Pair Production by Relativistic Electrons from an Intense Laser Focus*

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A preliminary discussion is given of electron-positron pair production by means of electrons accelerated to relativistic velocities at the focus of a laser beam. First, the pair-production cross section was numerically evaluated near its energy threshold. Then, two methods of relativistic electron production by focused laser light were considered: the coherent oscillation of electrons, and the acceleration of a few high-velocity electrons by plasma waves excited by laser-driven instabilities. It was found that the first method would produce pairs for neodymium-laser light at intensities close to the achievable limit ($10^{19}-10^{20}$ W/cm^2). The second method, however, may be capable of producing pairs at lower intensities.

I. INTRODUCTION

In recent years the brightness of pulsed laser light sources has increased from 10^{12} W/cm^2/sr up to the range of 10^{17}–10^{20} W/cm^2/sr at the neodymium-laser wavelength of 1.06 μ.[1,2] With a well-designed short-focus lens, such laser pulses can be focused to corresponding intensities of 10^{17}–10^{20} W/cm^2. At these intensities electron-positron pair production by the strong electromagnetic field at the focus might be possible either by direct vacuum pair production or indirectly from relativistic electrons accelerated by these strong fields.

Two calculations of the probability of vacuum pair production by multiphoton absorption have been published[3,4]; both of them indicate that the production of a detectable number of pairs from this process would require many more orders of magnitude of intensity than contemporary lasers provide. A simple order-of-magnitude estimate can show this. For an appreciable pair-production probability, the pair energy $2m_0c^2$ should be of the order of the electric-field potential energy at one Compton wavelength $\lambda\!\!\!^-$. That is,

$$eE_l \lambda\!\!\!^- \simeq 2m_0c^2 , \tag{1}$$

where E_l is the average electric field of the focused laser light. Therefore, the intensity I_l of the laser light in vacuum can be written

$$I_l = c\epsilon_0 E^2 \simeq \frac{m_0c^3}{\pi r_0 \lambda\!\!\!^{-2}}, \tag{2}$$

where r_0 is the classical radius of the electron:

$$r_0 = e^2/4\pi\epsilon_0 m_0c^2 . \tag{3}$$

Numerically, this is an intensity of 2×10^{30} W/cm^2, which is many orders of magnitude greater than available intensities, but is in rough agreement with values previously cited.[3,4]

However, it is well known that focused laser pulses create hot plasmas in matter. It has been pointed out that there exists the "wholly real possibility" of observing pairs produced in such laser-plasma experiments by means of the excitation of high-energy electrons.[5] When the electron kinetic energy E_v exceeds the pair-production threshold $2m_0c^2$, the fast electron can produce an electron-positron pair by scattering in the Coulomb potential of a nucleus as first calculated by Bhabha.[6] This is often called the "trident" process (see Fig. 1). In this paper we shall discuss some mechanisms by which energetic electrons can be created in the laser-plasma focus, and we shall attempt to evaluate the number of electron-positron pairs which can be produced by the subsequent trident process.

II. TRIDENT CROSS SECTION AT ELECTRON ENERGIES NEAR THE PAIR-PRODUCTION THRESHOLD

We have calculated the cross section σ_T for the trident pair-production process by two different methods; the results are plotted in Fig. 2. In the first method we integrated Eq. (30) of Bhabha's paper,[6] and obtained

$$\sigma_T = \frac{(\alpha r_0 Z)^2}{128}\left(\ln\frac{1}{\gamma_B^2} - \frac{161}{60} + c_1 + c_2 + c_3\right)\left(\frac{E_v}{m_0c^2} - 2\right)^4 , \tag{4}$$

where α is the fine-structure constant, r_0 is the classical electron radius, Z is the nuclear charge, and γ_B is defined by

$$\frac{1}{\gamma_B} \equiv 1 + \frac{E_v}{m_0c^2} . \tag{5}$$

The symbols c_1, c_2, and c_3 in Eq. (4) are lengthy algebraic functions of γ_B which are given in Eq. (28) of Bhabha's paper. Numerical calculations of

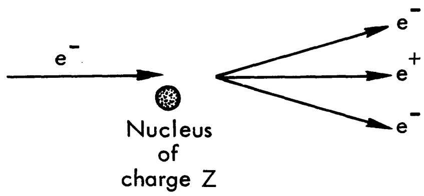

FIG. 1. Schematic of trident process of pair creation.

Eq. (4) are plotted in Fig. 2.

Bhabha's approach has the merit of yielding a convenient analytical expression for the cross section; however, he makes two approximations having attendant errors that are difficult to estimate. In the first place, the initial electron is described by a classical straight-line trajectory; secondly, the interference effects between the two electrons in the final state are neglected. Both approximations are avoided by evaluating the lowest-order Feynman diagrams which are applicable to the trident process. Our numerical calculation uses a program for evaluating these diagrams which was written by Brodsky and Ting.[7] The program numerically evaluates the necessary products of Dirac matrices and takes the trace to get the differential cross section. This procedure is more accurate than the usual one, which involves algebraic reduction of the matrix products. The differential cross section is then

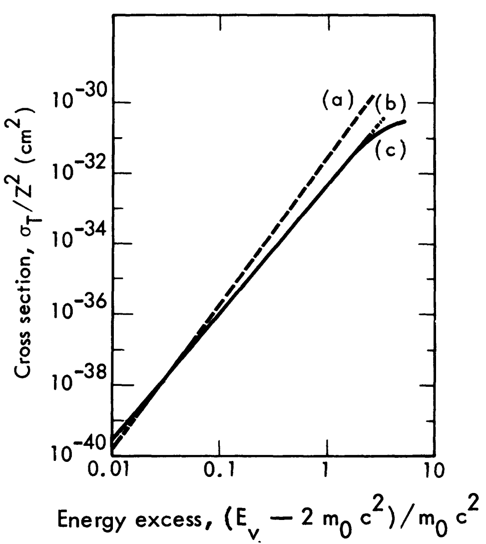

FIG. 2. Total cross section σ_T of trident process plotted vs the dimensionless energy excess above threshold, where E_v is kinetic energy of the incident electron. Curve (a) is Bhabha's analytical calculation (Ref. 6) for this range [see Eq. (4)]. Curve (b) is our computer calculation based on Brodsky and Ting's method (Ref. 7). Curve (c) is our approximate fit to the computer calculation.

numerically integrated to obtain the total cross section as a function of incident-electron energy. The results of these calculations are also plotted in Fig. 2.

It is seen that the second, more exact calculation gives lower cross-section values over most of the energy range of interest—near the trident-production threshold. The reduction in cross section is presumably due to inclusion of the interference between direct and exchange scattering, an effect which was omitted in Bhabha's calculation. For the subsequent portions of this paper we use the equation

$$\sigma_T \cong 9.6\times10^{-4}(\alpha r_0 Z)^2\left(\frac{E_v}{m_0c^2}-2\right)^{3.6}, \qquad (6)$$

which approximates the more exact cross-section curve. In Fig. 2 we see that the fit of the approximation is good up to kinetic energies of $\simeq 4m_0c^2$, which is a sufficient range for our problem. The possibility of pair production by bremsstrahlung from the electrons, mentioned in a footnote of Ref. 7, is not as likely as trident production in this energy range because it is a two-step process.

III. HIGH-INTENSITY CIRCULARLY POLARIZED LIGHT

We consider first the coherent motion of plasma electrons in the electromagnetic wave of the laser light. Because the pair-production threshold is twice the rest energy, this motion must be treated relativistically. This treatment is more difficult than the well-known nonrelativistic treatments.[8] We have not treated the linearly polarized wave, because it has been shown to be coupled to a longitudinal plasma wave.[9] However, the circularly polarized wave is a simple transverse wave at all intensities, and a convenient solution for this case has been obtained by Steiger and Woods.[10] We shall use these authors' results for the case which includes the relativistic mass change and the inverse Faraday effect, but we neglect energy losses caused by radiation by the electron.

Steiger and Woods[10] found that at high circularly polarized laser-beam intensities where the coherent electron-orbit velocity is relativistic, the electron kinetic energy E_v is a strong function of laser intensity, but is almost independent of plasma density. We replotted this result (Fig. 3) and found that a useful approximation is given by the following simple power law:

$$\frac{E_v}{m_0c^2} \cong \left(\frac{\lambda^2 I}{\lambda_0^2 I_0}\right)^n = \left(\frac{\lambda^2 I}{9\times10^{18}}\right)^{0.657}, \qquad (7)$$

where the laser wavelength λ is measured in μ and the intensity I is in W/cm^2.

An equation that describes the rate dN_p/dt of creation of pairs by means of the trident process in a volume whose characteristic dimension is approximately l times the wavelength λ is

$$\frac{dN_p}{dt}\ (\text{pairs/s}) = (l\lambda)^3 N_i N_e \sigma_T v_e\ , \tag{8}$$

where N_i is the ion (nucleus) density and v_e is the velocity of the electron. Assuming that the thermal velocity can be neglected in comparison with the relativistic coherent velocity of gyration in the intense electromagnetic field, we have

$$v_e \equiv \beta c = (c/\gamma)(\gamma^2-1)^{1/2}, \tag{9}$$

where the coefficient γ is the normalized total electron energy

$$\gamma = 1 + \frac{E_v}{m_0 c^2} = 1 + \left(\frac{\lambda^2 I}{9\times 10^{18}}\right)^{0.657}, \tag{10}$$

where we have substituted from Eq. (7).

Consider a plasma containing ions of charge $Z_i = N_e/N_i$, where Z_i is not necessarily the nuclear charge Z. Substitute Eqs. (6) and (9) into Eq. (8) to obtain the result

$$\frac{dN_p}{dt} = \frac{9.6\pi^2}{10^4}\left(\frac{c}{\lambda}\right) l^3 \alpha^2 \frac{Z^2}{Z_i}\left(\frac{N_e}{N_c}\right)^2 (\gamma-3)^{3.6}\frac{(\gamma^2-1)^{1/2}}{\gamma}, \tag{11}$$

where the density N_c,

$$N_c \equiv \epsilon_0 m\left(\frac{\omega_L}{e}\right)^2 = \frac{\pi}{r_0\lambda^2}, \tag{12}$$

is the nonrelativistic cutoff-density parameter at which the plasma frequency ω_p equals the laser frequency ω_L. In convenient units, Eq. (11) becomes

$$\frac{dN_p}{dt}\ (\text{pairs/ns}) = \frac{0.15}{\lambda(\mu)}\frac{Z^2}{Z_i} l^3\left(\frac{N_e}{N_c}\right)^2 (\gamma-3)^{3.6}\frac{(\gamma^2-1)^{1/2}}{\gamma}. \tag{13}$$

Equations (7) and (13) are presented in graphical form in Figs. 4 and 5.

The pair-production rate threshold ($\gamma = 3$) corresponds to a threshold laser beam intensity I_T of

$$I_T\left(\frac{\text{W}}{\text{cm}^2}\right) = \frac{2.6\times 10^{19}}{[\lambda(\mu)]^2}. \tag{14}$$

The pair-production rate then rises steeply to interesting values at somewhat higher intensities. These intensities (greater than 10^{19} W/cm^2 for neodymium lasers) are at the upper limit of current laser practice.[1,2]

Of course, this is an idealized single-particle calculation which neglects unstable collective effects in the plasma. One should regard it mainly as an order-of-magnitude estimate that trident-process pair production is hard to produce by means of the coherent electron motion alone.

When the electron motion is relativistic the laser beam can penetrate an overdense plasma,[9,10] because the plasma current is limited to the value $N_e ec$, instead of increasing with increasing beam intensity.[11] In such cases the pair-production rate will be enhanced, provided the intensity is above the threshold.

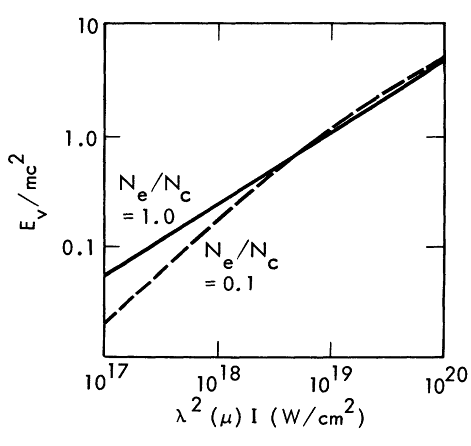

FIG. 3. Electron kinetic energy E_v (in units of m_0c^2) plotted as a function of laser beam intensity (normalized to the wavelength λ) as obtained from the circularly polarized transverse-wave solution of Steiger and Woods (Ref. 10) at two different plasma-density ratios.

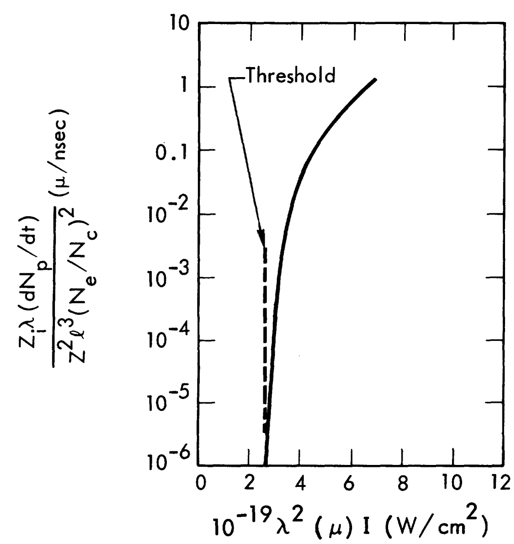

FIG. 4. Density-normalized pair-production rate plotted vs laser intensity, as computed from Eqs. (7) and (13).

IV. DISCUSSION OF PLASMA INSTABILITIES AT LOWER LIGHT INTENSITIES

Anomalous heating of plasmas by intense electromagnetic waves is now recognized as a significant effect both at radio-wave frequencies and at laser frequencies.[12,13] The incident wave energy is coupled into plasma waves by plasma instabilities.[14] One of these is the ion-acoustic decay instability[15] and the other is the oscillating two-stream instability.[16]

According to numerical computations of these processes,[13,17] the growth of the amplitude of the plasma waves is sufficiently rapid for the instability mechanism to saturate, causing the plasma to become turbulent. Many such calculations show a high-velocity, high-energy group of electrons, which is called a "suprathermal electron tail" on the Maxwellian electron-velocity distribution. One physical picture of this process is that some of the electrons are trapped by high-phase-velocity, high-amplitude plasma waves.[13,17]

Although these numerical calculations were nonrelativistic, the same qualitative arguments would be expected to hold for plasma waves whose phase velocity v_p is nearly the velocity of light ($v_p \to c$). Thus we must examine the plausibility of relativistic electron production by these plasma instabilities. If relativistic electrons ($E_v > 2m_0c^2$) are produced, the trident mechanism for pair production is possible.

The dispersion relation for longitudinal plasma waves can be written in the form[18]

$$v_p^2 = (\omega_p^2/K^2) + 3v_t^2\,, \qquad (15)$$

where K is the wave number and v_t is the thermal velocity of the electrons. Let us examine the wave whose phase velocity v_p is equal to the velocity of light, because waves which trap relativistic particles will have velocities which closely approach this limiting solution. Then for $v_p = c$, we find

$$(K_c\lambda_D)^2 = v_t^2/c^2 - 3v_t^2\,, \qquad (16)$$

where K_c is the corresponding wave number and λ_D is the Debye length of the plasma ($\lambda_D\omega_p = v_t$). From Eq. (16) we find that waves which are potentially capable of trapping electrons in the relativistic energy range tend to have small values of K_c, corresponding to longer-wavelength waves. As $v_t \to 0$, $K_c \to \omega_p/c$, as can be seen by eliminating λ_D from Eq. (16). Thus in the low-temperature limit, the wavelength of the longitudinal wave of velocity c is equal to the vacuum wavelength of an electromagnetic wave whose frequency is equal to the plasma frequency.

Now we ask whether plasma waves of wave number K_c are likely to be excited in the plasma by laser-driven instabilities. Consider first the threshold conditions for the ion-acoustic decay instability. The most unstable wave number K_p can be written[19]

$$K_p^2 = \frac{\omega_L^2 - \omega_p^2}{3v_t^2}\,. \qquad (17)$$

If we set $K_p = K_c$, and make the approximation $v_t \ll c$, we find from Eqs. (16) and (17) that

$$1 - \frac{\omega_p^2}{\omega_L^2} = 3\left(\frac{\omega_p^2}{\omega_L^2}\right)\frac{v_t^2}{c^2}\,. \qquad (18)$$

Here Eq. (12) can be used to find the plasma density

$$\Delta N/N_c \cong 3(v_t^2/c^2)\,, \qquad (19)$$

where ΔN is the difference between the cutoff density N_c and the electron density N_e and where $\Delta N \ll N_c$.

This result shows that electron plasma waves of velocity $v_p \simeq c$ can be excited by the parametric ion-acoustic instability at plasma densities close to the cutoff density. This is just the plasma-density regime where the threshold intensity for this instability is low.[20] For example, if the electron temperature T_e is 1 keV, we find that $N_e = 0.994\ N_c$. Also, we find that in this case the wavelength of this longitudinal wave is approximately the same as the vacuum wavelength of the laser radiation. Because most focal spots used

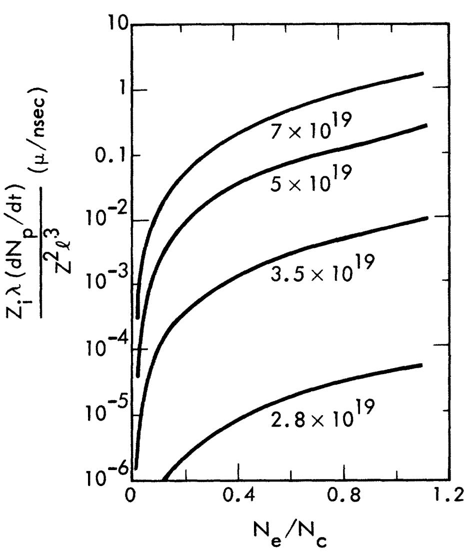

FIG. 5. Pair-production rate plotted vs density ratio for several values of the laser intensity. The figures on the curves are $\lambda^2 I$ [$(\mu)^2$ (W/cm^2)]. The curves continue beyond cutoff [$(N_e/N_c) > 1$] because of the possibility of relativistic beam penetration (Refs. 9 and 11).

in practice are at least several wavelengths in diameter, several wavelengths can build up inside the focus.

At incident laser intensities high above threshold, the ion-acoustic decay instability will be excited at lower plasma densities, where the most unstable mode will have a phase velocity less than the velocity of light. However, other modes will also be excited, particularly after saturation of the initial growth of the instability. Plasma-simulation calculations[17] show that the wave-number spectrum is enhanced in the low-K regime after saturation. Thus, long-wavelength plasma waves whose phase velocity is comparable to c should be excited over a wider range of densities in the plasma.

Another requirement for relativistic electron production is that the plasma-wave amplitude be sufficiently high to provide the necessary acceleration electric field. In order to reach the threshold energy for trident pair production ($2m_0c^2$), we must have

$$2m_0c^2 \cong eE_p(\lambda_p/2) \pm \tfrac{1}{2}m_0v_{ep}^2 , \tag{20}$$

where E_p is the average electric field of the plasma wave, λ_p is the wavelength, and v_{ep} is the velocity component of the individual electron in the direction of the electric field E_p. The number of electrons that will be accelerated thus depends not only on the wave intensity, but also on the shape of the velocity distribution on the "tail" of the distribution function. If the suprathermal tail is sufficiently large, an appreciable number of electrons can be accelerated to the threshold for trident pair production.

We conclude that it may be possible to obtain pair production from electrons accelerated in the turbulent plasma environment of anomalous absorption instabilities. However, we have not been able to estimate the probability of this process, or to make quantitative estimates of the production rate of the relativistic electrons. Such a capability awaits development of a relativistic theory or a relativistic plasma-simulation numerical code which can be applied to the inhomogeneous plasmas which are produced within the small dimensions of the focal-spot region of the focused laser light.

V. EXPERIMENTAL EVIDENCE FOR RELATIVISTIC ELECTRONS

In Sec. IV we have shown the plausibility of relativistic electron production by the plasma instabilities associated with the anomalous absorption of laser radiation. Experimental indications of such electrons have recently been seen at this laboratory.

These experiments were done with our "long-path" neodymium-glass-disk laser system.[21,22] A 6-ns double pulse was used whose intensity peaks were separated by 3 ns; the pulse shape was as shown in Ref. 21, not Ref. 22. The total output energy in each pulse was 80 ± 10 J. One difference from the older work was that the light path through the system made only five passes through the disks, rather than nine passes. Optical spectrometer measurements of this new pulse indicated a narrower over-all spectral width ($\simeq$ 60Å) than the previous pulse ($\simeq$ 100Å). A detailed account of the substructure of the output pulses from the long-path laser is available.[23]

Although the narrowing of the over-all spectral width seemed like a minor change, an unusually penetrating hard component of the x rays was seen when this new pulse was incident on our standard polyethylene target [approximate composition $(CH_2)_n$]. The detector was a plastic fluor (15-cm diam × 20 cm long) originally intended for neutron measurements with $(CD_2)_n$ targets; its front face was located 22 cm from the target. The absorption curve obtained from these measurements is plotted in Fig. 6.

The absorption coefficient μ of the hard component in Fig. 6 is approximately μ = 0.05 cm²/g, corresponding to x rays in the 1–10-MeV range

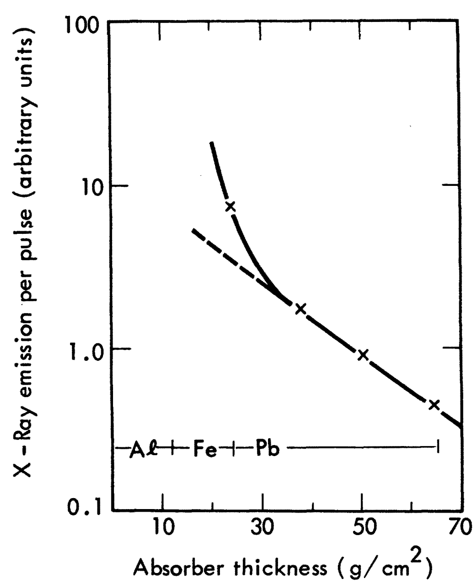

FIG. 6. X-ray absorption curve for laser-target experiment described in text. The ordinate scale is based upon the energy absorbed by the detector. The abscissa represents absorber thickness; the aluminum and iron were never removed.

(independent of the absorbers used).[24] Such extremely hard x rays were not seen in such abundance in earlier experiments[22]; the reasons for the difference are not known. Whatever the reasons, it appears that in at least one set of experimental conditions hard x rays were seen which are best interpreted as bremsstrahlung created by relativistic electrons of energy greater than $2m_0c^2$ (~1 MeV).

However, we are unable to estimate the absolute number of relativistic electrons produced, because we do not know where the x rays were produced. One-MeV electrons have a range in cold material of 0.4 g/cm^2 or more.[24] The range in hot plasma would be higher, but even the cold-material range is already much greater than the dimensions of the laser focal spot. Thus, relativistic electrons created at the target would be expected to escape into the vacuum chamber, where they would travel to the walls, creating bremsstrahlung x rays at many locations. Until further experimental work is done to isolate and measure these effects, it is not possible to estimate the absolute number of electrons produced.

These considerations also cast doubt on whether pair production takes place at the focus, since in some directions the distance from the focus that the relativistic electron travels is much greater than the focal-spot dimensions.

VI. COMPARISON OF PAIR PRODUCTION AND BREMSSTRAHLUNG BY FAST ELECTRONS

For experimental purposes it is of interest to compare the average energy loss of fast electrons by pair production to the average energy loss by bremsstrahlung x radiation. To do this, consider the probability dP that pairs are produced in distance dx:

$$dP = \sigma_T N_n\, dx , \tag{21}$$

where σ_T is the trident cross section and N_n is the density of nuclei per unit volume. Near the threshold we can assume that approximately all of the kinetic energy E_v is lost when the trident process occurs, so that the averaged energy loss (over many electrons) can be written

$$\left(\frac{dE}{dx}\right)_{\text{pairs}} \cong -E_v\,\frac{dP}{dx} = -9.6\times 10^{-4}\alpha^2 r_0^2 Z^2 N_n E_v \times\left(\frac{E_v}{m_0c^2}-2\right)^{3.6}, \tag{22}$$

where we have substituted from Eq. (6).

It is well known that a similar equation exists for the radiative energy loss due to bremsstrahlung;[25]

$$\left(\frac{dE}{dx}\right)_{\text{rad}} = -4\alpha r_0^2 Z^2 N_n E_v \ln\left[\left(\frac{183}{Z}\right)^{1/3}\right]. \tag{23}$$

This expression is the average energy loss at all x-ray frequencies. For comparison with the experiment described in Sec. V, however, we want to know the average radiation loss by emission of hard x rays ($E \gtrsim 2m_0c^2$). Because the x-ray spectrum is almost constant, this can be written approximately as

$$\left(\frac{dE}{dx}\right)_{\text{hard x}} \simeq -21\alpha r_0^2 Z^2 N_n (E_v - 2m_0c^2), \tag{24}$$

where we have put $Z = 1$ in the slowly varying logarithmic term.

The approximate ratio of average pair-production energy loss to average hard x-ray emission is then found from Eqs. (22) and (24):

$$\frac{(dE)_{\text{pairs}}}{(dE)_{\text{hard x}}} \simeq 4.6\times 10^{-5}\alpha\left(\frac{E_v}{m_0c^2}\right)\left(\frac{E_v}{m_0c^2}-2\right)^{2.6}. \tag{25}$$

This result can be used to make an estimate of the possibility of pair production in an experiment where bremsstrahlung x rays have been produced. In our earlier experiment,[22] 10^{-9} J of x-ray energy was emitted by the source as x rays of 100 keV or greater. One would expect much less energy to have been emitted as x rays of $2m_0c^2$ or greater. For electrons near threshold $[(E_v/m_0c^2-2)\ll 1]$, Eq. (25) predicts that the ratio of pair energy to hard x-ray energy would be much less than 10^{-6}. So one concludes that the average pair-production energy loss in this example was much less than 10^{-15} J. However, the threshold energy $2m_0c^2$ for production of a single pair is of the order of 10^{-13} J, which is still greater than this extreme upper limit. Thus we conclude that no pairs were produced in our early experiment,[22] and that one should not expect to see pair production in similar experiments unless orders-of-magnitude-greater x-ray bremsstrahlung intensities are detected.

In the more recent experiment, described above, in which more hard x rays were detected, we cannot tell whether pairs were produced, because we could not estimate the absolute x-ray intensity for that experiment.

VII. SUMMARY

We have examined various mechanisms for electron-positron pair production by intense, focused laser light pulses. Vacuum pair production was estimated to be unobservable, in agreement with previous authors. The trident process of pair production by high-energy electrons was then considered, and the cross section was calculated. The remaining question is how the high-energy (kinetic energy $> 2m_0c^2$) electrons can be created at the laser focus.

In one case, that of the coherent "quivering velocity" of an electron in a circularly polarized beam, we were able to obtain a result for the threshold of pair production which was at the extreme upper end of contemporary feasible focused intensities.

In another case, that of the parametric ion-acoustic instability which is excited in laser-produced plasmas, we have given qualitative arguments for the plausibility of production of at least a few relativistic electrons by the longitudinal plasma waves. On the basis of available experimental information, it seems unlikely that electron-positron pairs have been produced at experimental intensities of $10^{12}-10^{15}$ W/cm^2. When new experiments are done in the intensity range $10^{16}-10^{18}$ W/cm^2, however, a few pairs may possibly be produced. Thus, an experimental search for positron-electron pairs need not wait for a focused laser intensity as high as $10^{19}-10^{20}$ W/cm^2, as previously estimated.[5]

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