Bose-Einstein Condensation for Charged Particles*

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The self-consistent random-phase approximation is used to derive an integral equation for the quasiparticle energy of a charged Bose gas near a critical point. An approximate analytical solution, valid at long wavelengths, is used to compute the critical exponents which are those of an ideal Bose gas with the single-particle spectrum $\epsilon(p) = A p^{2-\eta}$, $\eta \simeq 0.2$. The accuracy of the analytical solution is checked by an exact numerical solution which is also used in a calculation of the transition curve. A necessary condition for the correctness of the theory is found to be $(T - T_c)/T_c \gtrsim 0.01$.

I. INTRODUCTION

Since the original work of Bogoliubov[1] most studies of the interacting Bose gas have concentrated on the low-temperature properties of systems with short-range interactions.[2-4] Work on the charged Bose gas was initiated by Foldy,[5] who calculated the ground-state energy, and has been continued in the same vein by a number of authors.[6-8] For neutral systems a theory of the Λ transition was proposed by Patashinskii and Pokrovskii,[9] who obtained a quasiparticle spectrum $\epsilon(p) \sim p^{3/2}$ for $T = T_\Lambda$. This work has been criticized by Migdal,[10] but a more satisfactory version has recently been given by Gould and Wong.[11] For charged systems two treatments have been given. The more recent work is due to Ma,[12] who uses Wilson's[13] approach to critical phenomena; and the earlier is due to Fetter,[14] who employs the Green's-function formalism in a calculation of the transition temperature. Ma's theory involves an expansion in $(1/N)$, where N is the number of real components of the charged Bose field; it is therefore not applicable to the two-component charged gas which is the subject of this work. On the other hand, Fetter's starting point and ours are the same; the difference is that we solve the relevant integral equation exactly, whereas Fetter stops at the first iteration. This difference does not strongly influence the calculation of the transition temperature, but it is very important for the calculation of the critical exponents.[15]

The plan of the paper is as follows: In Sec. II we present the truncated Dyson equation on which the theory is based, and in Sec. III we find the long-wavelength form of the solution which determines the critical exponents. The exact numerical solution, which is required for the calculation of the transition temperature, is discussed in Sec. IV; and an estimate of the domain of validity of the theory is given in Sec. V. In Sec. VI we discuss the results.

II. DYSON EQUATION

In order to simplify the formalism we imagine approaching the critical temperature from above; therefore, our starting point is the exact Dyson equation for the one-particle thermodynamic Green's function describing a normal system[16]:

$$G^{-1}(\vec{p}, \Omega_n) = \Omega_n + \mu - p^2 - \Sigma(\vec{p}, \Omega_n),$$

where μ is the chemical potential and the discrete, imaginary frequency $\Omega_n = 2\pi n i/\beta$. The self-energy Σ is expressed in terms of the dynamic screened potential V and the vertex function Γ by

$$\Sigma(\vec{p}, \Omega_n) = i \int \frac{d^3q}{(2\pi)^3} \frac{1}{(-i\beta)} \times \sum_m [G(\vec{p} - \vec{q}, \Omega_n - \Omega_m) V(\vec{q}, \Omega_m) \times \Gamma(\vec{p}, \Omega_n; \vec{q}, \Omega_m)]$$

and V is given by

$$V(\vec{q}, \Omega_m) = \frac{u(\vec{q})}{1 + u(\vec{q})\Lambda(\vec{q}, \Omega_m)},$$

$$u(\vec{q}) = 8\pi/q^2,$$

$$\Lambda(\vec{q}, \Omega_m) = -i \int \frac{d^3k}{(2\pi)^3} \frac{1}{-i\beta} \times \sum_l G(\vec{k}, \Omega_l) G(\vec{k} - \vec{q}, \Omega_l - \Omega_m) \times \Gamma(\vec{k}, \Omega_l; \vec{q}, \Omega_m).$$

We are using units such that $\hbar = 2m = \frac{1}{2}e^2 = 1$; thus lengths are measured in Bohr radii and energies in rydbergs.

In order to use these equations we must first truncate them by assuming an approximate form for Γ. We choose to set $\Gamma \equiv 1$, which leads to a self-consistent version of the random-phase approximation commonly employed in theories of charged systems. We next make use of the idea, originally suggested by Landau,[17] that critical phenomena are determined primarily by low-frequency excitations. We do this by neglecting the frequency dependence of the polarization function Λ; i.e., $\Lambda(\vec{q}, \Omega_n) \to \Lambda(\vec{q}) \equiv \Lambda(\vec{q}, 0)$. Since $\Lambda(\vec{q}, 0)$ is real, the screened potential is replaced by a real static potential $V(\vec{q}) \equiv V(\vec{q}, 0)$; furthermore, it follows from the definition of the self-energy that $\Sigma(\vec{p}, \Omega_n) \to \Sigma(\vec{p})$, which is real and independent of frequency. This means that $G(\vec{p}, \Omega_n)$ describes a stable quasiparticle; therefore the Dyson equation can be regarded as an equation for the quasiparticle energy which is conveniently written in terms of the quantities

$$\epsilon(p) \equiv \vec{p}^2 + \Sigma(\vec{p}) - \Sigma(0) ,$$

$$\bar{\mu} \equiv \Sigma(0) - \mu ,$$

We then find $G^{-1}(\vec{p}) = -[\bar{\mu} + \epsilon(\vec{p})]$.

$$\epsilon(\vec{p}) = p^2 + \int \frac{d^3q}{(2\pi)^3} f(\bar{\mu} + \epsilon(\vec{q}))[V(\vec{p}+\vec{q}) - V(\vec{q})], \tag{1}$$

$$V(\vec{q}) = \frac{u(\vec{q})}{1 + u(\vec{q})\Lambda(\vec{q})} , \tag{2}$$

$$\Lambda(\vec{q}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\epsilon(\vec{k}) - \epsilon(\vec{k}+\vec{q})} [f(\bar{\mu} + \epsilon(\vec{k}+\vec{q})) - f(\bar{\mu} + \epsilon(\vec{k}))] , \tag{3}$$

where $f(x) \equiv (e^{\beta x} - 1)^{-1}$ is the Bose function which appears when the frequency sums are performed in the standard way.[16]

The independent thermodynamic parameters in these equations are $\bar{\mu}$ and β. The relevant experimental parameters are density and temperature; therefore we add the equation for the density n,

$$n = \int \frac{d^3p}{(2\pi)^3} f(\bar{\mu} + \epsilon(\vec{p})) , \tag{4}$$

which is used to eliminate $\bar{\mu}$. The form of the transition curve in the density-temperature plane is obtained by setting $\bar{\mu} = 0$ in Eqs. (1)–(4). That is, one solves Eqs. (1)–(3) with $\bar{\mu} = 0$ and then uses the resulting $\epsilon(p)$ in Eq. (4); this yields n as a function of T, thereby defining the transition curve. In Sec. IV we present the transition curve resulting from an exact numerical solution for the quasiparticle energy.

III. LONG-WAVELENGTH SOLUTION

The values of the critical exponents are determined by the long-wavelength behavior of the various correlation functions which are in turn determined, at least in our theory, by the quasiparticle energy $\epsilon(\vec{p})$. We accordingly set $\bar{\mu} = 0$ and look for a solution satisfying $\epsilon(\vec{p}) \gg p^2$ as $p \to 0$. The last condition is essential since its violation would imply that the critical exponents are those of the ideal Bose gas. With this in mind we drop the kinetic-energy term in Eq. (1). Also note that the singularity of the Bose function can be isolated by the following expansion:

$$f(\epsilon(\vec{q})) = \frac{1}{\beta\epsilon(\vec{q})} - \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2\beta\epsilon(\vec{q})}{[\beta\epsilon(\vec{q})]^2 + (2\pi n)^2} .$$

The contribution of the nonsingular terms to the integral in Eq. (1) is analytic in p^2, and any such terms can be discarded in the long-wavelength limit. Thus we may replace the Bose function by its singular part. Next consider the potential $V(q)$; with the approximations already introduced, we have

$$\Lambda(0) \cong \frac{1}{\beta} \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{\epsilon(k)}\right)^2 > 0 .$$

Thus for small q we have $q^2 \ll \Lambda(q)$ and the long-wavelength limit of the screened potential is

$$V(\vec{q}) \to \frac{1}{\Lambda(\vec{q})} , \text{ as } \vec{q} \to 0 .$$

Combining these results yields the following asymptotic equations:

$$\epsilon(\vec{p}) = \frac{1}{\beta} \int \frac{d^3q}{(2\pi)^3} \frac{1}{\epsilon(\vec{q})} \left(\frac{1}{\Lambda(\vec{p}+\vec{q})} - \frac{1}{\Lambda(\vec{q})}\right) , \tag{5}$$

$$\Lambda(\vec{q}) = \frac{1}{\beta} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\epsilon(\vec{k})\epsilon(\vec{k}+\vec{q})} . \tag{6}$$

We see that Eq. (5) is homogeneous in ϵ; consequently we expect a solution of the form $\epsilon(p) = Ap^{2-\eta}$, where $\eta > 0$ is required for consistency with the assumption $\epsilon(p) \gg p^2$. With this assumed form for ϵ we have from Eq. (6)

$$\Lambda(\vec{q}) = C(\eta) q^{2\eta-1} / (2\pi)^2 \eta \beta A^2 ,$$

where

$$C(\eta) = \int_0^{\infty} dx\, x^{\eta-1}[(1+x)^{\eta} - |1-x|^{\eta}] .$$

The integral defining $C(\eta)$ diverges for $\eta \geq \frac{1}{2}$, so we must have $0 < \eta < \frac{1}{2}$. The value of η is determined by substituting the above form for Λ into Eq. (5); this leads to a transcendental equation

$$C(\eta) = \eta D(\eta) , \tag{7}$$

with

$$D(\eta) = \int_0^\infty dx\, x^{\eta-1} \left(\frac{(1+x)^{3-2\eta} - |1-x|^{3-2\eta}}{3-2\eta} - 2x^{2-2\eta} \right).$$

A numerical search revealed exactly one root for Eq. (7) at $\eta \cong 0.19$; therefore the various approximations introduced to obtain Eqs. (5) and (6) are justified.

We now turn to the calculation of the critical exponents. In general the procedure would be to express the various thermodynamic quantities (derivatives of the free energy) as integrals over the corresponding correlation functions. The singularities of the thermodynamic quantities as $T \to T_c$ would then arise from the small-wave-number parts of the integrals, so that the critical exponents would be determined by the long-wavelength form of the correlation functions. In our case a short cut is available which depends on the static approximation for $V(\vec{q}, \Omega)$. This approximation guarantees a stable quasiparticle; therefore the interacting Bose gas can be regarded as a noninteracting Bose gas composed of particles with spectrum $\epsilon(\vec{p})$. In fact we may use the long-wavelength form $\epsilon(\vec{p}) = Ap^{2-\eta}$, since the critical exponents are only sensitive to the small-wave-number behavior of ϵ. The critical exponents for this class of models have been worked out by Gunton and Buckingham,[18] and we only have to substitute our value of η into their expressions to obtain the results[15] listed in Table I, where the notation for the exponents is taken from Kadanoff.[19] We also include for comparison the exponents calculated from Landau mean-field theory and the theory of Patashinskii and Pokrovskii. We do not include Ma's theory since, according to Ma,[12] it does not give definitive results for a two-component Bose field in three dimensions.

TABLE I. Critical exponents.

Quantity	Critical exponent	Present paper $2-\eta=\frac{9}{5}$	Landau $2-\eta=2$	Patashinskii-Pokrovskii $2-\eta=\frac{3}{2}$
Correlation function	η	$\frac{1}{5}$	0	$\frac{1}{2}$
Correlation length	ν	$\frac{5}{9}$	$\frac{1}{2}$	$\frac{2}{3}$
Specific heat	α	$\frac{1}{3}$	0	$0(\ln\lvert T-T_c\rvert)$
Superfluid density	ξ	$\frac{2}{3}$	1	1

IV. EXACT SOLUTION

The coefficient A in $\epsilon(\vec{p}) = Ap^{2-\eta}$ is not determined by Eqs. (5) and (6) since they are homogeneous. This does not prevent the calculation of the critical exponents which only depend on the power η; however, from Eq. (4) we see that A enters into the density in a nontrivial way. For this reason the asymptotic equations do not fix the transition curve, and we are forced to solve the equation for ϵ exactly.

Our numerical procedure is to insert an assumed function $\epsilon(\vec{p})$ into the integrals occurring in Eqs. (1)–(3); we then evaluate the integrals numerically, and thereby produce a new $\epsilon(\vec{p})$ for the next round in an iterative procedure. The principal technical difficulty was the necessity for evaluating the double integrals in Eqs. (1) and (3) a very large number of times in each iteration; this necessitated developing an economical program for the evaluation of the integrals.

The numerical solution was used to compute the transition curve as outlined in Sec. II. The resulting curve is shown in Fig. 1 together with Fetter's result and the ideal-gas curve. Our self-consistent solution gives a small increase in the transition temperature. For the density region $0.4 < r_s < 0.8$ this increase is given approximately by

$$(T_c - T_c^0)/T_c^0 \cong 0.0422(na_0^3)^{-2/9} = 0.088 r_s^{2/3},$$

whereas Fetter finds a decrease in the transition

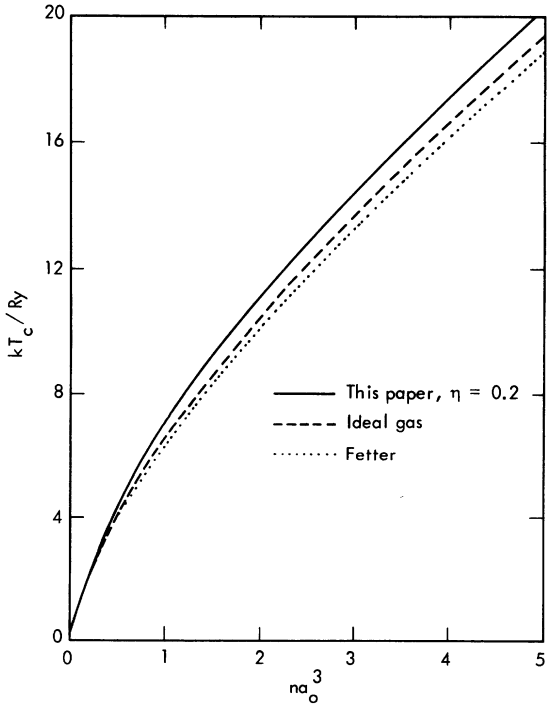

FIG. 1. Transition temperature vs density for the present theory, Fetter's theory, and the ideal Bose gas.

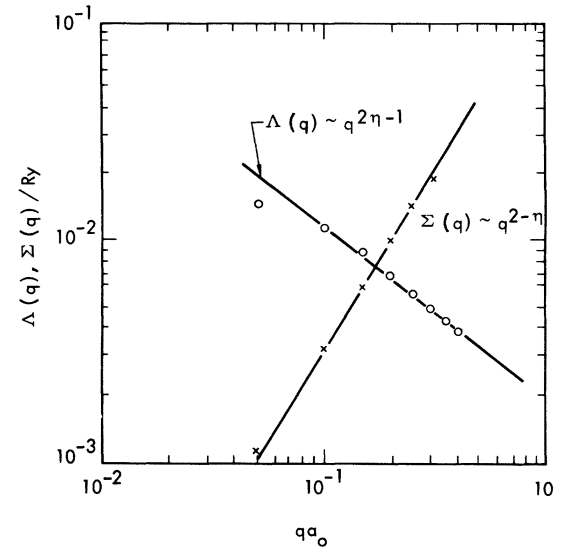

FIG. 2. Σ and Λ vs q at T_c from numerical solutions of Eqs. (1)–(3). The value of $2\eta-1$ from Λ is -0.76 and $2-\eta$ from Σ is 1.74.

temperature of the form $(T_c - T_c^0)/T_c^0 = -0.026 r_s^{1/3}$. The difference between our result and Fetter's seems to be that our $\epsilon(p)$, which goes as $p^{1.8}$ as $p \to 0$, is greater than p^2 for larger values of p than is the case with Fetter's calculation.

In Fig. 2 the power-law behavior of the subtracted self-energy $S(\vec{p}) \equiv \Sigma(\vec{p}) - \Sigma(0)$ can be seen explicitly. The power obtained from this graph is $\eta \cong 0.3$, which is in reasonable agreement with the analytical result $\eta \cong 0.2$; the discrepancy is due presumably to the finite step size used in the numerical calculation. The numerical solution therefore serves as a check on the validity of the long-wavelength solution obtained in Sec. III.

We have so far concentrated on solutions for the case $\bar{\mu} = 0$; however, the numerical program works even better for $\bar{\mu} \neq 0$; and we can exploit this fact to study the behavior of the solution in the vicinity of a critical point. In Fig. 3 we exhibit the $\bar{\mu}$ dependence of the self-energy for several values of $\vec{p}$. We see that the dependence is rather weak; this implies that for small $\bar{\mu}$ we may approximate the propagator by

$$G^{-1}(\vec{p}) = -\bar{\mu} - \epsilon(\vec{p}) ,$$

where $\epsilon(\vec{p})$ is the $\bar{\mu} = 0$ solution. Then for small $\vec{p}$ we have

$$G^{-1}(\vec{p}) = -\bar{\mu} - Ap^{2-\eta} ,$$
$$= \bar{\mu}[1 + A(p\xi)^{2-\eta}] ,$$

where $\xi \equiv \bar{\mu}^{-1/(2-\eta)}$ is to be interpreted as the correlation length. The divergence of the correlation length as $T \to T_c$ is then determined by the temperature dependence of $\bar{\mu}$. The usual assumption is $\bar{\mu} \sim (T - T_c)$, and this can be verified by means of our numerical solution. The value of $\bar{\mu}$ corresponding to a given temperature and density is found by solving Eq. (4) for $\bar{\mu}$. Having done this we allow $T \to T_c(n)$ and graph the corresponding $\bar{\mu}$ values. From Fig. 4 we see that $\bar{\mu}$ is indeed a linear function of $T - T_c$. Consequently the correlation length goes as $(T - T_c)^{-1/(2-\eta)}$ as $T \to T_c$. The question of vertex corrections to the linear temperature dependence of $\bar{\mu}$ will be considered in Sec. V.

A self-consistent formalism for $T < T_c$ has been developed; details are given in Ref. 20. The set of equations for $T < T_c$ analogous to those for $T > T_c$, Eqs. (1)–(3), are quite lengthy and no attempt has been made to solve them numerically. For $T \to T_c$ from below, these equations reduce to Eqs. (1)–(3) for $T = T_c$ from above.

V. RANGE OF VALIDITY

Since the basic dynamical approximation of our theory is the neglect of vertex corrections, we can get some idea of the domain of validity by

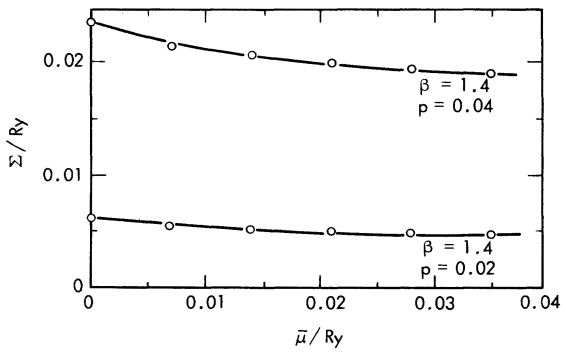

FIG. 3. Σ as a function of $\bar{\mu}$.

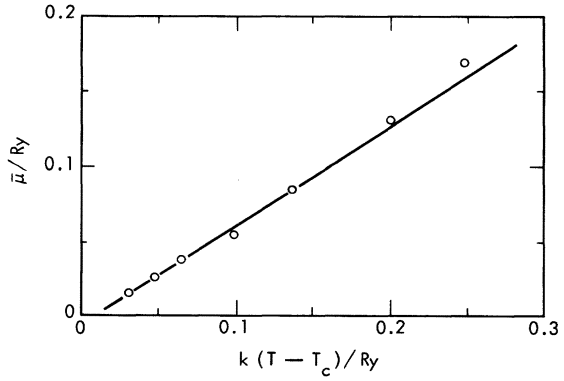

FIG. 4. $\bar{\mu}$ vs $(T - T_c)$ at $na_0^3 = 0.8$ and $kT_c/\mathrm{Ry} = 6.1$.

estimating the lowest-order correction to the bare vertex. The relevant diagram is shown in Fig. 5, where the solid lines represent particle propagators and the wiggly line represents the interaction V. In evaluating the diagram we again assume that the frequency sums are dominated by the zero-frequency terms. Then we have

$$\Gamma(\vec{p},\vec{q}) = 1 - \frac{1}{\beta}\int \frac{d^3k}{(2\pi)^3}\, V(\vec{k})G(\vec{p}-\vec{k})\, G(\vec{p}-\vec{k}-\vec{q}) + \cdots .$$

The integral can be seen to diverge for $\bar{\mu}=0$, so we have to keep $\bar{\mu}\neq 0$; thus we should use $G^{-1} = -\bar{\mu} - \epsilon(p)$ and the corresponding value for V. Unfortunately the expression for Λ cannot be evaluated in closed form if the correct propagator is used. To avoid this difficulty we adopt the usual expedient of imposing a cutoff for $|k| = \xi^{-1}$, and we use the $\bar{\mu}=0$ forms for the propagators when $|k| > \xi^{-1}$. Since the divergence arises from small $\vec{k}$ values we can also impose an upper cutoff; the deleted part of the integral does not contribute to the singularity. By choosing the upper cutoff sufficiently small we can use the long-wavelength forms for G and V. Furthermore, the most serious divergence occurs for $\vec{p}=\vec{q}=0$, so we confine our attention to that case:

$$\begin{aligned}\Gamma(0,0) - 1 &= \frac{1}{2\pi^2\beta}\int_{\xi^{-1}}^{k_0} dk\, k^2 V(\vec{k})[G(\vec{k})]^2 + \cdots \\ &= -\frac{2\eta}{C(\eta)}\ln\xi + \cdots \\ &= \frac{2\eta}{(2-\eta)C(\eta)}\ln\left(\frac{T-T_c}{T_c}\right) + \cdots . \quad (8)\end{aligned}$$

The right-hand side of this equation is an estimate of the error in our dynamical approximation

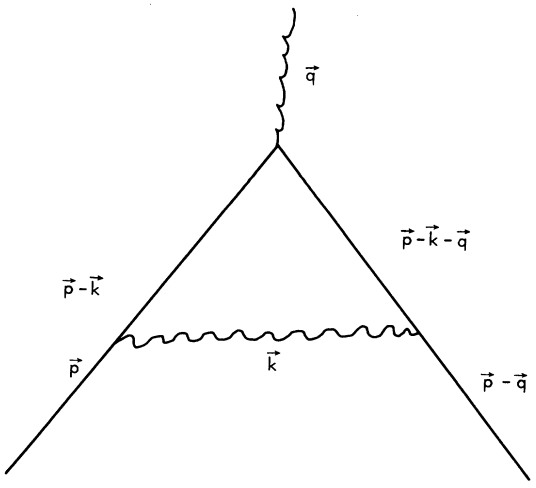

FIG. 5. Diagram for the lowest-order vertex function correction.

$\Gamma(\vec{p},\vec{q}) = 1$; therefore we must require it to be small. There is an additional consistency condition related to the assumption $\bar{\mu} \sim (T - T_c)$. More generally, one could assume $\bar{\mu} \sim \epsilon^{\gamma}$, where $\epsilon \equiv (T - T_c)/T_c$, so that $\xi \sim \epsilon^{-\nu}$ with $(2-\eta)\nu = \gamma$. The exponent γ can be related to the vertex function by means of a Ward's identity for the single-component charged Bose gas,

$$\Gamma(\vec{k}, 0) = \frac{\partial G^{-1}(\vec{k})}{\partial \mu} .$$

Putting $\vec{k}=0$ we have

$$\Gamma(0,0) = \frac{\partial \bar{\mu}}{\partial \mu} \propto \frac{\partial \bar{\mu}}{\partial T} \sim \epsilon^{\gamma-1}.$$

The coefficient of proportionality in the second of these relations is the slope of the critical curve. If $\gamma - 1$ is small we find

$$\Gamma(0,0) \sim 1 + (\gamma - 1)\ln\epsilon ,$$

and by comparison with Eq. (8) we obtain

$$\gamma - 1 = 2\eta/(2-\eta)C(\eta) = 0.21$$

for $\eta = 0.2$. Thus γ is indeed close to unity as we assumed. Finally we impose $|\Gamma - 1| \ll 1$, which gives the following condition on ϵ:

$$0.01 \ll \epsilon .$$

The lower limit shows that our theory is not valid for temperatures too close to the critical temperature. Similar difficulties arise in other theories[9,11,14] of critical phenomena; thus the power-law dependence on ϵ predicted by these theories, as well as ours, may fail for sufficiently small ϵ.

VI. DISCUSSION

We have presented a theory of the Bose transition for a charged system in which the main approximation is the neglect of vertex corrections. This leads to a self-consistent version of the random-phase approximation, and the first point to be emphasized is that the self-consistent feature is essential. By constructing the bubble function from dressed Green's functions we were led to a propagator with the long-wavelength form $G(p) = Ap^{-(2-\eta)}$, which is homogeneous as required by the scaling hypothesis. We may contrast this result with Fetter's[14] calculation which uses bare propagators to compute Λ; he obtains

$$G^{-1}(\vec{p}) = p^2/2m^* - Cp^2\ln p \quad \text{as } p \to 0 .$$

which clearly does not scale. In this connection it is interesting to note that the "screening approximation" applied to Landau's theory of phase transitions by Ferrell and Scalapino[21] leads to

results similar to Fetter's. The reason is that they also use bare propagators in constructing the function corresponding to Λ.

In spite of the fact that Ma's $1/N$ expansion cannot be applied to our problem, it is instructive to compare our results with his. In Ma's theory the function corresponding to our $\Lambda(q)$ vanishes as $q \to 0$; this is in marked contrast to our result $\Lambda(q) \to \infty$ as $q \to 0$. The source of the discrepancy is presumably that Ma's Λ involves certain vertex-correction diagrams which are missing in our approach. This suggests that the next step in the Green's-function approach is to invent a more elaborate approximation scheme which will self-consistently include some vertex corrections.

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[1]N. N. Bogoliubov, J. Phys. USSR 9, 23 (1947).

[2]S. T. Beliav, Zh. Eksp. Teor. Fiz. 34, 417 (1958) [Sov. Phys.-JETP 7, 289 (1958)].

[3]N. M. Hugenholtz and D. Pines, Phys. Rev. 116, 489 (1959).

[4]P. Hohenberg and P. Martin, Ann. Phys. (N.Y.) 34, 2291 (1965).

[5]L. L. Foldy, Phys. Rev. 124, 649 (1961).

[6]K. A. Bruckner, Phys. Rev. 156, 204 (1967).

[7]S. K. Ma and C. W. Woo, Phys. Rev. 159, 165 (1967).

[8]M. Girardeau, Phys. Rev. 127, 1809 (1962).

[9]A. Z. Patashinskii and V. L. Pokrovskii, Zh. Eksp. Teor. Fiz. 46, 994 (1963) [Sov. Phys.—JETP 19, 677 (1964)].

[10]A. A. Migdal, Zh. Eksp. Teor. Fiz. 55, 1964 (1968) [Sov. Phys.-JETP 28, 1036 (1969)].

[11]H. Gould and V. Wong, Phys. Rev. A 4, 719 (1971).

[12]S. Ma, Phys. Rev. Lett. 29, 1311 (1972).

[13]K. G. Wilson, Phys. Rev. Lett. 28, 548 (1972).

[14]A. L. Fetter, Ann. Phys. (Paris) 64, 1 (1971).

[15]P. V. Panat, H. E. DeWitt, and J. C. Garrison, Phys. Lett. 43A, 130 (1973).

[16]For a discussion of the Green's-function formalism see, for example, L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1961).

[17]See Ref. 9 for a discussion of this idea.

[18]J. P. Gunton and M. J. Buckingham, Phys. Rev. 166, 152 (1968).

[19]L. P. Kadanoff *et al.*, Rev. Mod. Phys. 39, 395 (1967).

[20]P. V. Panat, Ph.D. thesis (University of California at Berkeley, 1972) (unpublished).

[21]R. A. Ferrell and D. J. Scalapino, Phys. Rev. Lett. 29, 413 (1972).