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Charge Dependence in Multiple Scattering

S. Mukherjee
Department of Physics, Centre for Postgraduate Studies, Himachal Pradesh University, Simla-1, India
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It is shown that the multiple scattering of fast charged particles does not depend significantly on the sign of the charge at energies $\gtrsim$ 1 MeV, at least for scatterers of low atomic number.

I. INTRODUCTION

The multiple scattering (ms) of fast charged particles passing through matter has already been studied extensively.[1–4] However, the exact dependence of ms on the sign of the charge of the particle has not yet been fully understood. The interesting experiments of Strong and Roy,[5] who studied 2650 electron and 1887 positron tracks in nitrogen by cloud-chamber technique may be mentioned in this connection. They obtained a mean difference of $(3.4 \pm 2.6)\%$ between the angles of ms for particles with opposite charges. The theory of Molière,[1] which explains qualitatively most of the experimental results, does not predict any difference at all. Following the method of Nigam, Sundaresan, and Wu,[6] Reed[7] calculated the difference pertaining to the experiments of Strong and Roy and obtained a mean difference of 0.87%, in marginal agreement with the experiments. However, the accuracy of this method is open to question, since it fails to explain the experimental results when Thomas-Fermi (TF) potential is assumed for the scatterer, even of high-Z values. As an alternative, we calculated[2,3] the ms distribution function, making use of the distorted-wave Born approximation for the single-scattering cross section.[8] The results were in good agreement with the experimental results of Hanson *et al.*[9] In this note, we shall use the same method to study the dependence of ms on the sign of the charge $\delta (= \pm 1)$. It may be mentioned that the effect is small, and we are calculating it by a method which itself is based on two approximations. The approximation we made in calculating the single-scattering cross section[8] is slightly better than the second Born approximation. There is also the small-angle approximation, which is typical of the theory of ms. Whether one expects any significant δ dependence within the accuracy of the above two approximations, is the question we shall attempt to answer here. The conclusions drawn will, however, be of general validity.

II. CHARGE DEPENDENCE OF THE SCREENING ANGLE

One interesting aspect of Molière's theory is the fact that the entire dependence of the ms distribution function on the single-scattering cross section enters through a single parameter, the screening angle θ_α. The situation in our case is slightly more complex, there being another quantity λ. The screening angle nevertheless plays a major role. It will, therefore, be useful to first determine the dependence of θ_α on δ. We follow the method developed earlier[2] and obtain the following expression for θ_α for an electron beam:

$$\lambda \ln\theta_\alpha = \lambda \ln\theta_0 + e^{\pi a'(\epsilon/p)} \Bigg\{ \sum_{k=1}^{3} a_k^2 e^{-2a'(\epsilon/p)\tan^{-1}(2/\theta_k)} \ln b_k - 2\,\mathrm{Re} \sum_{k<l} a_k a_l \left(\frac{\theta_k^2 + 2i\theta_k}{\theta_l^2 - 2i\theta_l} \right)^{ia'(\epsilon/p)} \times \left[\tfrac{1}{2} - \frac{b_k^2 \ln b_k - b_l^2 \ln b_l}{b_k^2 - b_l^2} + ia'(\epsilon/p) \left(\frac{b_l^2}{b_k^2 - b_l^2} \ln^2 \frac{b_k}{b_l} - \tfrac{1}{2}\alpha_2\left(\frac{b_k^2 - b_l^2}{b_k^2} \right) \right) \right] \Bigg\}, \tag{1}$$

where we have assumed Molière's representation of the TF potential for the scatterer, viz.,

$$V(r) = (a'/r) \sum_{k=1}^{3} a_k e^{-b_k r / r_0}, \tag{2}$$

with $a' = \alpha Z$, the momentum p and the energy ϵ being measured in units of mc and mc^2, respectively. Also

$$\theta_0 = 1/p r_0; \quad \theta_k = b_k \theta_0 . \tag{3}$$

The quantity λ is given by

$$\lambda = e^{\pi a'(\epsilon/p)} \mathrm{Re} \sum_k \sum_l a_k a_l \left(\frac{\theta_k^2 + 2i\theta_k}{\theta_l^2 - 2i\theta_l} \right)^{ia'(\epsilon/p)} . \tag{4}$$

The function $\alpha_2(x)$ is Euler's dilogarithm defined by

$$\alpha_2(x) = -\int_0^x \frac{\ln(1-y)}{y}\,dy\,. \tag{5}$$

Our definition of θ_α is

$$\lambda \ln\theta_\alpha = -\tfrac{1}{2}\lambda - \lim_{\theta_s\to\infty}\left(\int_0^{\theta_s}\frac{q(\theta)}{\theta}d\theta - \lambda\ln\theta_s\right), \tag{6}$$

whereas Molière's θ_α is given by Eq. (6) with $\lambda = 1$. Noting that

$$\left(\frac{\theta_k^2 + 2i\theta_k}{\theta_l^2 - 2i\theta_l}\right)^{ia'(\epsilon/p)} \simeq e^{-\pi a'(\epsilon/p)} \times e^{ia'(\epsilon/p)\ln(b_k/b_l)}\, e^{a'(\epsilon/p)(\theta_k+\theta_l)/2}, \tag{7}$$

We can simplify the expression (1), viz.,

$$\lambda\ln\theta_\alpha = \lambda\ln\theta_0 + \sum_k a_k^2 \ln b_k - 2\sum_{k<l} a_k a_l\, e^{a'(\epsilon/p)(\theta_k+\theta_l)/2} \times\left\{\cos x_{kl}\left(\tfrac{1}{2} - \frac{b_k^2\ln b_k - b_l^2 \ln b_l}{b_k^2 - b_l^2}\right) - a'(\epsilon/p)\sin x_{kl} \times\left[\frac{b_l^2}{b_k^2 - b_l^2}\ln^2\frac{b_k}{b_l} - \tfrac{1}{2}\alpha_2\left(\frac{b_k^2 - b_l^2}{b_k^2}\right)\right]\right\}, \tag{8}$$

where

$$x_{kl} = a'(\epsilon/p)\ln(b_k/b_l)\,. \tag{9}$$

The expression for $\bar\theta_\alpha$, relevant for positron, is obtained by changing the sign of a' everywhere in the expression (8). The expression gives a clear criterion as to where a significant difference between θ_α and $\bar\theta_\alpha$ may be expected. Since all the terms within the curly brackets are even in a', the only difference comes through the exponential factor, $e^{a'(\epsilon/p)(\theta_k+\theta_l)/2}$. Hence, the δ dependence will be more pronounced either for large a' and/or at small momentum. Our method (as well as any other available method) is not, however, reliable for very-high Z or at very-low momenta. The general limitations of these methods are well known. The fact that the electron-positron difference comes through a term which is a product of two small quantities a' and θ_0 is indicative of the fact that we do not anticipate any significant δ dependence, at least in the region where these methods will work.

For small values of Z, the dependence of λ on δ is also negligible. For example, for electrons and positrons of energy 1.5 Mev, scattered from nitrogen (the effective Z being 7.05, the case studied by Strong and Roy), we have

$$\lambda = 0.997 \quad\text{and}\quad \bar\lambda = 0.995\,. \tag{10}$$

III. RESULTS

As a check on our arguments, we have made a model calculation for θ_α and $\bar\theta_\alpha$ for a particle momentum $p=1$ and $Z = 7.05$. The Euler dilogarithms $\alpha_2(x)$ have been evaluated by an IBM 1620 computer, and the calculated results are

$$\lambda\ln\theta_\alpha = \lambda\ln\theta_0 + 0.0743\,,$$
$$\bar\lambda\ln\theta_\alpha = \bar\lambda\ln\theta_0 + 0.0726\,. \tag{11}$$

These may be compared with the first Born approximation results

$$\ln\theta_\alpha = \ln\theta_0 + 0.0793 \tag{12}$$

and that of Molière,

$$\ln\theta_\alpha = \ln\theta_0 + 0.0699\,, \tag{13}$$

calculated for the identical problem.

These results agree with our expectations for low-Z values, although the three methods give results which differ considerably as Z increases. The electron-positron difference in θ_α, as shown in Eq. (11), does not lead to any significant difference in ms, which is described by the reduced distribution function

$$f(\varphi, t) = \frac{1}{2\pi}\int_0^\infty \eta\, d\eta\, J_0(\varphi\eta)\exp\left[-\frac{\eta^2}{4} + \left(\frac{\eta^2}{4B}\right)\ln\left(\frac{\eta^2}{4}\right)\right], \tag{14}$$

where $\varphi = x/\theta_c(B\lambda)^{1/2}$, x being the spatial angle of scattering and θ_c the characteristic scattering angle. The parameter B is determined by θ_α through the relation

$$B - \ln B = \ln(e\lambda\theta_c^2/\gamma^2\theta_\alpha^2)\,, \tag{15}$$

where $\ln\gamma = c$ is Euler's constant. From (15), we get

$$\frac{\delta\theta_\alpha}{\theta_\alpha} = -\tfrac{1}{2}(B-1)(\delta B/B)\,. \tag{16}$$

Since B is usually a large number (in the present case $B > 5$), a small fractional change in θ_α leads to a still smaller fractional change in B. Thus, while the reduced distribution function $f(\varphi, t)$ will show hardly any dependence on δ, even the scale of the reduced angle φ relative to the spatial angle of scattering x will exhibit very little dependence. In the case we are considering ($p = 1$), we found $B = 5.128$ and $B = 5.134$.

One may argue that for Z small, the TF potential is not realistic and a more accurate potential should be considered. However, use of a different potential will not alter the situation appreciably. This can be shown from (15), which we can write as

$$B - \ln B = 3.642 - 2\ln(\theta_\alpha/\theta_0)\,, \tag{17}$$

with $\theta_c = 0.106$. The second term, which accounts for the effect of screening, is of order ~0.15, and obviously no reasonable modification of the potential can bring in an appreciable δ dependence in the value of B.

In conclusion, we do not anticipate any significant δ dependence in multiple scattering by low-Z scatterers at energies $\gtrsim 1$ MeV. This is, of course, not an obvious result since the single-scattering cross section does show an appreciable dependence on δ. A small electron-positron difference for large values of Z is, however, not ruled out. Further experiments in this direction will be useful for a clarification of the situation.

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