(−1)	9.50(−2)	1.41(−1)
50	2.76(−2)	2.14(−2)	2.96(−2)	3.22(−2)	...	...	3.34(−2)
70	9.51(−3)	7.37(−3)	1.09(−2)	1.14(−2)	...	...	1.17(−2)
90	4.36(−3)	3.39(−3)	5.45(−3)	5.34(−3)	...	...	6.60(−3)
110	2.48(−3)	1.97(−3)	3.37(−3)	3.10(−3)	...	...	3.33(−3)
130	1.67(−3)	1.35(−3)	2.44(−3)	2.12(−3)	...	...	2.32(−3)
150	1.30(−3)	1.05(−3)	2.00(−3)	1.67(−3)	...	...	1.83(−3)
180	1.14(−3)	9.16(−4)	1.80(−3)	1.34(−3)	...	...	...

Fig. 2.

A more detailed account of our results is given in Tables I and II, where the first Born approximation, the Glauber[11] approximation, the EBS and SE differential cross sections are compared at incident electron energies of 200 and 400 eV for various scattering angles. Also included for comparison are the experimental data of Vriens *et al.*,[3] the same data as renormalized by Chamberlain *et al.*[5] and the measurements of Crooks and Rudd.[6] We note that the first Born results fall significantly below the experimental data. Although at 400 eV the disagreement between the first Born values and the large-angle data of Crooks and Rudd[6] is not too serious, this is rather misleading, since the Born amplitude is purely real, whereas the other theoretical amplitudes are predominantly imaginary. The Glauber method also yields results which are too low at all angles (except very near the forward direction, where it diverges). This is due to the fact, pointed out in I, that this approximation omits at each order in k_i^{-1} terms as large as those that it includes. For example, the Glauber approximation contains the term $\mathrm{Re}\bar{f}_{G3}$ but not the term $\mathrm{Re}\bar{f}_{B2}$; we showed in I that both terms are of order k_i^{-2}, except for very small momentum transfers where the term $\mathrm{Re}\bar{f}_{B2}$ is of order k_i^{-1} and actually yields the dominant correction to the first Born approximation.

Since higher-order terms play a significant role for large-angle scattering, we have also studied the relationship between the term $\mathrm{Im}\bar{f}_{B3}$ (missing from the Glauber series) and the term $\mathrm{Im}\bar{f}_{G4}$, both of order k_i^{-3}. The computation of $\mathrm{Im}\bar{f}_{G4}$ has been discussed in I. To obtain the quantity $\mathrm{Im}\bar{f}_{B3}$, we used the *unitarity relation* for the reduced T-matrix elements on the energy-momentum shell, that is

$$\mathrm{Im}T_{ba} = \pi \sum_n \delta(E_n - E_a)\delta(\vec{\mathrm{P}}_n - \vec{\mathrm{P}}_a)T_{bn}T_{an}^*, \qquad (1)$$

where E and $\vec{\mathrm{P}}$ refer, respectively, to the total energy and momentum. If we consider first the simple static potential model, Eq. (1) reduces to the well-known formula

$$\mathrm{Im}f_d^S(\theta) = \frac{k}{4\pi}\int f_d^S(\theta')^* f_d^S(\theta_0)\, d\Omega', \qquad (2)$$

where θ_0 is the angle between the directions $(\theta, 0)$ and (θ', ϕ') and f_d^S is the direct static amplitude. Then we have in third order

$$\mathrm{Im}\bar{f}_{B3}^S(\theta) = \frac{k}{2\pi}\int \mathrm{Re}\bar{f}_{B2}^S(\theta')f_{B1}(\theta_0)\, d\Omega', \qquad (3)$$

where the superscript S refers to the static part of the relevant amplitudes. Similarly, one may evaluate $\mathrm{Im}\bar{f}_{B3}$ by using only the state $|n\rangle = |0\rangle$ in Eq. (1), i.e., by making use of elastic unitarity. We have denoted the corresponding approximation to $\mathrm{Im}\bar{f}_{B3}$ by $\mathrm{Im}\bar{f}_{B3}^{\mathrm{el}}$. It is given by

$$\mathrm{Im}\bar{f}_{B3}^{\mathrm{el}}(\theta) = \frac{k}{2\pi}\int \mathrm{Re}\bar{f}_{B2}(\theta')f_{B1}(\theta_0)\, d\Omega', \qquad (4)$$

where $\bar{f}_{B2}$ is the complete (many-body) second Born amplitude that was evaluated in I by using closure. We may remark that the approximation of Eq. (3) consists in including only the ground state as an intermediate state in third order,

TABLE III. The quantities $\mathrm{Im}\bar{f}_{B3}^S$, $\mathrm{Im}\bar{f}_{B3}^{\mathrm{el}}$, and $\mathrm{Im}\bar{f}_{G4}$ for electron-helium elastic scattering at 400 eV.

θ	$\mathrm{Im}\bar{f}_{B3}^S$	$\mathrm{Im}\bar{f}_{B3}^{\mathrm{el}}$	$\mathrm{Im}\bar{f}_{G4}$
0	2.13(−2)	3.61(−2)	−3.71(−2)
30	1.92(−2)	2.82(−2)	−2.55(−2)
60	1.52(−2)	1.97(−2)	−2.13(−2)
90	1.19(−2)	1.45(−2)	−1.78(−2)
120	9.82(−3)	1.16(−2)	−1.61(−2)
150	8.72(−3)	1.01(−2)	−1.50(−2)
180	8.38(−3)	9.70(−3)	−1.47(−2)

while the approximation of Eq. (4) includes, in addition, all third-order processes that contain a single excited intermediate state followed or preceded by a ground intermediate state.

We present in Table III the results we have obtained for $\mathrm{Im}\bar{f}_{B3}^{S}$, $\mathrm{Im}\bar{f}_{B3}^{el}$, and $\mathrm{Im}\bar{f}_{G4}$ at an incident energy of 400 eV. We note that there is a very significant cancellation between $\mathrm{Im}\bar{f}_{B3}$ and $\mathrm{Im}\bar{f}_{G4}$, the cancellation being particularly striking in the case of $\mathrm{Im}\bar{f}_{B3}^{el}$. We note also that for scattering angles greater than 60°, $\mathrm{Im}\bar{f}_{B3}^{S}$ and $\mathrm{Im}\bar{f}_{B3}^{el}$ agree to within 20%, reinforcing our earlier remark about the dominance of the static potential.

The cancellation illustrated in Table III makes the Glauber approximation unreliable for this problem. For the same reason, one should be cautious in interpreting the success of the EBS method at large angles. Terms of higher order (in k_i^{-1}) than $\mathrm{Re}\bar{f}_{B3}$ are not included in our EBS calculations, so at large angles, where higher terms have significant magnitudes, its accuracy must depend precisely on the type of cancellation discussed above. Such cancellations, although present in the low orders of perturbation theory discussed here in the context of electron-helium scattering, cannot be expected to occur in another situation.

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[7]That is, $V_S = \langle 0|V|0\rangle$ where V is the full electron-atom interaction and $|0\rangle$ is the ground eigenket of the target.

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