Glauber Cross Sections for Excitation of the 2 ^1S State of Helium by Electron and Positron Impact

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The recently proposed analytic methods, which reduce the Glauber amplitude for charged-particle–neutral-atom collisions to a one-dimensional integral representation involving modified Lommel functions, are used to evaluate the cross sections for the direct excitation of the 2 ^1S state of helium by electron and positron impact. Comparison is made with the Coulomb-projected Born approximation of Hidalgo and Geltman and the truncated eigenfunction-expansion method of Berrington, Bransden, and Coleman.

In a recent paper,[1] new reduction techniques for the Glauber e^--He elastic scattering amplitudes are proposed. The results obtained using these methods are in good agreement with those obtained by Franco[2] from a three-dimensional integral representation. The purpose of this note is to show that the generating function $I(\lambda_1, \lambda_2; q)$ obtained in Ref. 1 can be used to calculate the cross sections for direct excitation of helium by electron and positron impact.

The scattering amplitude $F_{fi}(\vec{q})$, where the He atom is excited from the ground state $\Psi_{1s}(\vec{r}_1, \vec{r}_2)$ to the final state $\Psi_{2^1S}(\vec{r}_1, \vec{r}_2)$ by an incident charged particle $Z_i e$ with velocity v_i, is given according to the Glauber theory by

$$F_{fi}(\vec{q}) = \frac{ik_i}{2\pi} \int \Psi^*_{2^1S}(\vec{r}_1, \vec{r}_2)\Gamma(\vec{b}; \vec{r}_1, \vec{r}_2) \times \Psi_{1s}(\vec{r}_1, \vec{r}_2) e^{i\vec{q}\cdot\vec{b}} d^2b\, d\vec{r}_1\, d\vec{r}_2 , \qquad (1)$$

where

$$\Gamma(\vec{b}; \vec{r}_1, \vec{r}_2) = 1 - \left(\frac{|\vec{b}-\vec{s}_1|}{b}\right)^{2i\eta} \left(\frac{|\vec{b}-\vec{s}_2|}{b}\right)^{2i\eta} \qquad (2)$$

and

$$\eta \equiv -Z_i/v_i \text{ (in atomic unit).}$$

In Eqs. (1) and (2), $\vec{b}$, $\vec{s}_1$, $\vec{s}_2$ are the respective projections of the position vectors of the incident particle and the bound electrons ($\vec{r}_1$ and $\vec{r}_2$) onto the plane perpendicular to the direction of the Glauber path integration. The approximate atomic wave functions we choose (in atomic units) are the orthonomal set used by van den Bos[3] for calculations on proton-helium scattering and Hidalgo and Geltman[4] for calculations on e^--helium collisions:

$$\Psi_{1s}(\vec{r}_1, \vec{r}_2) = \frac{2.605^2}{4\pi} \left(e^{-1.41r_1-1.41r_2} + 0.799e^{-2.61r_1-1.41r_2} + 0.799e^{-1.41r_1-2.61r_2} + 0.799^2 e^{-2.61r_1-2.61r_2}\right) \qquad (3)$$

and

$$\Psi_{2^1S}(\vec{r}_1, \vec{r}_2) = \frac{0.6451}{\pi(1+0.06996^2)^{1/2}} \left(e^{-2r_1-1.136r_2} + e^{-1.136r_1-2r_2} - 0.2806 r_1 e^{-0.464r_1-2r_2} - 0.2806 r_2 e^{-2r_1-0.464r_2}\right). \qquad (4)$$

Substituting Eqs. (3) and (4) into Eq. (1), we obtain the amplitude $F(\vec{q})$ in terms of the generating function $I(\lambda_1, \lambda_2; q)$,

$$F(\vec{q}) = 1.092 i k_i \Bigg[\left(\frac{\partial^2 I}{\partial\lambda_1\partial\lambda_2}\right)_{\substack{\lambda_1=3.41\\ \lambda_2=2.546}} + 0.799\left(\frac{\partial^2 I}{\partial\lambda_1\partial\lambda_2}\right)_{\substack{\lambda_1=4.61\\ \lambda_2=2.546}} + 0.799\left(\frac{\partial^2 I}{\partial\lambda_1\partial\lambda_2}\right)_{\substack{\lambda_1=3.41\\ \lambda_2=3.746}} + 0.6384\left(\frac{\partial^2 I}{\partial\lambda_1\partial\lambda_2}\right)_{\substack{\lambda_1=4.61\\ \lambda_2=3.746}}$$
$$+ 0.2806\left(\frac{\partial^3 I}{\partial\lambda_1^2\partial\lambda_2}\right)_{\substack{\lambda_1=1.874\\ \lambda_2=3.41}} + 0.2242\left(\frac{\partial^3 I}{\partial\lambda_1^2\partial\lambda_2}\right)_{\substack{\lambda_1=3.074\\ \lambda_2=3.41}} + 0.2242\left(\frac{\partial^3 I}{\partial\lambda_1^2\partial\lambda_2}\right)_{\substack{\lambda_1=1.874\\ \lambda_2=4.61}}$$
$$+ 0.1792\left(\frac{\partial^3 I}{\partial\lambda_1^2\partial\lambda_2}\right)_{\substack{\lambda_1=3.074\\ \lambda_2=4.61}} \Bigg], \qquad (5)$$

where I is defined and given in Eq. (19) of Ref. 1:

$$
\begin{aligned}
I &\equiv I(\lambda_1, \lambda_2; q) \\
&= -2^4(2i\eta)^2\Gamma(i\eta)\Gamma(1-i\eta)q^{2i\eta-2}[\lambda_2^{-2}\lambda_1^{-2i\eta-2}\,{}_2F_1(1-i\eta, 1-i\eta; 1; -\lambda_1^2/q^2) \\
&\qquad + \lambda_1^{-2}\lambda_2^{-2i\eta-2}\,{}_2F_1(1-i\eta, 1-i\eta; 1; -\lambda_2^2/q^2) - 2^5(2i\eta)^4] \\
&\quad \times \int_0^\infty b^5\, db\, J_0(qb)(i\lambda_1 b)^{-2i\eta-2}L_{2i\eta-1,0}(i\lambda_1 b)(i\lambda_2 b)^{-2i\eta-2}L_{2i\eta-1,0}(i\lambda_2 b),
\end{aligned} \tag{6}
$$

where $L_{\mu,\nu}(ix)$ is the modified Lommel function.[1] From Eq. (6) and Eq. (A12) of Ref. 1, we can obtain the expressions for $\partial^2 I/\partial\lambda_1\partial\lambda_2$ and $\partial^3 I/\partial\lambda_1^2\partial\lambda_2$. The procedures for numerical computation of $\partial^2 I/\partial\lambda_1\partial\lambda_2$ and $\partial^3 I/\partial\lambda_1^2\partial\lambda_2$ are the same as those of $I(\lambda_1, \lambda_2; q)$ and are described in detail in Ref. 1.

We have calculated the differential cross sections $d\sigma/d\Omega$ by means of Eq. (5) and the expressions for $\partial^2 I/\partial\lambda_1\partial\lambda_2$ and $\partial^3 I/\partial\lambda_1^2\partial\lambda_2$ for various incident electron energies, as a function of the scattering angle. In Fig. 1 the results are compared with those obtained using the truncated-eigenfunction-expansion method of Berrington, Bransden, and Coleman (BBC).[5] We note from Fig. 1 that both produce results which are in reasonable agreement with the corresponding experimental measurements,[6-8] although the Glauber-predicted $d\sigma/d\Omega$ are about half of those of the BBC at the forward direction, and as the scattering angle is increased (>3°) our results are slightly larger than those of BBC. From Fig. 2 we see that the Glauber predicted $d\sigma/d\Omega$ are very close (0.5–13%) to those of the Coulomb-projected Born approximation of Hidalgo and Geltman (HG) at 15°; however, the Glauber $d\sigma/d\Omega$ fall off more rapidly than those of HG as energy is increased at 30°. We have also integrated the differential cross section and therefore obtained the total cross sections as a function of the incident electron energy, and the results are shown in Fig. 3. We notice that only a small difference between our results and those of HG exists, since the most important contribution to the integration comes from the region from 3° to 25° where both are very close to each other. We have also evaluated the total cross section at 82 eV with the result $\sigma^{\text{theor}} = (1.68 \times 10^{-2})\pi a_0^2$, which is also in reasonable agreement with the experimental value,[9] $\sigma^{\text{expt}} = (3.4\times10^{-2})\pi a_0^2$ (estimated error 58%). We are not very serious about this agreement, since 82 eV is probably too low an energy to expect validity of the Glauber approximation.

For positron-helium scattering, $\eta_{e^+} = -\eta_{e^-}$, from (5), one gets $F(q)|_{e^+-\text{He}} = F^*(q)|_{e^--\text{He}}$. Therefore $(d\sigma/d\Omega)_{e^+-\text{He}} = (d\sigma/d\Omega)_{e^--\text{He}}$ is valid in the Glauber approximation. As a check on numerical accuracy we have evaluated the positron-He scattering cross section for various scattering angles at 300 eV and found that it agrees to about 0.7% with the electron-He scattering cross section. Thus our numerical results are presumable accurate to this level of precision. In Fig. 4 the results are compared with the BBC, which, however, predicted the different results between e^--He and e^+-He scattering at 300 eV.

Finally, we would like to mention that the van

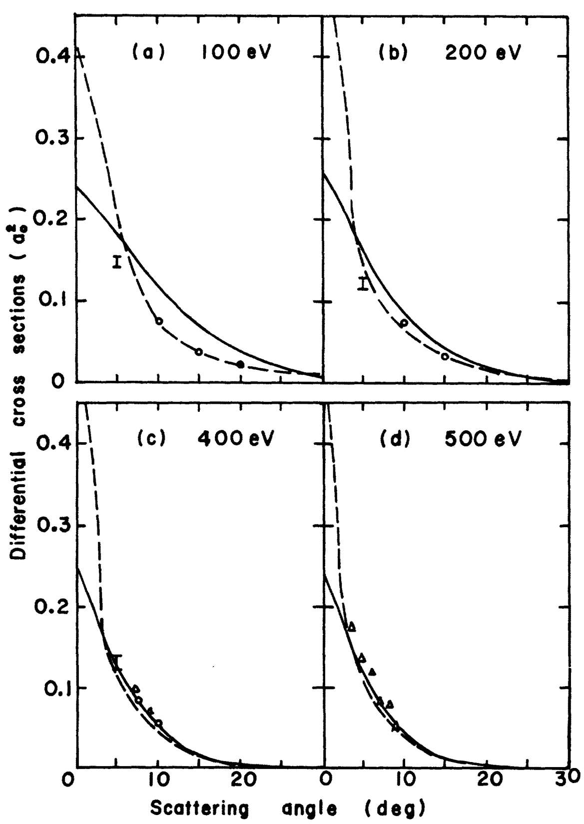

FIG. 1. Differential cross sections for the $2\,^1S$ excitation of helium by electrons at (a) 100 eV, (b) 200 eV, (c) 400 eV; (d) 500 eV. Solid line, this work; dashed line, BBC; ○, Vriens *et al*; renormalized at 5° to Chamberlain *et al.*; △, Lassettre *et al.* at 417 and 511 eV; I, Chamberlain *et al.*

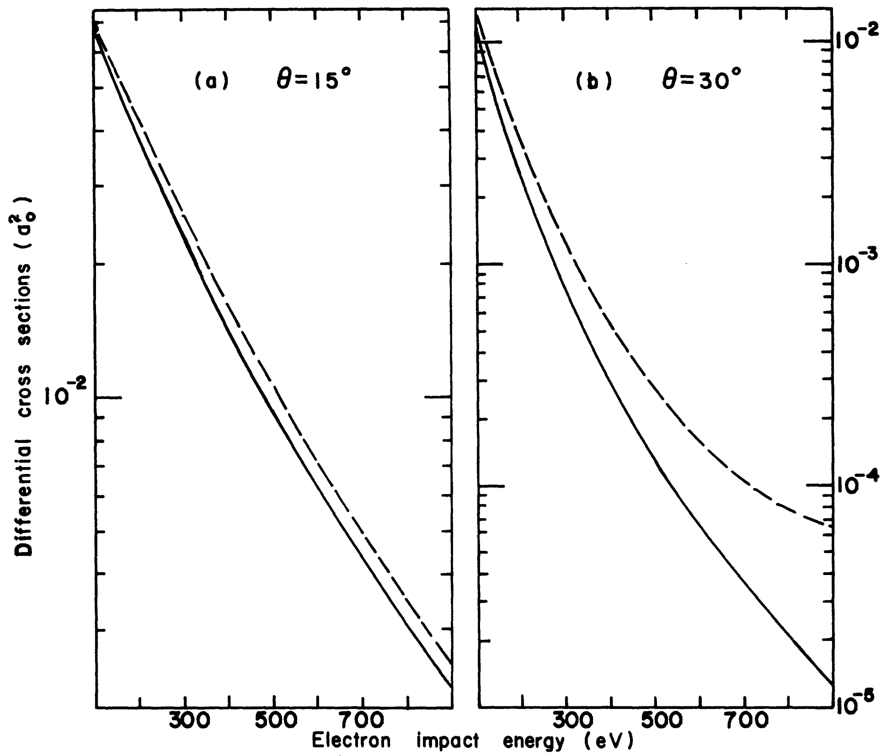

FIG. 2. Differential cross sections for the $2\,^1S$ excitation of helium by electrons as a function of electron impact energy at (a) 15°, (b) 30°. Solid line, this work; dashed line, HG.

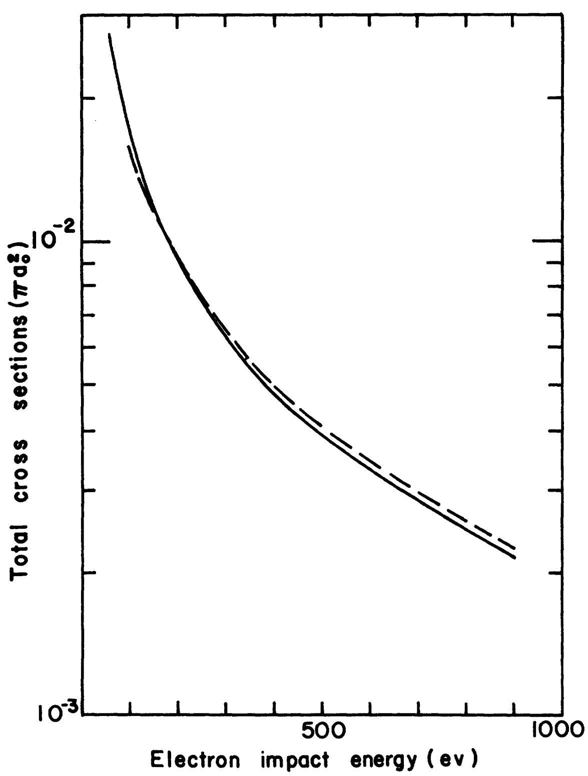

FIG. 3. Total cross sections for the $2\,^1S$ excitation of helium by electrons as a function of electron impact energy. Solid line, this work; dashed line, HG.

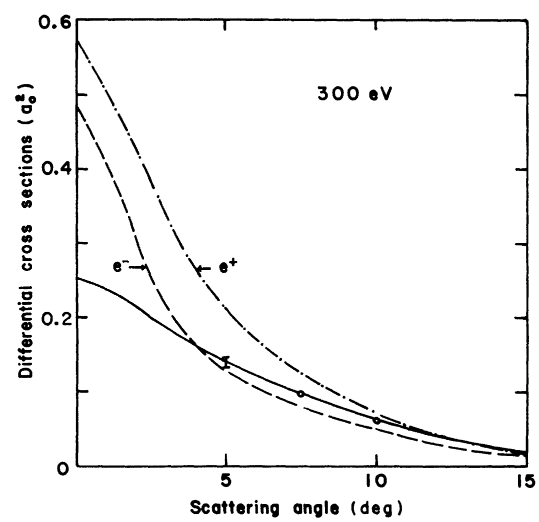

FIG. 4. Differential cross sections for the $2\,^1S$ excitation of helium by electrons and positrons at 300 eV. Solid line, this work; dashed line, BBC; I, Chamberlain *et al.*; O, Vriens *et al.*, renormalized at 5° to Chamberlain *et al.*

den Bos wave functions for He used in this paper are certainly not the most accurate helium wave functions available, but we cannot conveniently use correlated wave functions as they would lead to intractable integrals in Eq. (1). We have also used the orthogonal helium wave functions for the 2^1S state described by Flannery[10] to evaluate the differential cross sections from 0° to 30° at 300 eV. We found that the results differ from the work reported here by about 10%.

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