Quantum-Mechanical and Experimental Study of the Excitation of the $2\,^1P$ State of He by Electron Impact at 29–40 eV

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The close-coupling approximation (c.c.), the Vainshtein-Presnyakov-Sobelman approximation (VPS), and various first-order approximations, including the first Born approximation (B), are compared to accurate normalized experimental differential cross sections (estimated error less than 20%) for excitation of the $2\,^1P$ state of helium for impact energies 29.6, 34, and 40.1 eV, and scattering angles 3°–138°. The most accurate new measurements are those at 29.6 and 40.1 eV. These measurements were normalized to the experimental integral cross sections of Donaldson *et al*. The experimental differential cross sections of Hall *et al*. are in good agreement with these present measurements. All the calculations except B include electron exchange. The VPS has the most accurate magnitude of the present calculations at small scattering angles but it is still too large (by a factor which at 45° is about 2.5 at 29.6 and 34 eV and is about 1.5 at 40.1 eV). The c.c. has the most accurate magnitude at large scattering angles and the most accurate angular dependence at all scattering angles, but it is not in good agreement with the measurements at the largest scattering angles.

I. INTRODUCTION

Excitation of the $2\,^1P$ state of He by electron impact at impact energies of 29–40 eV (the energy range 8–19 eV above threshold) has been studied in many ways. The integral cross sections have been measured in absolute units by measuring the intensities of spectral lines excited by electron impact.[1-4] Less-reliable estimates of the integral cross sections had earlier been obtained by extrapolating the integral cross sections for excitation of the $n\,^1P$ states with $n \geq 3$.[5-7] The differential cross section as a function of angle has been measured using electron-impact spectrometers.[8-10] Experimental work done at lower- and higher-impact energies than the 29–40 eV region under study here is also discussed in these references, and further references to such work may be found in several places.[11-13] Many Born-approximation calculations have been carried out, culminating most recently in very accurate calculations of the generalized oscillator strengths.[13-16] The Born-approximation integral and differential cross sections may easily be obtained from these generalized oscillator strengths.[9,13-15] However, the Born approximation is not expected to be accurate at such a low-impact energy because of the assumptions involved.[16] There have been some calculations of the integral[17-26,9] and differential[18,9,22,23,26] cross sections using theories better than the Born approximation. These theories have sometimes been applied in the 29–40 eV energy region, but most of the work has been directed to higher energies. Inokuti has recently given a review of the theory.[16]

Prior to the present work and that of Hall *et al.*,[10] measurements of the differential cross section for excitation of the $2\,^1P$ state in the energy range 29–40 eV have been limited to scattering angles $\theta \leq 80°$.

Recently, Opal and Beatty[27] and Crooks and Rudd[28] have extended to angles as large as 150° the angular range over which differential cross sections for excitation of the $2\,^1P$ state have been measured for impact energies of 50 eV or higher. In the present article we report new experimental differential cross sections for the excitation of the $2\,^1P$ state at 29.6–40.1 eV. These differential cross sections are more accurate and cover a wider angular range (3°–138°) than those previously reported by ourselves and co-workers in this energy range.[8,9] Very recently, Hall *et al.*[10] measured the differential cross sections for excitation of the $2\,^1P$ state in the 10°–125° angular range at 29.2 and 39.2 eV. They normalized their cross sections to the absolute scale by determining the over-all instrumental efficiency of their apparatus with the help of the absolute elastic-scattering differential cross section of Andrick and Bitsch.[29]

So far there have been three sets of close-coupling calculations on excitation of the $2\,^1P$ state of He. The calculations of Vainshtein and Dolgov[21] included the $1\,^1S$ and $2\,^1P$ states but neglected exchange of the scattering electron. They did calculations only for the lowest-few partial waves and do not present enough information for the calculation of differential cross sections. The calculations of Burke *et al.*,[24,25] included the $1\,^1S$, $2\,^3S$, $2\,^1S$, $2\,^3P$, and $2\,^1P$ states and exchange, but were limited to energies within 1 eV of threshold, and to S, P, and D waves, except for calculations in whose expansions the ground state was not included. The calculations of Chung and Lin[30] were primarily directed to the study of excitations of higher-energy states at an impact energy of 100 eV. There have been very few close-coupling calculations on any systems in the intermediate energy range of interest here. Since the close-coupling theory was so successful at lower energies, it is of interest to see how successful it will be in the intermediate-energy range. Further, in only four cases have differential cross sections been presented for close-coupling calculations on $S \to P$ excitations for electron scattering by any atom; these were for the $1\,^2S \to 2\,^2P$ excitation of H[31], the $4\,^2S \to 4\,^2P$ excitation of K,[32] the $3s$-$3p$ excitation of Na,[33] and the $6s$-$6p$ excitation of Hg.[33] In the present article we present differential cross sections for the $1\,^1S \to 2\,^1P$ excitation of He from close-coupling calculations, including the $1\,^1S$ and $2\,^1P$ states and electron exchange at 29, 34, and 39 eV.

While simpler perturbation treatments of electron scattering are less appropriate at 29–40 eV than at higher energies, they have achieved some success at intermediate energies as low as 34 eV. The method of Vainshtein, Presnyakov, and Sobelman (VPS)[34,35] has been shown to give good integral cross sections even at the energies considered in this article.[34-36] The Born approximation (B), the Born-Ochkur-Rudge approximation (BOR), and some similar theories have been shown to predict the angle dependence of the small-angle differential cross sections accurately, even at intermediate energies.[9] In this article we present calculations in the VPS, B, BOR, and related approximations for the differential cross sections at 29, 34, and 39 eV.

II. EXPERIMENTS

A. Measurements of Differential Cross Sections in Arbitrary Units at 29.6 and 40.1 eV

A recently constructed apparatus[37] featuring an electron beam crossed with a molecular beam was utilized for the measurements at 29.6 and 40.1 eV impact energies. The He beam was generated by a multichannel capillary array and the signal was detected by pulse-counting techniques. The coverage of scattering angles obtainable with this instrument is $-15°$ to $+140°$. For the present experiments, the angular resolution[38] was about 2°, the solid angle subtended by the detector apertures at the scattering center was 6.8×10^{-4} sr, and the energy resolution was about 45 meV.

The intensity of scattered electrons as a function of energy loss was measured in the neighborhood of the $2\,^1P$ feature. At a given impact energy and fixed scattering angle, repetitive energy-loss scans were recorded in a multichannel scaler until an adequate signal was obtained. The scattering angle was then changed by intervals of about 5°. The background was well defined and the scattering intensities were assumed proportional to the peak heights (minus the background), since the peak shape was independent of scattering angle; occasional checks made by integrating over the whole peaked feature (minus the background) gave the same results. The angular coverage was achieved in a few hours, during which the instrumental conditions did not change significantly; these conditions were checked by returning to a reference scattering angle (30°) periodically during the measurements.

The angular distribution of the intensity of scattering was also determined in experiments which measured the intensity at only four energy-loss points at each scattering angle with a single-channel scaler. One point corresponded to the background region and the other points were the peak and points 5 meV below and above the peak. By this method, the angular coverage was achieved in a shorter time period, during which no observable instrumental drift occurred. These measurements were repeated with different instrumental tunings (under tuning we include all instrumental conditions that could influence the measurements) and no differences were found. The statistical error in the measured intensity was less than 10%.

Special attention has to be paid to the experimental conditions; otherwise serious deviations can be introduced into the angular distributions. Besides the trivial points of keeping all control voltages, external field and surface conditions, detector efficiency, etc., independent of scattering angle and time, the most serious source of error could come from the changing of scattering geometry with scattering angle. The basic requirement to avoid this latter error is to have the electron-beam axis and the signal-view-cone axis in the same plane, crossing at the same point independent of scattering angle. This was achieved for the present instrument by careful alignment utilizing sensitive height gauges and optical tech-

TABLE I. Error estimation at 29.6 and 40.1 eV impact energies.

Type of error	Error in % From 10° to 90°	From 3° to 10° From 90° to 138°
Error in experimental intensity measurement [a]	10	15
Error due to change of scattering geometry with angle	5	5
Normalization errors		
(i) Error in the optical excitation function	8	8
(ii) Error due to the extrapolation of DCS	5	5
Total errors [b]	15	18

[a] Includes statistical error in the signal and reproducibility of data points.
[b] Square root of sum of squares of component errors.

niques and by measuring the electron-beam current periodically during the experiment with precisely located Faraday cups at 0° and 90°. At the beginning of the experiment the electron-beam current to these two Faraday cups was optimized to assure correct axial position. There is, however, some uncertainty in relating the scattering intensities to the differential cross sections even if the axial alignments are perfect. This is due to the fact that the distribution of scattering points, the solid angle of signal acceptance associated with these points, the electron-beam density distribution as viewed from the detector, and the DCS's within the signal view cone all change with scattering angle. In other words, the "effective path length" changes somewhat with scattering angle even in a beam/beam experiment. We cannot estimate the "effective-path-length" corrections accurately, but preliminary calculations[39] carried out with simplified beam-density distributions indicate that this error is not greater than a few per cent. For the purpose of estimating the total error in Table I, it was taken as 5%.

The symmetry of the scattering intensity was checked in the angular region −15° to +15°. The intensity was found to be symmetric within the scatter of experimental data.

The impact energy was calibrated by observing the 19.35-eV resonance in elastic scattering by He at 90°. The contact potential was thus found to be +0.60 eV.

B. Normalization of Experimental Differential Cross Sections at 29.6 and 40.1 eV

The measurements yield the differential cross sections in arbitrary units in the 3°–138° range with estimated errors of about 15% (see Table I and Sec. II A for summary of errors). The results were extrapolated to 0° and 180° using quantum-mechanical calculations as a rough guide. The differential cross sections were then integrated over the whole angular range to obtain the integral cross section. Studies of the effect of using different methods for the extrapolation to 180° on the integral cross sections have been presented previously.[9,40] The estimated error due to extrapolation (5%) listed in Table I was based on a similar study. The extrapolation error has been reduced as compared to our previous measurements, since we now have to extrapolate only over the small angular regions 0°–3° and 138°–180° and the differential cross section is very small in the latter region. Although the integrated cross section is expected to change very little even for extreme choices of extrapolation, we have indicated precisely what values were used for our integration by asterisks in Figs. 2 and 4. The integral cross sections were then normalized to the experimental integral cross sections (with estimated errors of 8%) of Donaldson *et al.*[3] These integral cross sections were taken as $0.129a_0^2$ and $0.243a_0^2$ at 29.6 and 40.1 eV, respectively. For comparison the values obtained by Hall *et al.*[10] by arbitrarily extrapolating their normalized differential cross sections (discussed in Sec. I) to 0° and 180° and integrating are $0.14a_0^2$ and $0.24a_0^2$ at 29.2 and 39.2 eV, respectively.

C. Cross Sections for Other Processes Determined Using New Apparatus

The experiments also yielded the ratios of the differential cross sections for elastic scattering and excitation of the $2\,^3S$, $2\,^3P$, and $2\,^1S$ states to the differential cross section for excitation of the $2\,^1P$ state at both impact energies. These ratios plus the differential cross sections in absolute units for excitation of the $2\,^1P$ state yield the elastic scattering differential cross section and the differential cross sections for excitation of the other $n=2$ states in absolute units. These other cross sections will be presented separately.[38,41]

D. Measurement and Normalization of Differential Cross Section at 34 eV

One set of differential cross sections for excitation of the $2\,^1P$ state has already been published at 34 eV impact energy.[9] A second set of data was taken at 29, 34, and 39 eV using another apparatus and techniques similar to those[42,43] used for the previous 34-eV measurements. This second set of data was for the 20°–85° region. It was extrapolated, integrated, and normalized to the experimental integral cross sections of Jobe and St. John.[1] While the differential cross sections obtained in this way are less accurate than

TABLE II. Abbreviations used for scattering theory calculations.

Direct f	Exchange g	Scattering amplitude Total $f-g$
B	...	B (Born approx.)
B	OR	BOR (Born-Ochkur-Rudge approx.)
B	ORP	BORP (post BOR)
B	ORB. I	BORB or BORB. I (symmetrized BOR)
VPS	VPS	VPS (Vainshtein-Presnyakov-Sobelman approx.)
B	TVPS	BTVPS (Born-transferred-VPS)

those determined with the new apparatus and discussed in Secs. II A and II B above, these differential cross sections are the most-accurate data available at 34 eV. Comparison of the 29 eV data with the differential cross sections determined using the new (third) apparatus (Sec. II A; third set of data) indicates the second set of data is accurate for the angle dependence for scattering angles up to 50° within 20%. The agreement at 39 eV is worse, but still within a factor of 2 for the 20°–50° angular region. We will thus use the second set of data at 34 eV in this angle range for the purpose of comparison with quantum-mechanical theories, and it is presented in this article.

III. QUANTUM-MECHANICAL CALCULATIONS

A. First-Order Theories

The differential cross section $I(\theta)$ for excitation of the $2\,^1P$ state is given in the perturbation-theory calculations by[44]

$$I(\theta)=(k_m/k_0)|A_{m0}|^2\,, \tag{1}$$

where $\vec{k}_0$ and $\vec{k}_m$ are the initial and final wave-number vectors of the scattering electron, and

$$k_m^2=k_0^2-2\Delta E_m\,, \tag{2}$$

where $\Delta E_0=0$, $\Delta E_m=0.7797$. The scattering amplitude A_{m0} is given by

$$A_{m0}=f-g\;. \tag{3}$$

The direct scattering amplitude f is given by the Born approximation (B) or the Vainshtein-Presnyakov-Sobelman approximation (VPS). The exchange-scattering amplitude g is set equal to zero, is given by the prior, post, or symmetrized Ochkur-Rudge approximation (OR, ORP, or ORB.I, respectively), or is given by the VPS theory or the transferred VPS theory (TVPS). The various combinations considered here are summarized in Table II.

For excitation of the $2\,^1P$ state, cross sections were calculated in the B, BOR, BORP, BORB, VPS, and BTVPS approximations using the accurate generalized oscillator strengths of Kim and Inokuti, as described in Ref. 9. Calculations were also performed using the Ochkur method,[9] but these will not be presented here because the results for that method turned out to be very similar to the results for the BORB method.

B. Close-Coupling Calculations

The close-coupling calculations presented here include the $1\,^1S$ and $2\,^1P$ states in the wave-function expansion and they include electron exchange. Thus these are 2-state calculations.[45] Calculations were done for total angular momentum $L\leq 9$. For $L=0$, they involve two channels and for $L>0$ they involve three channels.[45] This type of calculation is sometimes referred to as the "strong-coupling" approximation. Except for not including the $2\,^1S$, $2\,^3S$, and $2\,^3P$ states in the wave-function expansion, the calculation of the reactance matrix and the integral cross sections follow the procedures[25] and use the programs[46] described in detail elsewhere.

The transition matrix $\vec{T}$ is obtained from the reactance matrix $\vec{R}$ using

$$\vec{T}=(\vec{1}-i\vec{R})^{-1}(\vec{1}+i\vec{R})-\vec{1}\;. \tag{4}$$

and the differential cross section is then calculated using[47–49]

$$I(\theta)=\sum_{J=0}^{\infty}B_J P_J(\cos\,\theta)\,, \tag{5}$$

where

$$\begin{aligned}B_J=-(4k_1^2)^{-1}\sum_{L=0}^{\infty}\sum_{K=|J-L|}^{J+L}&Z(L,L,K,K;0J)\\ \times[&Z(L-1,L,K-1,K;\;1J)\;\mathrm{Re}\;T_{21}^L\;T_{21}^{K*}\\ &+Z(L-1,L,K+1,K;\;1J)\;\mathrm{Re}\;T_{21}^L\;T_{31}^{K*}\\ &+Z(L+1,L,K-1,K;\;1J)\;\mathrm{Re}\;T_{31}^L\;T_{21}^{K*}\\ &+Z(L+1,L,K+1,K;\;1J)\;\mathrm{Re}\;T_{31}^L\;T_{31}^{K*}]\,,\end{aligned} \tag{6}$$

where the Z quantities are defined in Ref. 47.

From this expression one obtains the integral cross section[50]

$$Q=4\pi B_0=\frac{\pi}{k_1^2}\sum_{L=0}^{\infty}(2L+1)(|T_{21}^L|^2+|T_{31}{}^L|^2)\,. \tag{7}$$

As a check the differential and integral cross sections were also calculated from the more general formulas of Blatt and Biedenharn,[47] using an independent program.[51]

In practice, the sum over J in Eq. (5) was trun-

TABLE III. Differential cross sections $I(\theta)$ for excitation of the $2\,^1P$ state calculated by the $1\,^1S$-$2\,^1P$ close-coupling method assuming $T^J_{ij} = 0$ for $J > J_{max}$. [a]

E (eV)	θ (deg)	$I(\theta)(a_0^2/\mathrm{sr})$ $J_{max}=9$	$J_{max}=8$	$J_{max}=7$
29	0	3.13(−1)	3.06(−1)	2.93(−1)
	10	2.73(−1)	2.68(−1)	2.60(−1)
	20	1.84(−1)	1.84(−1)	1.84(−1)
	30	1.02(−1)	1.04(−1)	1.06(−1)
	45	3.58(−2)	3.58(−2)	3.66(−2)
	75	6.31(−3)	6.26(−3)	5.96(−3)
	90	3.93(−3)	4.01(−3)	3.97(−3)
	105	3.71(−3)	3.78(−3)	3.72(−3)
	135	8.31(−3)	8.00(−3)	8.19(−3)
	180	1.54(−2)	1.38(−2)	1.67(−2)
39	0	6.90(−1)	6.27(−1)	5.45(−1)
	10	5.75(−1)	5.35(−1)	4.78(−1)
	20	3.29(−1)	3.27(−1)	3.14(−1)
	30	1.32(−1)	1.41(−1)	1.49(−1)
	45	2.74(−2)	2.94(−2)	3.40(−2)
	75	6.40(−3)	5.73(−3)	4.15(−3)
	90	3.44(−3)	3.97(−3)	3.85(−3)
	105	2.65(−3)	2.98(−3)	2.95(−3)
	135	4.42(−3)	3.47(−3)	3.91(−3)
	180	8.68(−3)	3.97(−3)	1.12(−2)

[a] Numbers in parentheses are multiplicative powers of 10.

cated at $J = 18$ and the sums over L and K in Eqs. (6) and (7) were truncated at 9, since T^J_{ij} was assumed zero for $J > 9$. We tested this assumption by repeating the calculations, assuming $T^J_{ij} = 0$ for $J > 8$ and $J > 7$. In no case did the integral cross section for excitation of the $2\,^1P$ state change by more than 2%. The changes in the differential cross sections are illustrated in Table III. The changes are largest at scattering angles near to 0° and 180° and the tests show that for scattering angles close to these extremities higher values of total angular momentum J should be considered for convergence of the differential cross sections.

IV. RESULTS

Figure 1 shows the results of the present calculations for integral cross sections and compares them to the previous experimental results.[1–3, 52]

Figures 2–4 show the results of the present experiments and calculations for differential cross sections, and they also show the extrapolations which were used. For comparison the very recent values of Hall *et al.*[10] are also shown in Figs. 2 and 4. Since the present experimental results at 29.6 and 40.1 eV are more accurate than previously published[8, 9] results in this energy region and may be useful standards for future work, the numerical values are tabulated in Table IV.

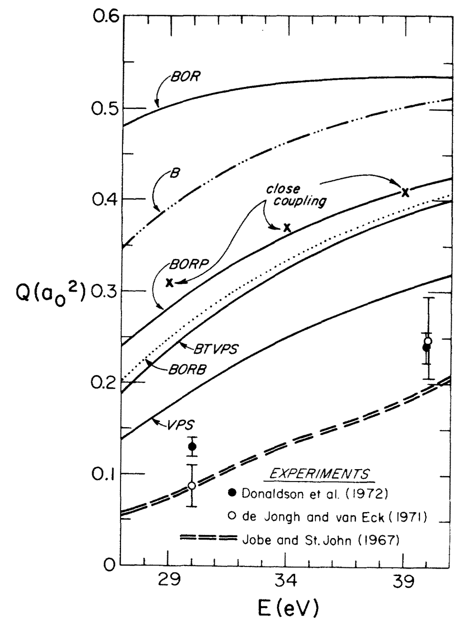

FIG. 1. Integral cross sections for excitation of the helium $2\,^1P$ state from three experiments (Ref. 1–3, as indicated) and seven calculations as functions of impact energy. The abbreviations used for the theories are BOR: prior Born-Ochkur-Rudge approximation, B: Born approximation, close-coupling: 2-state close-coupling approximation including exchange, BORP: post Born-Ochkur-Rudge approximation, BORB: symmetrized Born-Ochkur-Rudge approximation, BTVPS: Born-transferred-Presnyakov-Sobelman approximation, and VPS: Vainshtein-Presnyakov-Sobelman approximation (see Table II, sec. III, and Ref. 9 for further explanations of the theories).

V. DISCUSSION

A. Integral Cross Sections

The VPS approximation is in best agreement with the experimental integral cross section, but because of the large number[12] of approximations in this theory we believe this is a fortuitous result. The other first-order theories, which all use the Born approximation for the direct scattering amplitude, all give large overestimates of the cross sections, as we have previously shown.[9] We can conclude that merely adding a first-order exchange amplitude to the Born approximation does not correct the magnitude of the integral cross section.

Although the close-coupling approximation improves upon the Born approximation by including distortion of the scattering electron wave function from a plane wave state, complete electron exchange, back coupling to all orders, competition with elastic scattering, and some of the polarizability of the target (that due to the $2\,^1P$ state), it does not yield very much improvement in the in-

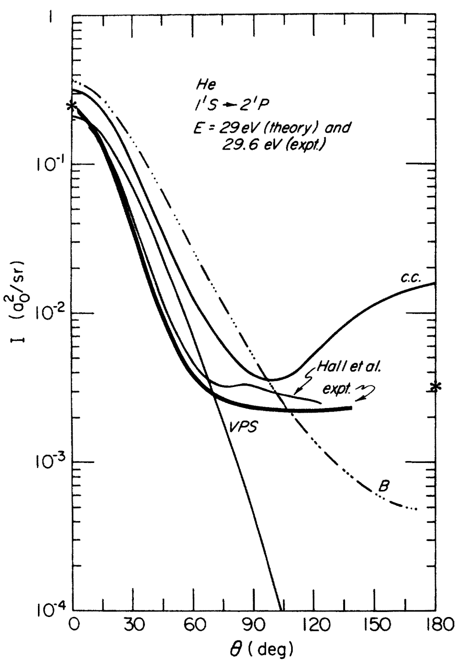

FIG. 2. Theoretical (curves) and experimental (thick curve) differential cross sections for excitation of the $2\,^1P$ state of helium at 29 eV (theory) and 29.6 eV (experiment) as functions of scattering angle. The solid lines are the close coupling (c.c.) and Vainshtein-Presnyakov-Sobelman (VPS) approximations. The dash-tripledot line is the Born approximation (B). The thick line is the present experimental result as discussed in Secs. II A and II B. The asterisks at 0° and 180° denote extrapolated values used for the calculation of integral cross sections in the normalization procedure. The experimental results of Hall *et al.*[10] at 29.2 eV are also shown for comparison.

tegral cross sections. Part of the error in the magnitude of the cross sections predicted by the close-coupling calculation could be in the bound-state wave function but that source of error does not appear likely to account for most of the error. Note that, except for $L = 0$, the close-coupling calculation is a three-channel calculation, since it includes the $1\,^1S$ channel and two out of three components of the $2\,^1P$ state which have the correct parity to be coupled to the $1\,^1S$ channel. Evidently it is necessary to improve on the three-channel description of the target, even to predict the magnitude of the intermediate-energy integral cross section for this process.

Although two-channel calculations were not carried out, they would be expected to give less-accurate results, barring accidental cancellation of errors. Since the $1\,^1S \to 2\,^1P$ excitation is a strongly allowed process, i.e., it is electric-dipole allowed, we expect that a two-state or two-channel representation of the target would be more appropriate than for other processes (such

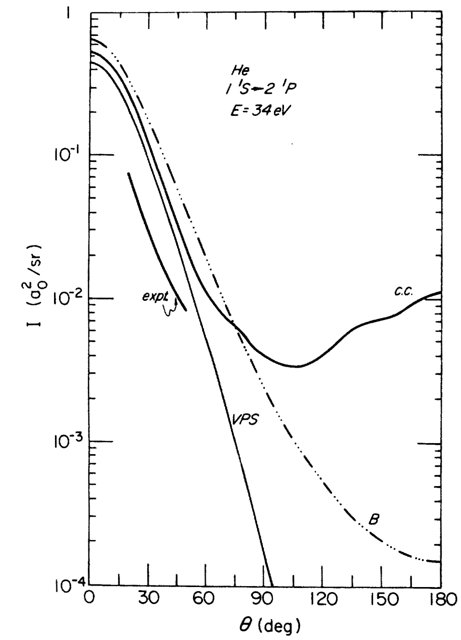

FIG. 3. Theoretical (curves) and experimental (thick curves) differential cross sections for excitation of the $2\,^1P$ state of helium at 34 eV as functions of scattering angle. The curves are explained in Fig. 2, except that the experiments are described in Sec. II D.

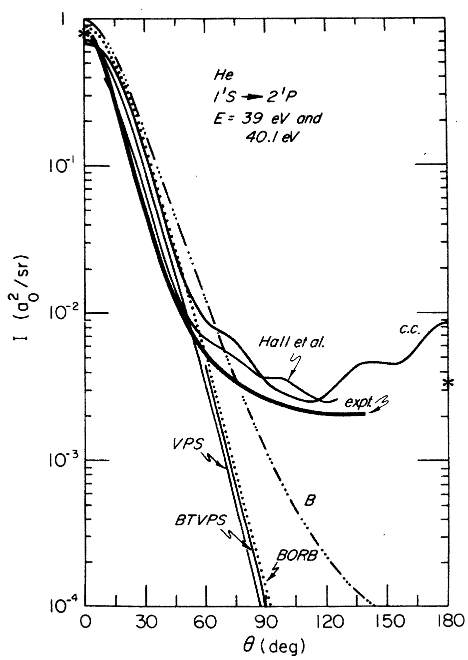

FIG. 4. Same as Fig. 2 except at 39 eV (theory), 40.1 eV (present experiment), and 39.2 eV (experiment of Hall *et al.*[10]); two additional calculations are shown: the dotted-line symmetrized Born-Ochkur-Rudge (BORB) approximation and the solid-line Born-transferred-Vainshtein-Presnyakov-Sobelman (BTVPS) approximation. The latter is not shown for $\theta < 60°$, since it agrees with the former within 10% there.

TABLE IV. Experimental differential cross sections in a_0^2/sr.

θ(deg)\E(eV)	29.6 eV	40.1 eV
3	0.230	0.792
5	0.216	0.707
10	0.169	0.525
20	0.083	0.157
30	0.037	0.0546
40	0.015	0.0196
50	0.0069	0.0091
60	0.0039	0.0055
70	0.0029	0.0040
80	0.0026	0.0031
90	0.0024	0.0027
100	0.0023	0.0024
110	0.0022	0.0022
120	0.0023	0.0021
130	0.0023	0.0021
138	0.0024	0.0022

as the $1\,^1S \to 2\,^1S$ excitation), where intermediate states are expected to be important.[53,54] Thus the present results indicate that the two-state and two-channel coupling approximations will not in general be useful tools for the prediction of the magnitude of the integral cross section at energies $1\frac{1}{2}-2$ times the threshold energy. Further, inclusion of the coupling and the competition between degenerate components of the final channel, e.g., using a three-channel approximation in the present case, does not correct the error.

B. Differential Cross Sections

Figures 2 and 4 show that there is good agreement between the present experimental results and those of Hall *et al.*[10] for the angular dependence of the differential cross sections. The largest discrepancies in the shapes of the curves are in the angular regions 10°–20° and 80°–125°. The generally excellent agreement between the two measurements, however, is reassuring since the measurements were performed with different types of instruments and completely independent normalization procedures.

Figures 2–4 show, as previously demonstrated,[9] that the Born approximation and the Born approximation augmented by various first-order exchange amplitudes predict the correct angle dependence (but predict too large a magnitude) for the differential cross section at small-momentum transfers (which correspond to small scattering angles). However, they predict orders-of-magnitude that are too small for differential cross sections at large-momentum transfer. The figures show that the VPS approximation leads to similar results, although the magnitudes of the small-angle differential cross sections are a little improved.

Figures 2–4 also show that the close-coupling-approximation differential cross sections have the qualitatively correct angle dependence at scattering angles of 90° or less (except very close to 0°, where these calculations are apparently not yet converged; see Sec. III B). However, the close-coupling approximation is not accurate at larger angles. This is most obvious in Fig. 2, where the close-coupling-approximation differential cross section has a large slope in the 115°–138° angular range, whereas the measurements are essentially flat. The situation is less clear at 40 eV, since Table III shows that larger values of J may be necessary for complete convergence in that case. The close-coupling approximation overestimates the differential cross section at all scattering angles. The success of the close-coupling approximation for the qualitative angle dependence at $\theta \leq 90°$ is an important conclusion, since it is the shape of the differential cross section in this angular range which has been shown to be important for identifying states in electron-impact spectroscopy.[42]

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