Possibility of Ω^--Hyperonic Atoms and the Electric Quadrupole Moment of the Ω^- Particle*

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(Received 13 June 1973)

The only relatively long-lived fundamental particle which may possess an electric quadrupole moment is the Ω^- hyperon, since it has spin 3/2. We consider the possibility that Ω^- hyperons may be captured into atomic orbits at some future date. If this possibility should be realized (in spite of great technical difficulties), then it may be possible to determine simultaneously the electric quadrupole moment Q_Ω and the magnetic dipole moment μ_Ω of the Ω^- hyperon. In the present paper, we have calculated the fine structure of the Ω^--Pb atom for reasonable values of Q_Ω and μ_Ω. The $n = 11$, $l = 10 \rightarrow n = 10$, $l = 9$ transition has been considered, since there is some indication from an experiment on antiprotonic atoms that the capture of the Ω^- by the nucleus will take place from a hydrogenic state with n of the order of 10 for lead, and $l \sim n - 1$. It is shown that for $Q_\Omega = 2 \times 10^{-26}$ cm^2, the quadrupole splittings should be resolvable experimentally. The magnetic dipole splittings have also been calculated.

I. INTRODUCTION

Although at present only about 30 Ω^- particles[1] have been observed,[2] we wish to envisage the possibility that at some future time Ω^- beams of at least moderate intensities may become available. If one considers that only about 15 years have elapsed between the observation of the first 50 antiprotons[3] and the detection of antiprotonic atoms by Bamberger *et al.*,[4] one may hope that in the not too distant future, the observation of Ω^--hyperonic atoms may actually be realized.

If such Ω^--hyperonic atoms should be observed, a very interesting and important possibility will occur, namely, the observation of a possible electric quadrupole moment Q_Ω of the Ω^- hyperon. Of course, a nonvanishing Q_Ω is possible because of the spin $\frac{3}{2}$ of the Ω^- particle, which is the only weakly decaying member of the SU(3) decuplet.

In a recent paper, Fox *et al.*[5] have reported the magnetic moment of the antiproton from an observation of a structure (splitting) in a transition in both lead and uranium antiprotonic atoms. The structure occurs in the $n=11, l=10$ to $n=10$, $l=9$ transition of these atoms. It turns out that the $\bar{p}$ is captured by the nucleus from the $n=9$ level, but the highest-energy transition $n=10 \rightarrow n=9$ is obscured by the $13 \rightarrow 11$ transition which is close in energy; therefore, the $n=11 \rightarrow n=10$ transition was used to detect the fine-structure splitting due to the magnetic moment of the $\bar{p}$.

In the absence of accurate information about the interaction of Ω^- with a nucleus, and for definiteness, we shall assume that the observed transition for the Ω^--hyperonic lead atom is also $n=11$, $l=10 \rightarrow n=10$, $l=9$. The results would not be very different if a neighboring transition were considered, and they are also expected to be similar for Ω^--lead and Ω^--uranium. It should be noted that according to the analysis of Martin[6] on K^- mesic atoms, one expects that on slowing down, the Ω^- will be captured into an orbit with principal quantum number $n \sim (m_{\Omega^-}/m_e)^{1/2} \approx 57$, for which the radius of the Ω^--Pb atom is of the order of the K shell radius of the ordinary (electronic) Pb atom. From the capturing level $n \sim 57$, the Ω^- cascades down into states with successively decreasing n values, and with l values close to n (classically circular orbits). Thus, it is not unreasonable that when nuclear capture does occur, it is from a state with the maximum value of l, i.e., $l=n-1$, as has been observed by Fox *et al.*[5] for antiprotonic atoms of lead and uranium.

Two effects contribute to the fine structure of the levels in Ω^--Pb atoms, namely the magnetic moment μ_Ω of the Ω^- and its electric quadrupole moment Q_Ω. There is little theoretical information on either of these quantities, and therefore we will make some reasonable assumptions, in order to obtain at least the order of magnitude of the effects (splittings) involved. It turns out, as will be shown below, that the more important effect is that of Q_Ω, provided that its value is not unreasonably small. As a crude assumption, we may take Q_Ω to be of the order of $[m_\Omega/m(\mathrm{Li}^7)]^{2/3} = 0.40$ times the quadrupole moment of the Li^7 nucleus (because of the approximate proportionality of $\langle r^2 \rangle$ and $A^{2/3}$). Since $Q(\mathrm{Li}^7) \approx -0.044$ b,[7] we will use in our calculation an estimate $Q_\Omega = 0.02$ b $= 2 \times 10^{-26}$ cm^2. The actual fine structure due to Q_Ω is proportional to Q_Ω. As concerns the magnetic moment μ_Ω, we shall assume, for definiteness for the anomalous part of the magnetic moment $g_1 = +1$. The splitting is proportional to $g_0 + 2g_1$, with $g_0 = +1$. Thus, even if g_1 were larger, say $g_1 = +2$, the effect would be increased by only a factor of $\frac{5}{3}$, and it would still be relatively small compared to the effect due to Q_Ω for $Q_\Omega = 2 \times 10^{-26}$ cm^2.

II. EVALUATION OF QUADRUPOLE FINE STRUCTURE

The Ω^- hyperonic atom is an interesting system, because the Ω^- particle has spin $S=\frac{3}{2}$. Thus, the fine structure is essentially different from that of ordinary electronic atoms with $S=\frac{1}{2}$. Obviously, each level n, l is split into a quartet of levels with $j=l+\frac{3}{2}$, $j=l+\frac{1}{2}$, $j=l-\frac{1}{2}$, and $j=l-\frac{3}{2}$.

The quadrupole interaction is due to the effect of the field gradient due to the nucleus V_{zz} on the quadrupole moment Q_Ω. (For simplicity, we assume that the nucleus has no magnetic or quadrupole moment, so that the nuclear hyperfine structure is absent. This situation is, of course, realized for the spin-0 nucleus Pb^{208}.)

The field gradient V_{zz} due to the nucleus is obtained by differentiating the field Ze/r^2, or rather the field along the z direction, namely, $E_z = Zez/r^3$. We thus obtain

$$V_{zz} = (2Ze/r^3)\,P_2(\cos\theta), \tag{1}$$

where θ is the angle between the radius vector $\vec{r}$ and the z axis, and P_2 is the Legendre polynomial.

It should be noted that, in analogy to the situation for K^--mesic atoms and antiprotonic atoms, the orbit at nuclear capture is so far inside the K shell of the atom, that the wave function of the Ω^- is hydrogenic to a very high accuracy. However, we must use the appropriate reduced mass $m_{\rm red}$, which for ${}^2m_{\Omega^-} = 1672.5$ MeV and a Pb^{208} nucleus becomes $m_{\rm red} = 1657.9$ MeV. Thus, we find

$$m_{\rm red}/m_e = 1657.9/0.5110 = 3244.4 \tag{2}$$

and the appropriate Bohr radius a_Ω is given by

$$a_\Omega = (5.292\times10^{-9})/3244.4 = 1.631\times10^{-12}\ \text{cm}. \tag{3}$$

For hydrogenic wave functions, the average value $\langle r^{-3}\rangle$ is given by[8]

$$\langle r^{-3}\rangle = [Z^3/n^3 l\,(l+\tfrac{1}{2})(l+1)]\,a_\Omega^{-3}. \tag{4}$$

The equivalent of the Rydberg unit is given by

$$\mathcal{R}_\Omega = 13.605\times(m_{\rm red}/m_e) = 44.14\ \text{keV}. \tag{5}$$

We first consider the lower states, having $n=10$, $l=9$. We find

$$\langle r^{-3}\rangle = \frac{82^3}{10^3(9)(9.5)(10)}\,a_\Omega^{-3} = 1.487\times10^{35}\ \text{cm}^{-3}. \tag{6}$$

It was pointed out by Blume[9] that the equation for the interaction energy contains besides $2Ze^2 \times\langle r^{-3}\rangle\,Q_\Omega$ [as obtained from Eq. (1)], an additional factor[10] $l/(2l+3)$, so that the energy $E_Q^{(0)}(n,l)$ is given by

$$E_Q^{(0)}(n=10, l=9) = 2Ze^2\,\langle r^{-3}\rangle\,Q_\Omega l/(2l+3)$$
$$= 4.822\times10^{-8}\ \text{erg} = 30.10\ \text{keV}, \tag{7}$$

where we have used Eq. (6) and $Q_\Omega = 2\times10^{-26}\ \text{cm}^2$. This quantity $E_Q^{(0)}$ must still be multiplied by appropriate J-dependent factors to obtain the actual energy shifts due to Q_Ω for the lower $(n=10, l=9)$ state.

Before proceeding to this calculation, we note that the zero-order energy of the transition $n=11 \to n=10$ is given by

$$E_{11} - E_{10} = 44.14\ \text{keV}\ (82)^2(\tfrac{1}{100} - \tfrac{1}{121})$$
$$= 2968 - 2453 = 515\ \text{keV}. \tag{8}$$

The $n=10$, $l=9$ level splits into four levels with $J=\frac{21}{2}$, $\frac{19}{2}$, $\frac{17}{2}$, and $\frac{15}{2}$, respectively.

By analogy with the theory of nuclear quadrupole hyperfine structure, we find that the J-dependent factor which multiplies $E_Q^{(0)}$ is given by[11]

$$D = \frac{3C(C+1) - 4L(L+1)S(S+1)}{8S(2S-1)L(2L-1)}, \tag{9}$$

where, of course, $L=9$, $S=\frac{3}{2}$, and the J-dependent quantity C is given by

$$C = J(J+1) - L(L+1) - S(S+1). \tag{10}$$

We thus obtain for $C(J)$; $C(\frac{21}{2}) = 27$, $C(\frac{19}{2}) = 6$, $C(\frac{17}{2}) = -13$, and $C(\frac{15}{2}) = -30$. The resulting values of $D(J)$ are as follows: $D(\frac{21}{2}) = +\frac{1}{4}$, $D(\frac{19}{2}) = -\frac{1}{3}$, $D(\frac{17}{2}) = -\frac{49}{204} = -0.2402$, and $D(\frac{15}{2}) = +\frac{35}{102} = +0.3431$.

Similarly, the upper level, with $n=11$, $L=10$, splits into four levels having $J=\frac{23}{2}$, $\frac{21}{2}$, $\frac{19}{2}$, and $\frac{17}{2}$, respectively. We note that for the upper level, $E_Q^{(0)}$ is smaller than the value given by Eq. (7), on account of the larger values of n and L in the denominator for $\langle r^{-3}\rangle$ [Eq. (6)]. We thus obtain

$$E_Q^{(0)}\,(n=11, L=10) = 30.10\times0.5643 = 16.99\ \text{keV}. \tag{11}$$

The actual energy levels (referred to the zero-order value) are given by $E_Q^{(0)}(n,L)D(J)$ in each case.

We thus obtain, for the upper levels,

$$E_Q(\tfrac{23}{2}) = 4.25\ \text{keV},$$
$$E_Q(\tfrac{21}{2}) = -5.52\ \text{keV},$$
$$E_Q(\tfrac{19}{2}) = -4.11\ \text{keV}, \tag{12}$$

and

$$E_Q(\tfrac{17}{2}) = 5.66\ \text{keV}.$$

For the lower levels we obtain, by means of Eq. (7),

$$E_Q(\tfrac{21}{2}) = 7.53\ \text{keV},$$
$$E_Q(\tfrac{19}{2}) = -10.03\ \text{keV},$$
$$E_Q(\tfrac{17}{2}) = -7.23\ \text{keV}, \tag{13}$$

and

$$E_Q(\tfrac{15}{2}) = 10.33 \text{ keV}.$$

These levels are shown schematically in Fig. 1, in which the most prominent transitions are also indicated (a, b, c, and d). The energy difference between the centroids of the upper and lower levels is 515 keV, as obtained from Eq. (8).

In order to obtain the relative intensities of the various transitions, we have used the expressions given in the book of Kuhn,[12] which were originally derived by Dirac.[13] We note that all transitions with $\Delta J = \pm 1$ and 0 are allowed. However, it turns out that for the five transitions with $\Delta J \neq \Delta L (= -1)$, the intensities are always less than 2% of that of the most prominent transition, namely, $J = \frac{23}{2} \to \frac{21}{2}$, which has been labeled as transition a. The four strong transitions are those for which $\Delta J = \Delta L = -1$, namely, $\frac{23}{2} \to \frac{21}{2}$ (transition a) (intensity taken as 1 arbitrarily), $\frac{21}{2} \to \frac{19}{2}$ (transition b; relative intensity $= 0.904$), $\frac{19}{2} \to \frac{17}{2}$ (transition c; relative intensity $= 0.816$), and $\frac{17}{2} \to \frac{15}{2}$ (transition d; relative intensity $= 0.737$).

In order to calculate the energies of the transitions a, b, c, and d, or rather the energy deviations ΔE_Q from the central value $\Delta E_0 = 515$ keV [see Eq. (8)], we use the results given in Eqs. (12) and (13), from which we obtain

$$\Delta E_Q(a) = \Delta E(\tfrac{23}{2} \to \tfrac{21}{2}) = 4.25 - 7.53 = -3.28 \text{ keV}, \tag{14}$$

and similarly

$$\Delta E_Q(b) = +4.51 \text{ keV},$$
$$\Delta E_Q(c) = +3.12 \text{ keV}, \tag{15}$$
$$\Delta E_Q(d) = -4.67 \text{ keV}.$$

These are the transition energy shifts in the absence of magnetic dipole fine structure (see Sec. III), and they are shown as the full lines in Fig. 2. The height of each line is proportional to the intensity of the transition, as calculated above. The complete pattern extends over an energy interval of $4.67 + 4.51 = 9.18$ keV, and thus the existence of the pattern, and probably the individual details, i.e., the existence of four separate lines, should be readily observable.

Of course, it should be noted that the very existence of four lines would demonstrate at once that the spin of the Ω^- is $S = \frac{3}{2}$, as is expected from the SU(3) theory. (For the antiproton with spin $\frac{1}{2}$, two lines are observed in the experiment of Fox *et al.*[5]) Incidentally, the smallest energy splittings, namely, those between transitions (a) and (d) and between (b) and (c) are both 1.39 keV. If these transitions are resolvable, then all four lines could be detected separately.

The preceding values for $\Delta E_Q(\alpha)(\alpha = a, b, c, d)$ are, of course, modified to some extent by the presence of the fine structure due to the magnet-

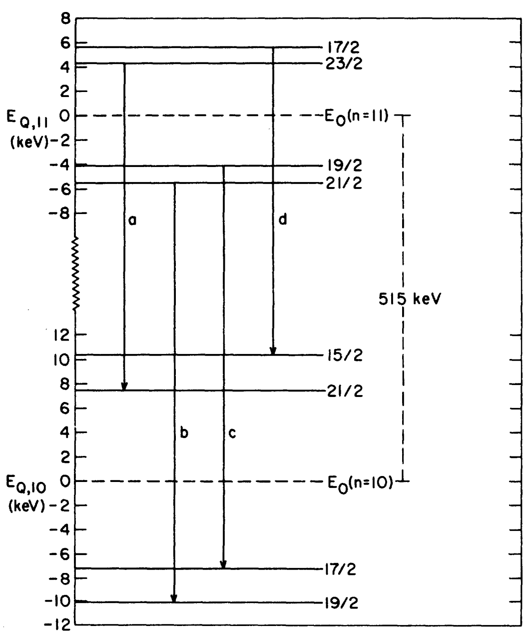

FIG. 1. Energy levels of the states $J = \frac{17}{2}, \frac{19}{2}, \frac{21}{2}$, and $\frac{23}{2}$ for $n = 11$, $l = 10$, and the levels of the states $J = \frac{15}{2}, \frac{17}{2}, \frac{19}{2}$, and $\frac{21}{2}$ for $n = 10$, $l = 9$ for a Ω^--Pb atom assuming only an electric quadrupole moment of the Ω^-, $Q_\Omega = 2 \times 10^{-26}$ cm^2 (i.e., assuming $\mu_\Omega = 0$). The zero-order energy difference between the $n = 11$ and $n = 10$ levels, $\Delta E_0 = 515$ keV, is indicated by the dashed vertical line [see Eq. (8)].

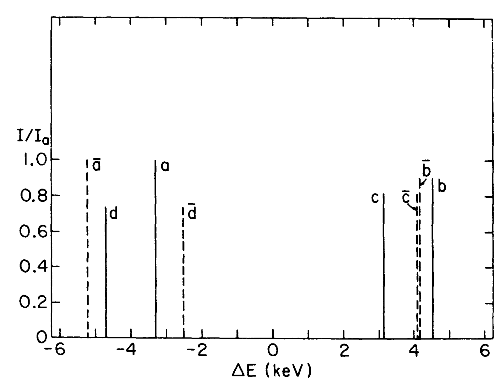

FIG. 2. Energy difference ΔE (with respect to $\Delta E_0 = 515$ keV) for the four transitions marked a, b, c, and d in Fig. 1 for $Q_\Omega = +2 \times 10^{-26}$ cm^2. The heights of the lines correspond to the intensities of the transitions relative to that of transition a, i.e., the values of I/I_a. The full lines marked (a, b, c, d) correspond to the values of ΔE_Q of Fig. 1, without magnetic interaction [Eqs. (14) and (15)]. The broken lines (marked $\bar{a}, \bar{b}, \bar{c}, \bar{d}$) correspond to the values of ΔE_{total}, including the interaction due to a magnetic moment [Eqs. (30)–(33)].

ic moment μ_Ω of the Ω^- particle, which will be estimated in Sec. III.

III. ESTIMATE OF MAGNETIC DIPOLE FINE STRUCTURE

A discussion of the fine structure due to the spin-orbit coupling is given, for example, in Kuhn's book.[14] Thus for a particle of spin $\vec{S}$, the energy associated with the spin-orbit coupling, after applying the relativistic Thomas factor of $\frac{1}{2}$, is given by

$$E' = \langle \xi(r) \rangle \vec{L} \cdot \vec{S}, \quad (16)$$

where $\xi(r)$ is given by

$$\xi(r) = -\frac{e}{2}\left(\frac{\hbar}{m_0 c}\right)^2 \frac{1}{r}\frac{dV}{dr}, \quad (17)$$

where V is the central potential ($=Ze/r$ in the present case) and m_0 is the rest mass of the particle. Equation (17) actually differs by a factor $\hbar^2$ from Eq. (III.53) of Kuhn, because we define L and S as the quantum numbers associated with the orbital and spin angular momentum, respectively, i.e., $1/\hbar$ times the quantities L and S considered by Kuhn.

Upon using $V(r) = Ze/r$, Eq. (17) becomes

$$\langle \xi(r) \rangle = +\frac{e^2}{2} Z \left(\frac{\hbar}{m_0 c}\right)^2 \langle r^{-3} \rangle . \quad (18)$$

where the average value of r^{-3} is to be taken for the appropriate hydrogenic wave function. For m_0, we use the reduced mass $m_{\text{red}} = 1657.9$ MeV appropriate for the Ω^--Pb atom.

As was pointed out in Ref. 5, if the particle considered has an anomalous magnetic moment $3g_1\mu_N$, where μ_N is a nuclear magneton (appropriate to the mass $m_0 = m_{\text{red}}$), then E' is multiplied by a factor $(g_0 + 2g_1)$, where $g_0 = +1$. We note, of course, that with $m_{\text{red}} = 1657.9$ MeV $= 1.767 m_p$ (m_p is the mass of proton), the "nuclear magneton" $e\hbar/2m_{\text{red}}c$ is appreciably smaller (by a factor 1.767) than the proton magneton which is $e\hbar/2m_p c$.

Thus we shall use for the energy E_{mag} due to the magnetic dipole moment, the expression

$$E_{\text{mag}} = \langle \xi(r) \rangle \vec{L} \cdot \vec{S} (g_0 + 2g_1), \quad (19)$$

with $\langle \xi(r) \rangle$ given by Eq. (18). In Eq. (19), the appropriate value of $\vec{L} \cdot \vec{S}$ is obtained as usual from the relation

$$\vec{L} \cdot \vec{S} = \tfrac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$$
$$= \tfrac{1}{2}[J(J+1) - L(L+1) - S(S+1)] = \tfrac{1}{2}C, \quad (20)$$

with C as defined in Eq. (10).

With Eq. (4) for $\langle r^{-3} \rangle$, we obtain

$$\langle \xi(r) \rangle = +\frac{e^2}{2} \frac{Z^4}{n^3 L(L+\frac{1}{2})(L+1)} \frac{(\hbar/m_0 c)^2}{a_\Omega^3} . \quad (21)$$

Now $(\hbar/m_0 c)/a_\Omega = \alpha$, the fine-structure constant, and $e^2/2a_\Omega = \mathcal{R}_\Omega = 44.14$ keV, so that

$$\langle \xi(r) \rangle = \mathcal{R}_\Omega \frac{Z^4 \alpha^2}{n^3 L(L+\frac{1}{2})(L+1)} . \quad (22)$$

(Incidentally, we note that l and L have been used interchangeably for the orbital angular momentum quantum number.)

It is convenient to define the quantity $\bar{C}$ (the quantum-mechanical equivalent of the cosine of the angle between $\vec{L}$ and $\vec{S}$) as follows:

$$\bar{C} = \frac{\vec{L} \cdot \vec{S}}{LS} = \frac{1}{2}\frac{C}{LS} = \frac{C}{3L}, \quad (23)$$

in view of $S = \frac{3}{2}$. Then the magnetic energy becomes

$$E_{\text{mag}}(J) = \langle \xi(r) \rangle \bar{C} \, \tfrac{3}{2} L (g_0 + 2g_1) . \quad (24)$$

With the estimate $g_0 + 2g_1 \cong +3$ (as discussed in Sec. I), and upon inserting Eq. (22), we obtain

$$E_{\text{mag}}(J) = \left(\frac{9}{2} \mathcal{R}_\Omega \frac{Z^4 \alpha^2}{n^3 (L+\frac{1}{2})(L+1)}\right) \bar{C}(J) . \quad (25)$$

The values of $\bar{C}(J)$, as derived from the values of $C(J)$, are as follows:

(i) For the lower levels, with $\bar{C} = \frac{1}{27}C$, we find

$$\bar{C}(\tfrac{21}{2}) = +1, \quad \bar{C}(\tfrac{19}{2}) = +\tfrac{2}{9} = +0.222,$$
$$\bar{C}(\tfrac{17}{2}) = -\tfrac{13}{27} = -0.481,$$

and

$$\bar{C}(\tfrac{15}{2}) = -\tfrac{10}{9} = -1.111 .$$

(ii) For the upper levels, with $\bar{C} = \frac{1}{30}C$, we find

$$\bar{C}(\tfrac{23}{2}) = +1, \quad \bar{C}(\tfrac{21}{2}) = \tfrac{7}{30} = +0.233,$$
$$\bar{C}(\tfrac{19}{2}) = -\tfrac{7}{15} = -0.467,$$

and

$$\bar{C}(\tfrac{17}{2}) = -\tfrac{11}{10} = -1.10 .$$

For the case of Pb, the factor B in the large parentheses of Eq. (25) has the following values: (i) for the lower levels, with $n = 10$, $L = 9$: $B = 5.04$ keV; (ii) for the upper levels, with $n = 11$, $L = 10$: $B = 3.11$ keV.

We thus obtain, for $E_{\text{mag}}(J)$,

(i) for the lower levels,

$$E_{\text{mag}}(\tfrac{21}{2}) = +5.04 \text{ keV}, \quad E_{\text{mag}}(\tfrac{19}{2}) = +1.12 \text{ keV},$$
$$E_{\text{mag}}(\tfrac{17}{2}) = -2.43 \text{ keV}, \quad E_{\text{mag}}(\tfrac{15}{2}) = -5.60 \text{ keV}, \quad (26)$$

(ii) for the upper levels,

$$E_{\text{mag}}(\tfrac{23}{2}) = +3.11 \text{ keV}, \quad E_{\text{mag}}(\tfrac{21}{2}) = +0.73 \text{ keV},$$
$$E_{\text{mag}}(\tfrac{19}{2}) = -1.45 \text{ keV}, \quad E_{\text{mag}}(\tfrac{17}{2}) = -3.43 \text{ keV}. \quad (27)$$

The change in energy of the transitions (*a*), (*b*), (*c*), and (*d*) is thus given by

$$\Delta E_{mag}(a) = +3.11 - 5.04 = -1.93 \text{ keV}, \quad (28)$$

and similarly,

$$\Delta E_{mag}(b) = -0.39 \text{ keV},$$
$$\Delta E_{mag}(c) = +0.98 \text{ keV}, \quad (29)$$
$$\Delta E_{mag}(d) = +2.17 \text{ keV}.$$

These values may be compared with those of $\Delta E_Q(\alpha)$ in Eqs. (14) and (15), which were obtained by neglecting the magnetic interaction. The values of $\Delta E_{mag}(\alpha)$ are rather small compared to those of $\Delta E_Q(\alpha)$, in particular for transitions (*b*) and (*c*).

By adding the two results [Eqs. (14) and (15) and Eqs. (28) and (29)], we obtain the total ΔE, denoted by ΔE_{total}, on the assumption of $Q_\Omega = 2\times10^{-26}$ cm^2 and $g_1 = +1$:

$$\Delta E_{total}(a) = -3.28 - 1.93 = -5.21 \text{ keV}, \quad (30)$$
$$\Delta E_{total}(b) = +4.51 - 0.39 = +4.12 \text{ keV}, \quad (31)$$
$$\Delta E_{total}(c) = +3.12 + 0.98 = +4.10 \text{ keV}, \quad (32)$$
$$\Delta E_{total}(d) = -4.67 + 2.17 = -2.50 \text{ keV}. \quad (33)$$

Thus, the pattern extends over a region of 5.21 + 4.12 = 9.33 keV. In this case, the transitions (*b*) and (*c*) have essentially the same energy ($\approx +4.10$ keV), but they are clearly separated from ΔE_{total} (*a*) and $\Delta E_{total}(d)$, which are also adequately separated from each other. Thus we would have two energy intervals, from which the values of Q_Ω and $g_0 + 2g_1$ can be determined.

In Fig. 2, we have shown the four transition energies given by Eqs. (30)–(33) (dashed lines), together with the transition energies in the absence of magnetic effects (full lines). The heights of the lines are proportional to the relative intensities of the transitions.

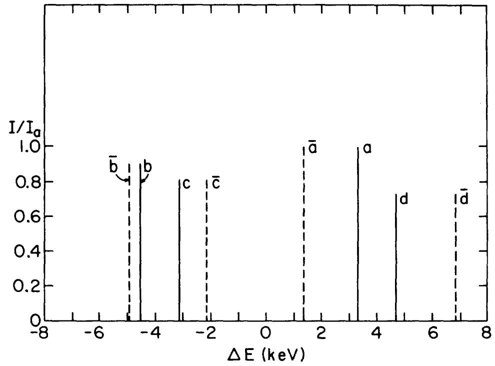

FIG. 3. Same plot as in Fig. 2, but with Q_Ω negative, i.e., $Q_\Omega = -2\times10^{-26}$ cm^2 [see Eqs. (34) and (35)].

We note that the values of $|\Delta E_{total}(\alpha)|$ depend only on the relative sign of Q_Ω and (g_0, g_1) [for the given absolute values of Q_Ω and (g_0, g_1)]. Thus, if the sign of Q_Ω should be reversed (i.e., negative), i.e., $Q_\Omega = -2\times10^{-26}$ cm^2, we would obtain for the value of $\Delta E_{total}(a)$:

$$\Delta E_{total}(a) = +3.28 - 1.93 = +1.35 \text{ keV}, \quad (34)$$

and similarly [in view of Eqs. (31)–(33)],

$$\Delta E_{total}(b) = -4.90 \text{ keV}, \quad \Delta E_{total}(c) = -2.14 \text{ keV},$$

and (35)

$$\Delta E_{total}(d) = +6.84 \text{ keV}.$$

In this case, the pattern extends over a region of 6.84 + 4.90 = 11.74 keV, and it consists of four well-separated lines. Thus there are three measurable energy intervals, and the system of linear equations for the two constants Q_Ω and g_1 will be overdetermined, leading to an additional check on the calculated values of Q_Ω and g_1. Figure 3 shows the pattern of the lines for $Q_\Omega < 0$, demonstrating the clear separation of the four transitions $\bar{a}$, $\bar{b}$, $\bar{c}$, and $\bar{d}$.

IV. SUMMARY AND DISCUSSION

We have calculated the fine structure of the Ω^--Pb atom, expected for representative values of the electric quadrupole moment Q_Ω and the magnetic dipole moment μ_Ω of the Ω^- hyperon. The $n=11$, $l=10 \to n=10$, $l=9$ transition has been considered, since an experiment on antiprotonic atoms of lead[5] has made it likely that the capture of the Ω^- by the nucleus will take place from a hydrogenic state with n of the order of 10 for the case of lead, and with l close to or equal to its maximum value[15] $n-1$. It has been shown that for a value of $Q_\Omega = 2\times10^{-26}$ cm^2, the quadrupole splittings are large enough so that it should be possible to detect them experimentally. The expected spin $S=\frac{3}{2}$ of the Ω^- particle gives rise to four strong lines, and the very existence of four lines (or possibly three observable maxima, with two lines being unresolved) would demonstrate at once that the spin S_Ω is indeed $\frac{3}{2}$, as expected from SU(3). It should also be pointed out that a measurement of the intrinsic moments Q_Ω and μ_Ω would be of great interest for models of the SU(3) decuplet containing the Ω^- (e.g., quark models; compound models[16] in which Ω^- is regarded as a bound system consisting of either $\Xi^0 + K^-$ or $\Xi^- + \bar{K}^0$).

We may remark that the experiment to produce Ω^--hyperonic atoms will, of course, be technically very complicated, because of the difficulties of stopping the Ω^- particles, on account of their strong interactions. Nevertheless, with the expected operation of Ξ^- beams, it may be possible to attempt the experiment.

In any case, quite aside from the feasibility (or infeasibility) of the experiment, it is obvious that the Ω^--hyperonic atom is an interesting possible object of theoretical investigation. The main reason is, of course, the spin $S=\frac{3}{2}$ of the Ω^- hyperon. Thus s states will be $s_{3/2}$, while p states will be $p_{1/2}$, $p_{3/2}$, and $p_{5/2}$, etc. For the Ω^--protonic atom (Ω^--p), there is the added interesting feature that the masses of the two constituent particles are comparable, so that the reduced mass $m_{\rm red}=0.601$ GeV is appreciably less than either m_p or m_Ω. Similar results are obtained for the Ω^--d and Ω^--α atoms, for which the reduced-mass values are $m_{\rm red}=0.884$ and 1.154 GeV, respectively.

From Eqs. (6) and (7), it is seen that $E_Q^{(0)}$, and hence E_Q is proportional to a_Ω^{-3}, i.e., to $(m_{\rm red}/m_\Omega)^3$. Thus, the quadrupole fine structure goes as $(m_{\rm red}/m_\Omega)^3$, while the zero-order energy of the transition, being proportional to $\mathcal{R}_\Omega$, is proportional to $m_{\rm red}/m_\Omega$. On the other hand, for the magnetic dipole fine structure, upon referring to the derivation of the spin-orbit coupling,[14] one finds that the factor $(\hbar/m_0c)^2$ in Eq. (17) for $\xi(r)$ should be replaced by $(\hbar^2/m_{\rm red}m_\Omega c^2)$, so that $E_{\rm mag}$ is proportional to $(m_{\rm red}/m_\Omega)^2$. Thus for a given transition $n\to n-1$, the magnetic fine structure and especially the quadrupole fine structure become relatively less important compared to the transition energy, for those cases in which $m_{\rm red}/m_\Omega$ is appreciably less than one.

In connection with Eq. (17), which is equivalent to Eq. (III.53) of Kuhn,[14] we have made the implicit assumption that the Thomas relativistic precession factor is $\frac{1}{2}$, the same as for a particle of spin $S=\frac{1}{2}$. The fact that the Thomas factor represents essentially a classical effect, and is independent of spin, is evident from the derivation given by Furry.[17]

There are, of course, other theoretical problems connected with the spin $S=\frac{3}{2}$ of the orbiting particle, in particular the treatment of relativistic velocities, since the $S=\frac{3}{2}$ Ω^- hyperon obviously does not obey the Dirac equation. A wave equation for particles of spin $\frac{3}{2}$ was first proposed by Rarita and Schwinger.[18] It would be of interest to determine the intrinsic quadrupole moment and the intrinsic magnetic moment of a spin-$\frac{3}{2}$ particle from such wave equations, analogously to the magnetic moment $e\hbar/2mc$ derived from the Dirac equation. However, because of the strong interactions of the Ω^- particle, there is expected to be a large "anomalous" contribution to both the quadrupole moment and the magnetic moment, which could not be predicted from the wave equation for a free spin-$\frac{3}{2}$ particle.[19]

For our present estimates of ΔE_Q and $\Delta E_{\rm mag}$, and in particular in connection with the determination of $\langle r^{-3}\rangle$ [Eq. (4)], we do not have a relativistic situation, so that complications due to the generalization of the Dirac equation do not occur. Thus the velocity in a $n=10$ orbit for Ω^--Pb is of the order of $\alpha c(Z/n)=8.2\alpha c=0.06c$, which is small compared to c.

It should be pointed out that the present calculations may be of interest if a heavy lepton with spin $\frac{3}{2}$ (or higher) should be discovered. Although present theories do not seem to envisage this possibility, one may note that for the hadronic particles, an increase in mass is usually accompanied by an increase of spin of the particle. If the heavy lepton should have a sufficiently long lifetime, e.g., $\tau \gtrsim 10^{-9}$ sec, then it would have an advantage over the Ω^-, in that it does not interact strongly, and therefore could be readily stopped in an absorber. This would make it relatively easy to capture the heavy lepton in an atomic orbit, and thus observe its magnetic and quadrupole moments.

ACKNOWLEDGMENTS

We wish to thank Dr. M. Blume, Dr. T. F. Kycia, and Dr. R. F. Peierls for helpful discussions.

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*Work performed under the auspices of the U.S. Atomic Energy Commission.

[1] V. E. Barnes *et al.*, Phys. Rev. Lett. 12, 204 (1964).

[2] Particle Data Group, Rev. Mod. Phys. 45, S1 (1973).

[3] O. Chamberlain, E. Segrè, C. Wiegand, and T. Ypsilantis, Phys. Rev. 100, 947 (1955).

[4] A. Bamberger, U. Lynen, H. Piekarz, J. Piekarz, B. Povh, H. G. Ritter, G. Backenstoss, T. Bunaciu, J. Egger, W. D. Hamilton, and H. Koch, Phys. Lett. B 33, 233 (1970).

[5] J. D. Fox, P. D. Barnes, R. A. Eisenstein, W. C. Lam, J. Miller, R. B. Sutton, D. A. Jenkins, R. J. Powers, M. Eckhause, J. R. Kane, B. L. Roberts, M. E. Vislay, R. E. Welsh, and A. R. Kunselman, Phys. Rev. Lett. 29, 193 (1972).

[6] A. D. Martin, Nuovo Cimento 27, 1359 (1963).

[7] S. L. Kahalas and R. K. Nesbet, J. Chem. Phys. 39, 529 (1963).

[8] See, for example, H. G. Kuhn, *Atomic Spectra*, 2nd ed. (Academic, New York, 1969), p. 119.

[9]M. Blume (private communication).

[10]This can also be seen from the derivation given by A. Abragam, *Principles of Nuclear Magnetism* (Oxford U. P., London, 1961), p. 165.

[11]H. G. Kuhn, Ref. 8, p. 351.

[12]H. G. Kuhn, Ref. 8, p. 191.

[13]P. A. M. Dirac, Proc. R. Soc. 111, 281 (1926).

[14]H. G. Kuhn, Ref. 8, p. 117.

[15]It should be pointed out that if there were an appreciable population of n levels in lower l states (i.e., $l < n-1$), this effect would tend to blur the fine structure calculated in the present paper. However, there seems to be no appreciable indication of this effect in the antiprotonic-atom experiment of J. D. Fox *et al.*, Ref. 5, who observed only the $l = n-1$ states in both $\bar{p}$-Pb and $\bar{p}$-U. A similar conclusion can also be drawn from the calculations of Martin on K^- mesic atoms (Ref. 6). This author found for the case of K^- mesic atoms that when we consider a low-lying initial state of K^--Ag, such as $n_1 = 6$, $l_1 = 5$, the dominant electromagnetic transition goes to the "circular" state $n_2 = 5$, $l_2 = 4$. In fact, even for a highly excited initial state such as $n_1 = 25$, $l_1 = 5$, the dominant transition goes to the nearly "circular" state, $n_2 = 6$, $l_2 = 4$. (These results are given in Table II of Ref. 6.) Thus, it seems reasonable to restrict our attention to the transitions between states with $l = n-1$.

[16]M. Goldhaber, Phys. Rev. 92, 1279 (1953).

[17]W. H. Furry, Am. J. Phys. 23, 517 (1955).

[18]W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941); see also Y. Takahashi and H. Umezawa, Prog. Theor. Phys. 9, 1 (1953).

[19]After this paper was submitted for publication, a paper by L. W. Alvarez [Phys. Rev. D (to be published)] came to our attention. In this paper, Alvarez describes three old cosmic-ray emulsion events as Ω^- decays and interactions. In one of the events, the Ω^- is "a clear-cut example of an Ω^- decaying in orbit, bound to an emulsion nucleus."