Comments and Addenda

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in **Physical Review Letters** *and are not appropriate for regular Articles. It includes only the following types of communications: (1) comments on papers previously published in* **The Physical Review** *or* **Physical Review Letters**; *(2) addenda to papers previously published in* **The Physical Review** *or* **Physical Review Letters**, *in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section should be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.*

Anisotropic Superfluidity in ^{3}He: Consequences of the Spin-Fluctuation Model

W. F. Brinkman
Bell Telephone Laboratories, Incorporated, Holmdel, New Jersey 07733

P. W. Anderson
Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey 07974
(Received 22 May 1973)

Recently we have shown that an anistropic Anderson-Morel state may be stabilized by feedback effects in the effective interaction arising from introduction of a gap in the spin-fluctuation spectrum. In this paper we determine the general form of the fourth-order term in the free energy owing to the above mechanism and use this result to make predictions regarding the phase diagram in the presence of a magnetic field and as a function of pressure.

In a recent letter[1] the present authors proposed a mechanism for stabilizing an anisotropic Anderson-Morel[2] (AM)-type state as the A phase of ^{3}He below 2.7 °mK.[3] The stabilization results from the fact that the introduction of a BCS-type gap changes the nature of the low-energy excitation spectra of the liquid. The most important low-energy excitations, the spin fluctuations, are assumed to give rise to the attractive interaction between two ^{3}He atoms.[4] Changes in the spin-fluctuation spectrum consequently affect the effective interaction and, depending on the type of state, enhance or reduce the free energy. It was shown[1] that, if the pairing is such that the spins of the pairs are in a triplet state with $S_z=0$, the effective interaction is in fact enhanced. In contrast, for the isotropic Balian-Werthamer (BW) state the interaction is reduced. The net result is that the AM state is stabilized by this feedback effect, provided the spin-fluctuation contribution to the free energy is large enough. An estimate made from the properties of the normal state indicated that stabilization was indeed possible.

In this Addendum we determine the form of fourth-order term which the above spin-fluctuation effect introduces into the free energy. Then, *assuming* that all other contributions to the free energy are adequately described within weak-coupling theory, we deduce a value for the magnitude of the spin-fluctuation contribution to the free energy from the discontinuity in the specific heat as determined by Webb *et al.*[5] Having determined the spin-fluctuation parameter it is then possible to calculate the phase diagram in a magnetic field in the same way that Ambegaokar and Mermin[6] did in the weak-coupling limit. Our results indicate (a) that there are only two transitions, (A_1, A_2); (b) that the two transition temperatures vary linearly with magnetic field and their variation is such that the slope of the second transition A_2 is −0.5 that of the first; (c) that the specific-heat discontinuity ΔC_v at the A_1 transition is 0.6 that of the normal-state specific heat C_N. The remaining discontinuity at A_2 brings C_v up to $2.65-2.9C_N$, as in the absence of a magnetic field. At the present time it appears that the best one can say is that the experimental data are not inconsistent with these predictions.[7] A definitive observation of the third transition predicted by Ambegaokar and Mermin[6] would clearly contradict the present results. However, it is not clear at this time whether a third transition near A_2 has been observed.

In order to obtain the above results we consider first the general form of the free energy that is fourth order in the order parameter $\Delta_{\alpha\beta}(\vec{k}) = \langle a^\dagger_{k\alpha} a^\dagger_{-k\beta}\rangle$.[8] ($a^\dagger_{k\alpha}$ creates a ^{3}He atom in the state k with spin α.) To do this, we introduce the vector $\vec{d}(\vec{k})$ such that $\vec{d}(\vec{k})\cdot\vec{\sigma}\sigma_y = \Delta(k)$.[9] By restricting ourselves to p-wave solutions we can write

$$d_i(\vec{k}) = d_{\alpha i}\hat{k}_\alpha , \tag{1}$$

where $\hat{k}$ is a unit vector in the direction of $\vec{k}$. The $d_{\alpha i}$ transforms like a vector with respect to the index α under space rotations and, similarly, like a vector under spin rotations with respect to the index i. (Spin-orbit coupling is neglected here.) One then considers the irreducible representations of the group of all spin and space rotations formed from the product $d_{\alpha i} d_{\beta j}$. There are five such irreducible representations and one fourth-order invariant is obtained from each. After some simplification the resulting expression for the free energy reduces to

$$F^4 = \frac{a_1}{2}\left| \sum_{\alpha i} d_{\alpha i}^2 \right|^2 + \frac{a_2}{2} d^*_{\beta i} d^*_{\beta j} d_{\alpha i} d_{\alpha j} + \frac{a_3}{2} d^*_{\alpha i} d^*_{\beta i} d_{\alpha j} d_{\beta j}$$

$$+ \frac{a_4}{2}\left(\sum_{\alpha i} |d_{\alpha i}|^2 \right)^2 + \frac{a_5}{2} d^*_{\alpha i} d^*_{\beta j} d_{\alpha j} d_{\beta i} . \tag{2}$$

The a_i's are phenomenological constants which must either be determined experimentally or from a microscopic calculation. We proceed via the following argument. In weak-coupling theory the form of the fourth-order invariant is given.[6] In fact, in weak coupling, letting $a_1 = -s$, we have that $a_2 = a_4 = a_5 = -a_3 = 2s$. Normally, corrections to these relations are expected to be small: $O[(T_c/\epsilon_F)^2]$ (ϵ_F is the Fermi energy). However, the spin-fluctuation effect previously calculated[1] was shown to give rise to corrections of first order. Therefore, we calculate the change in a_i due to the spin fluctuations and *assume* that all other deviations from weak-coupling theory are unimportant. The simplest way to do this is to calculate the spin-fluctuation effect, using our previous results for a sufficient number of states to uniquely determine the variation of the a_i. We introduce the parameter

$$\delta = (\Delta F^s)_{\rm BW}/(\Delta F^0)_{\rm AM\text{-}BW} ,$$

where $(\Delta F^s)_{\rm BW}$ is the fourth-order contribution to the free energy due to spin fluctuations in the BW state and $(\Delta F^0)_{\rm AM\text{-}BW}$ is the difference in the fourth-order free energy of the AM and BW states in weak-coupling theory.[1] After some algebra we find that the values of a_1 and a_2 remain unchanged while $a_3 = (-s)(2+\delta)$, $a_4 = s(2+\delta)$, and $a_5 = s(2-\delta)$. A value for δ can be obtained by noting that the discontinuity in specific heat is proportional to the reciprocal of the coefficient of the fourth-order term. Therefore, by using the fact that the weak-coupling theory gives a discontinuity equal to the BCS result, i.e., that $\Delta C_v/C_N = 1.43$ for the BW state, we can write

$$\frac{\Delta C_v}{C_N} = \frac{5}{3}\frac{(1.43)}{(2-\delta)} . \tag{3}$$

The experimental value of $\Delta C_v/C_N$ appears to be somewhat uncertain. Webb *et al.*'s[5] best value is 1.65, but their extrapolation of $C_v(T)$ to T_c gives 1.9. The corresponding values of δ are 0.56 and 0.7. Both are sufficient to stabilize the AM state.

The effect of the presence of a magnetic field on the expression for the free energy has been discussed previously in Ref. 6. We simply adopt their result and consider the expression for the free energy when the order parameter is of the form

$$\Delta(\vec{k}) = (k_x + ik_y)\begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{pmatrix} . \tag{4}$$

If $\Delta_1 = \Delta_2$ the above state is the original AM state. It is an $S_{z'} = 0$ state with the z' axis in the plane perpendicular to the field. Inserting this form for Δ, the free energy reduces to

$$F = (t - \eta h)|\Delta_1|^2 + (t + \eta h)|\Delta_2|^2 - 2s\delta|\Delta_1|^2|\Delta_2|^2$$
$$+ 2s(|\Delta_1|^4 + |\Delta_2|^4) ,$$

where $t = (T - T_c)/T_c$, h is the magnetic field in units of T_c and η should be regarded as a parameter of the theory. As in Ref. 6, the A_1 transition occurs at $T/T_c = 1 + \eta h$. However, because of the coupling between Δ_1 and Δ_2 the second transition occurs at

$$\frac{T}{T_c} = 1 - \eta h\left(\frac{1-\delta/2}{1+\delta/2}\right)$$
$$\simeq 1 - 0.5\eta h .$$

The discontinuity in the specific heat at A_1 can be obtained by again comparing fourth-order terms. The result,

$$\left.\frac{\Delta C_v}{C_N}\right|_{A_1} = \frac{5}{12}\left(\frac{C}{C_N}\right)_{\rm BCS} \simeq 0.6 ,$$

is independent of the spin-fluctuation contributions, i.e., is given by weak-coupling theory. It is straightforward to verify that the discontinuity in specific heat at the A_2 transition is such that the sum of the two discontinuities equals the discontinuity in zero field.

Experimentally, it appears that it is presently impossible to determine the direction of the mean temperature of the A_1 and A_2 transitions.[7] Crude estimates of the specific-heat discontinuities, however, appear to be consistent with the above results.[10]

Finally, we consider the question of whether the AM state being used here is actually the lowest state for the deduced value of δ. We can show the following. There are only four distinct unitary solutions to the gap equation. Three of the solutions are those with $d_{\alpha i}$ proportional to one-, two-, and three-dimensional unit matrices. The three-dimensional solution is the BW state; the two-dimensional state is the AM state used by BW[9] and one of us in recent calculations.[11] The one-dimensional state is a new solution discovered simultaneously by the present authors and others.[8,12] The fourth solution is the AM state used in the above calculations [$\Delta_1 = \Delta_2$ in Eq. (4)]. It is the lowest state among these four states when $\delta > \frac{1}{4}$. It can also easily be shown that it is the lowest state among all states, nonunitary included, if δ is sufficiently large. We have extensively looked for other more stable nonunitary states with $\delta \sim \frac{1}{4}$ but have not found any. Therefore, we conclude that we have the correct state, given that the parameters in the free energy vary as assumed above.

Finally, we should like to comment on what might happen if the main effect of pressure is to enhance the spin-fluctuation effect. As long as the system is in the AM state as δ is reduced with decreasing pressure, the discontinuity in specific heat is decreased until the volume becomes such that $\delta = \frac{1}{4}$. At this point the system will change over to the BW state and the discontinuity will again increase. There should be no observable change in dT_c^A/dP at this transition point; the first-order B transition between the AM and BW states would just touch the $T_c^A(P)$. The discontinuities in entropy and volume at the first-order transition between the AM and BW states should grow in proportion to $(T_c^B - T_c^0)^2$, where T_c^0 is the critical point where the three phases meet. The slope of the first-order transition dP/dT involves the sixth-order terms in an expansion of the free energy in powers of the order parameter, and consequently is difficult to estimate. Since the properties of the phase diagram where the AM and BW states join are consequences of thermodynamics, they do not depend explicitly on the spin-fluctuation model. However, if the lower-temperature anomaly in the thermal-conductivity measurements by Wheatley and coworkers[13] is indeed to be associated with the B transition observed in the compression experiments, it would be rather encouraging.

Note added in proof. The latter statement has now been established experimentally by J. C. Wheatley and his group.

The authors would like to acknowledge stimulating correspondence with N. D. Mermin and a number of useful conversations with C. M. Varma and N. R. Werthamer. We also would like to acknowledge many useful explanations of the experimental data by D. D. Osheroff and several discussions with J. C. Wheatley as well as the Cornell experimental group.

[1]P. W. Anderson and F. W. Brinkman, Phys. Rev. Lett. 30, 1108 (1973).
[2]P. W. Anderson and P. Morel, Physica 26, 671 (1960); Phys. Rev. 123, 1911 (1961).
[3]D. D. Osheroff, W. J. Gully, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 29, 920 (1972).
[4]A. Layser and D. Fay, Intern. J. Mag. 1, 135 (1971).
[5]R. E. Webb, T. J. Greytak, R. T. Johnson, and J. C. Wheatley, Phys. Rev. Lett. 30, 210 (1973).
[6]V. Ambegaokar and N. D. Mermin, Phys. Rev. Lett. 30, 81 (1973).
[7]W. J. Gully, D. D. Osheroff, D. T. Lawson, R. C. Richardson, and D. M. Lee (unpublished).
[8]N. D. Mermin and G. Stare have recently written down similar results.
[9]R. Balian and N. R. Werthamer, Phys. Rev. 131, 1553 (1963).
[10]D. D. Osheroff (private communication).
[11]P. W. Anderson, Phys. Rev. Lett. 30, 368 (1973).
[12]G. Barton and M. A. Moore (unpublished).
[13]Reported in an invited talk at the Washington APS meeting.