

Absolute Measurements of Total Cross Sections for Electron Scattering by Sodium Atoms (0.5–50 eV)*

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Absolute measurements have been made of total cross sections for the scattering of electrons by sodium over the electron energy range 0.5–50 eV; some new data for potassium at 10, 20, and 50 eV are also presented. The measured cross sections for sodium range from 352 \AA^2 at 0.5 eV, to 57 \AA^2 at 50 eV, with over-all uncertainty of 13%. At very low energies, the 250-meV energy spread in our experiment introduces additional uncertainty. As with earlier measurements from this laboratory using potassium, we find excellent agreement with close-coupling calculations.

I. INTRODUCTION

Because alkali-metal-atom beams are relatively easy to produce and detect, and because the alkalis are effectively one-electron systems, they should provide a common meeting ground for experiment and theory in electron-atom scattering studies. Indeed, we believe that experimental and theoretical results obtained in the past few years indicate the electron-potassium total cross section to be one of the most accurately known total cross sections for electron-atom scattering in the energy range covered by the measurements 0.5–50 eV. There are several groups now working on the electron-alkali-metal-atom scattering problem, and they have already produced many total and differential measurements of the collision cross sections^{1–9} including some with spin analysis.^{1–5} Theorists have been equally active.^{10–19} Besides the fundamental goal of understanding the electron-atom interaction, electron-alkali-metal studies are of considerable practical value. The low-ionization potentials of the alkali metals make them useful candidates as sources of quiescent plasmas; their high electric dipole polarizabilities²⁰ result in large elastic, inelastic, ionization, and reactive cross sections, ideal for many applications.

In 1929, Brode²¹ measured total cross sections for electron-alkali-metal-atom scattering (except lithium) using a modified Ramsauer technique. Earlier articles^{1,6,22} have already discussed at length the probable errors in Brode's alkali measurements. In 1962, Perel, Englander, and Bederson²³ reported relative measurements for sodium and lithium scattering. A comparison was made to potassium at one energy, which in turn was normalized to Brode's potassium curve, the only absolute data available at the time. Recently, Collins, Bederson, and Goldstein¹ reported new absolute total cross sections for electron-potassium scattering, and Visconti, Slevin, and Rubin⁶

reported absolute total cross sections for electron scattering by the heavier alkalis, potassium, rubidium, and cesium. These measurements have shown that the Brode cross-section results are roughly a factor of 2 too large, and display spurious structure around the n_0S-n_0P inelastic threshold, where n_0 is the principal quantum number of the ground state. In this article, we present absolute measurements of total electron scattering cross sections for sodium over the electron energy range 0.5–50 eV.²⁴ These are the first absolute measurements aside from those of Brode for sodium. Some new data are also presented for potassium at 10, 20, and 50 eV. Our results for sodium show the same excellent agreement with close-coupling calculations as did the earlier potassium measurements made in our laboratory (Collins *et al.*).

The theoretical work on electron-alkali scattering is mostly of two types: adiabatic potential (polarized orbitals) and close-coupling techniques. Various formulations of the adiabatic model can be found in the literature. Stone and Reitz¹³ and Garrett and Mann¹⁴ have used an adiabatic model to calculate electron-cesium cross sections. Garrett¹⁵ has also calculated electron-lithium and electron-sodium cross sections. These calculations yielded results, which in light of more recent theoretical and experimental work, are too large. The adiabatic calculations of Crown and Russek¹⁶ for cesium, and of Balling¹⁷ for rubidium, are close to the experimental values of Visconti *et al.*⁶ at higher energies, but differ both qualitatively and quantitatively at low energies (<3 eV).

In recent years, close-coupling calculations have seen much success with electron-alkali elastic scattering.⁵ The two-state calculation of Karule¹⁰ and Karule and Peterkop¹⁰ in 1965 was the first to be reported for the alkalis. Hartree-Fock wave functions were used for lithium, and semiempirical wave functions were used for sodi-

um, potassium, and cesium. The calculations covered the energy range 0–5 eV for elastic and n_0S-n_0P inelastic scattering. Burke and Taylor¹¹ performed a $2s-2p$ and a $2s-2p-3d$ close-coupling calculation for electron-lithium scattering, and except at very low energies, their results are in excellent agreement with Karule, and Karule and Peterkop. The most recent close-coupling calculations are those of Norcross¹² for sodium and lithium, and Moores and Norcross¹² for sodium. The latter is a four-state ($3s-3p-4s-3d$) calculation using wave functions obtained from a scaled Thomas-Fermi potential which includes core polarization. We will compare our experimental measurements to the close-coupling results in Sec. III. Lately, new computations by Sinfaillam and Nesbet¹⁸ have been published for elastic electron-alkali scattering. They use a variational formulation of the continuum Bethe-Goldstone equations and have results for electron energies below about 1 eV. Their total cross-section results for lithium, sodium, and potassium are in excellent agreement with the close-coupling calculations of Karule for electron energies relevant to the present experiment (>0.5 eV). The Glauber approximation has been used by Tripathi, Mathur, and Joshi¹⁹ for electron-sodium scattering. Their low-energy results are as much as a factor of 2 above the close-coupling values.

II. METHOD AND APPARATUS

Our atom-beam recoil technique for measuring total cross sections has been previously described^{1, 4, 22} We observe the attenuation in the forward atom beam intensity due to electron scattering. Figure 1 is a schematic diagram of the apparatus, showing slit sizes and other relevant dimensions. The beam source is a conventional alkali oven, heated to 400 °C for sodium and 300 °C for potassium, in both cases yielding a vapor

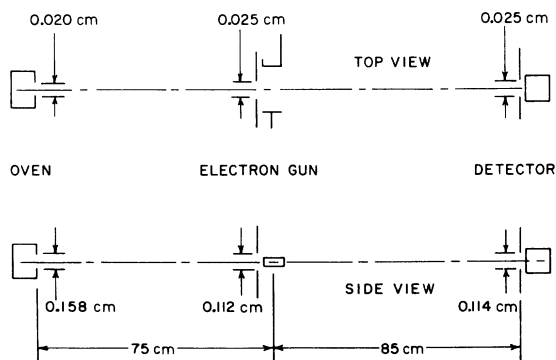


FIG. 1. Schematic diagram of the apparatus, giving relevant dimensions.

pressure in the oven of a few hundred millitorr. The most critical element in this experiment is the electron gun. A complete description of our electron gun is contained in Ref. 25. The gun is used to produce a uniform reasonably parallel beam of electrons 0.08×2.51 cm² in cross section. A 1.1-kG magnetic field collimates the beam and reduces transverse velocity components due to mutual repulsion. Retarding potential measurements determine the electron energies, which are checked (above the n_0S-n_0P threshold) by moving the detector off-axis to observe inelastically scattered atoms corresponding to 0° inelastic electron scattering; the technique is described in Refs. 1 and 6. The electron energy spreads are about 250-meV full width at half-maximum below 3-eV electron energy, and 350–400 meV at 10 eV. The detector is a hot-wire surface-ionization detector, followed by a simple magnetic mass spectrometer and a continuous-channel electron multiplier. Platinum and oxygenated tungsten hot wires were used for beam detection. The purpose of the mass spectrometer was to remove the potassium background noise from the hot wire when studying sodium. In both cases the mass-analyzed background current was several orders of magnitude less than the full beam current.

The detector output current was digitized with a voltage-to-frequency converter and the output pulse rate, proportional to the beam intensity, was accumulated by a multichannel analyzer operating in the multiscaling mode. The electron gun was cycled on and off with a 6-sec period by

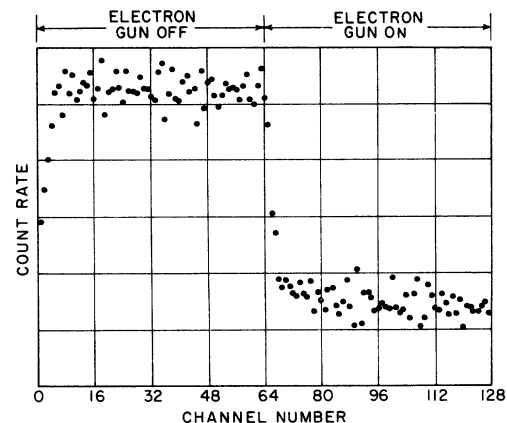


FIG. 2. Typical output of the multichannel scaler. The zero level is suppressed. The left-hand half of the figure represents the top 0.1% of the full atom beam, and the right-hand half is the beam attenuated by scattering events. The difference between the two beam levels, divided by the "electron-gun-off" count rate, gives the fraction of atoms scattered during the integration time.

the multichannel analyzer, and after $\frac{1}{2}$ -1 h of integration, reasonable signal-to-noise ratios (~ 40) were obtained. The signal in this case is the difference between the "gun-off" and "gun-on" levels in the analyzer, being proportional to the number of atoms scattered during the integration time. An example of the raw data is given in Fig. 2. In principle, two scalers could be used instead of the 128-channel multiscaler actually used, but the latter allows for invaluable diagnostic checks. The time display makes for easy observation of possible problems involving the hot-wire time constant, background gas scattering, or fluctuations in the beam intensity during data runs. In addition, it permits statistical calculations such as standard deviations and the signal-to-noise ratio for each datum.

The measured total cross section is given in terms of experimental quantities by^{1, 4, 22}

$$\sigma_T = h \bar{v} I_s / I_a I_e, \quad (1)$$

where h is the atom-beam height at the interaction region; \bar{v} is the average atom speed in the beam source; I_s is the measured scattering signal proportional to the scattered atom intensity; I_a is the full beam signal as given by the multichannel scaler, proportional to the true intensity; and I_e is the electron-particle current passing through the atom beam. Since I_s and I_a are measured with the same detection system, all gain factors cancel. This is one principal advantage of the atom-beam recoil technique. The measurements are absolute in the sense that it is not necessary to have knowledge of the atom-beam density in the interaction region provided the detector is linear. The factors h , \bar{v} , and I_e enter Eq. (1) from an evaluation of the electron- and atom-beam overlap integral, which is simplified by assuming that the atom beam is uniform over its height and by arranging slit sizes so that all of the electrons pass through the atom beam. Thermocouple measurements of the oven temperature were used to obtain \bar{v} . The angular resolution error in Eq. (1) will be discussed in Sec. IV.

III. DISCUSSION OF RESULTS

As a check on the present experimental setup, which was substantially renovated and improved since earlier potassium measurements (Collins *et al.*¹), we first remeasured the electron-potassium total cross section. The results are presented in Fig. 3, along with the data of Collins *et al.* and Visconti *et al.*,⁶ and the close-coupling results of Karule¹⁰ and Karule and Peterkop.¹⁰ All three of the experiments compared in Fig. 3 are the atom-beam recoil type, and are seen to

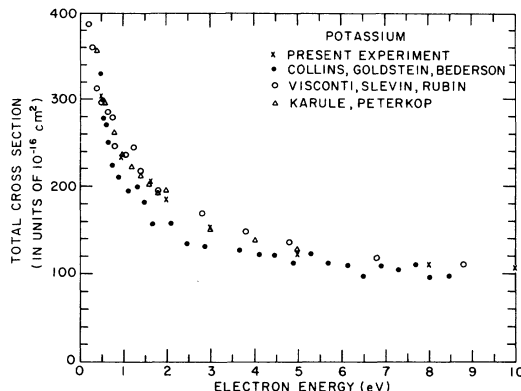


FIG. 3. Comparison of our absolute-total-cross-section measurements for electron-potassium scattering with other experiments (Collins *et al.* and Visconti *et al.*) and a two-state close-coupling calculation of Karule, and Karule and Peterkop. Not shown are our data at 20 and 50 eV: 93 and 75 Å², respectively.

be in good agreement with each other. Moreover, there is good agreement with the two-state close-coupling calculation. In making this comparison, as well as others below, we have plotted theoretical total elastic cross sections below the first inelastic threshold, and

$$\int \sigma_{\text{elastic}}(\theta) d\Omega + \int \sigma_{n_0S-n_0P}(\theta) d\Omega \quad (2)$$

for electron energies above the threshold for excitation of the n_0S-n_0P resonance transition in the alkalis. In the case of the sodium calculation of Moores and Norcross, contributions from 3S-4S and 3S-3D excitations are also included. The impressive agreement suggests that the electron-potassium total cross section is among the most accurately known total cross sections for electron-atom scattering in this energy range, although there is close competition from electron-helium scattering measurements.²² Many of the uncertainties (Sec. IV) associated with our measurement could be reduced considerably, and absolute results accurate to $\pm 5\%$ could be obtained, if warranted.

In Fig. 4 we show our electron-sodium total cross-section data, the two-state close-coupling calculation of Karule¹⁰ and Karule and Peterkop,¹⁰ and the four-state close-coupling calculation of Moores and Norcross.¹² The only absolute measurements for sodium besides the present work are those of Brode²¹ which are found to be, just as with potassium, rubidium, and cesium,^{1, 6} a factor of 2 too large, with greatly exaggerated structure near the 3S-3P excitation threshold. (Renormalizing the data of Perel *et al.*²³ to the present potassium cross sections yields values in excellent agreement with the present sodium cross

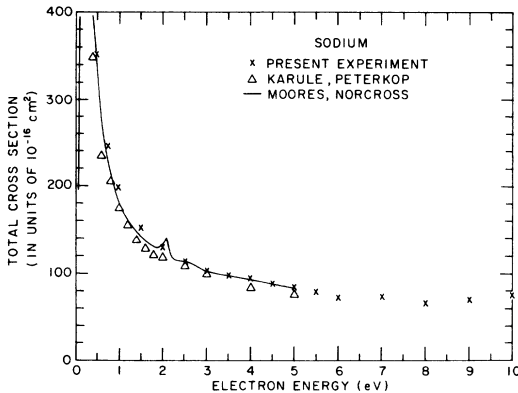


FIG. 4. Comparison of our absolute-total-cross-section measurements for electron-sodium scattering with a two-state close-coupling calculation of Karule, and Karule and Peterkop, and a four-state close-coupling calculation of Moores and Norcross. Not shown are our data at 20 and 50 eV: 65 and 57 \AA^2 , respectively.

sections.) Moores and Norcross predict a small cusp feature in the total cross section at the opening of the first inelastic channel. We used our multichannel analyzer to scan the electron energy range from 1.5 to 3 eV in an attempt to observe the cusp, but saw no departure from a slowly decreasing cross section, amidst noise corresponding to about $\pm 4 \text{\AA}^2$. The predicted width of the cusp is equal to about one-half of our electron energy spread, and if the cusp is present, it would seem to be masked by the electron energy width. (The cusp is more evident in optical experiments.^{5, 12, 26}) Otherwise, there is excellent agreement between our measurements and the close-coupling results.

In Table I we tabulate our total cross-section results.

IV. ANALYSIS OF UNCERTAINTIES

We have made a number of experimental consistency tests, the main ones being a test of the independence of the measured cross sections on electron current and on atom-beam intensity. The present potassium data are remeasurements of cross sections previously published from this laboratory, although the present work is somewhat more accurate and includes points at 10, 20, and 50 eV. At all electron energies used in our sodium work, data were obtained for at least two values of the electron current, one roughly one-half of that of the other. (In Figs. 3 and 4, the data at each electron energy value are shown averaged, weighted by the signal-to-noise ratio for each individual datum.) In some cases, several electron-current values were used and results

for sodium at 5 eV are shown in Fig. 5.

Figure 6 is an example of measurements of the total cross section for different oven temperatures, for electron-sodium scattering at 5 eV. The apparent cross section is seen to be independent of the oven temperature. The range of temperatures is such that the beam intensity changes by a factor of 15, and the average beam velocity changes by 14%.

In Fig. 7 we show measurements of apparent electron-sodium total cross sections at 5 eV for different values of the electron-gun anode voltage. At low anode voltages, the suppression of secondary electrons produced at the anode and reentering the interaction region is not complete, thereby introducing anomalous scattering. The presence of such scattering can also be checked by moving the detector off axis in directions both parallel and antiparallel to the direction of the electron beam. Recoil-scattered atoms should only be seen on the parallel side; this is indeed the case in our present experiment. In Fig. 7 it can be seen that at the usual operating point of 35 V on the anode, suppression of this spurious effect is essentially complete.

The electron gun was always operated at a low enough current to avoid space-charge depression of the electron energy, from 20 to 30 μA at 0.5 eV up to 200 μA at 10 eV. The electron-current mea-

TABLE I. A tabulation of our absolute total cross sections for electron scattering by sodium and potassium atoms. The uncertainty in the sodium data is 13% below 4 eV, and 12% above 4 eV. The uncertainty in the potassium data is 15% below 4 eV and 12% above 4 eV.

Electron energy (eV)	$\sigma_T(\text{Na})$ (\AA^2)	$\sigma_T(\text{K})$ (\AA^2)
0.5	352	304
0.75	246	...
1	197	234
1.5	152	...
1.6	...	205
2	129	185
2.5	113	...
3	102	152
3.5	97	...
4	93	...
4.5	88	...
5	82	122
5.5	79	...
6	72	...
7	73	...
8	66	109
9	69	...
10	75	105
20	65	93
50	57	75

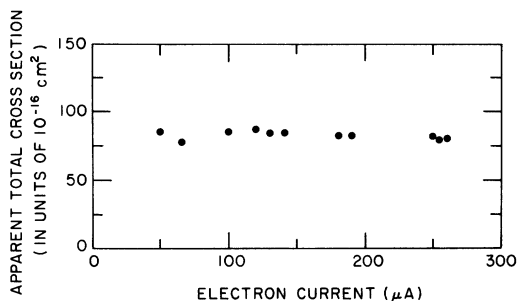


FIG. 5. Consistency check on the effect of the magnitude of the electron-beam current for sodium at 5-eV electron energy.

surement is assigned an uncertainty of 1%.

The atom-beam height h in the interaction region is determined by a slit in front of the electron gun, and we assign an uncertainty of 1% to this quantity, which directly enters the cross-section expression, Eq. (1).

The average atom speed \bar{v} in Eq. (1) is determined from the alkali oven temperature as measured with a thermocouple embedded in the oven block near the exit slit. The oven vapor pressure is low enough to assure free molecular flow from the oven (Sec. II). The beam speed determination may be checked in two ways: (a) by the constancy of the apparent cross section for different oven temperatures, as in Fig. 6; and (b) with a velocity-selected beam measurement of the cross section, obtaining the velocity from the onset of 0° inelastic scattering (Sec. II). The first method results in a 14% variation of \bar{v} for a temperature range practical for the experiment. The second method is poor statistically ($\sim 5\%$) but serves as a gross consistency check. In all, we ascribe a 5% uncertainty to the factor \bar{v} in Eq. (1).

The presence of molecules in the beam can be

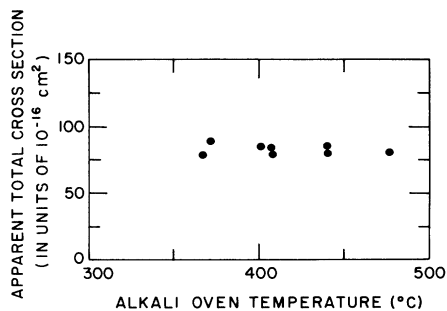


FIG. 6. Consistency check on the effect of the beam intensity and speed for sodium scattering at 5-eV electron energy. The oven-temperature range covered corresponds to an over-all change in the beam intensity of a factor of 15, and a change in the average beam speed of about 14%.

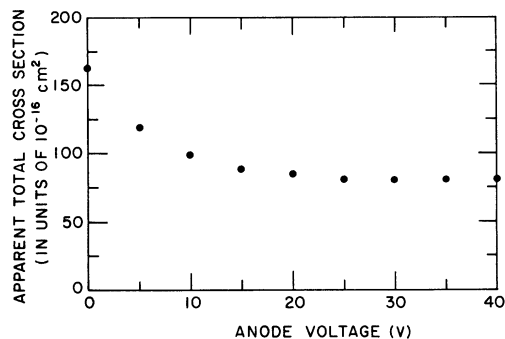


FIG. 7. Effects of secondary and reflected electrons on the electron-sodium scattering signal, at 5-eV electron energy. Such effects are insignificant at our normal operating point of 35 V.

demonstrated by deflecting the atomic component out of the beam with a Stern-Gerlach magnet inserted following the alkali oven. The molecular content of the alkali beam was always about 1% or less; however, the molecular total cross section is roughly 50% greater than that of the corresponding atom,²⁷ and the molecular speed is 0.7 that of the corresponding atom, resulting in an increase in the effective atomic cross-section measurements of about 1%.

We have calculated the angular-resolution error of our apparatus (see Appendix) and give representative values in Table II. The finite resolution of the recoil experiment results from scattering through angles which are too small to cause scattering out of the forward beam, thereby not contributing to the measured total cross section. The error caused by this finite resolution can be estimated from the known geometry of the experiment, using the theoretical elastic differential

TABLE II. Results of an angular-resolution calculation for our apparatus. The angles given, at three different energies, are the electron-polar-scattering angle at which our detection efficiency is 0.5. For smaller angles the detection efficiency worsens, and for larger angles it rapidly increases to unity. The corresponding errors in the total cross sections were calculated from the angular-detection efficiencies and theoretical differential cross sections of Karule (Ref. 10). The error always results in a measured cross section which is less than the correct cross section.

Atom	At 0.5 eV	At 1 eV	At 5 eV
Sodium			
Angular resolution	7.5°	5.5°	2.8°
Total cross section error	1.8%	1.7%	0.5%
Potassium			
Angular resolution	9°	6.5°	3.3°
Total cross section error	2.9%	3.8%	0.7%

cross sections, which are certainly sufficiently reliable for this purpose. The error always results in a measured value which is less than the correct cross section, as opposed to that attributed to molecules in the beam. Our calculations indicate a maximum angular resolution error in the sodium data of 2% below 4 eV, and 1% above 4 eV. For the potassium data, the maximum angular resolution error is 4% below 4 eV, and 1% above 4 eV.

The statistical uncertainty in the data is at worst 3%. A total of 60 data were obtained for sodium and 12 for potassium. The combined statistical and systematic uncertainty for the sodium results, without regard to sign, is a maximum of 13% below 4 eV, and 12% above 4 eV. For the potassium results the over-all uncertainty is a maximum of 15% below 4 eV, and 12% above 4 eV. In addition, in examining our results one should consider our electron energies uncertain by 0.15 eV, and at very low energies our energy spread of 250-meV full width at half-maximum is important. Above 3 eV, the energy spread increases, to 350–400 meV at 50 eV.

V. CONCLUSIONS

In summary, our absolute measurements of the electron-sodium and electron-potassium cross sections are in excellent agreement with existing close-coupling calculations. In other work, agreement with close-coupling results for elastic electron scattering by sodium and potassium has been found in differential-cross-section measurements,^{1, 3-5, 7-9} even those yielding spin information.^{1, 3-5} (Differential *inelastic*-scattering results,^{2, 5} for potassium, do not compare so favorably with two-state close-coupling calculations.) We are attempting differential spin-exchange cross-section measurements for electron-sodium scattering, and plan to extend all of these measurements to lithium.

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APPENDIX

In order to calculate the angular-resolution error in our results we have used an unpublished analysis by Eisner.^{2b} The analysis will be outlined below. Atoms scattered through angles χ and ψ are considered, to determine if they are scattered out of either the side of the detector slit or the top or bottom of the detector slit. If

so, then they are indeed counted as having been scattered and no error in the total cross section occurs. If χ and ψ are too small then an error results. The efficiencies for observation of a scattering event are averaged over the finite slit widths and heights. Beam divergence is not taken into account, and it is assumed that the detector slit is not smaller than the atom-beam size in the interaction region. Inelastically scattered atoms need not be considered as they are all scattered outside of the detector slit. The expressions for the efficiencies are simple in terms of the atom scattering angles χ and ψ but take on a complicated appearance in terms of the more conventional electron scattering angles θ and φ because of the transformation equations.

Define a reference plane by the direction of the incoming atom and electron beams. After an elastic collision the scattered atom direction may be described by a polar angle ψ in the reference plane and an azimuthal angle χ in a perpendicular plane containing the scattered atom ray. The scattered electron direction may be described by a polar angle θ between the incident and scattered electron directions and an azimuthal angle φ in a plane perpendicular to the reference plane and perpendicular to the incident electron direction.

The angles are related by the momentum-conservation equations:

$$MV = M'V' \cos\chi \cos\psi + mv' \sin\theta \cos\varphi, \quad (3)$$

$$mv = M'V' \cos\chi \sin\psi + mv' \cos\theta, \quad (4)$$

$$0 = M'V' \sin\chi - mv' \sin\theta \sin\varphi, \quad (5)$$

where upper-case letters refer to the atom, lower-case letters refer to the electron, V and v are the respective velocities before the collision, and V' and v' are the respective velocities after the collision. Solving these equations for ψ and χ and introducing small-angle approximations (since χ and ψ are small and we are only interested in small values of θ), we find

$$\chi = \alpha \theta \sin\varphi, \quad (6)$$

$$\Psi = \frac{1}{2}\theta^2 \alpha, \quad (7)$$

where α is mv/MV .

If w is the width of the detector slit and s is the width of the atom beam at the interaction region, then the largest angle ψ at which an atom can be detected is $\psi = (w + s)/2L$, where L is the distance between the interaction region and the detector slit. For angles $\psi < (w - s)/2L$, all atoms are detected regardless of their position in the beam. In a recoil attenuation experiment a detected atom is not counted as having been scattered. Thus for $\psi < (w - s)/2L$ the experimental efficiency for

observation of a scattering event across the width of the detector is $\kappa_w = 0$ and for $\psi > (w + s)/2L$, $\kappa_w = 1$. In between, κ_w is taken to be linear in ψ , in effect an averaging over the slit width. Using Eq. (7) to express κ_w in terms of the electron-polar-scattering angle,

$$\begin{aligned}\kappa_w(\theta) &= 0, \quad \theta \leq \theta_{\min}, \\ \kappa_w(\theta) &= L\alpha\theta^2/2s - (w - s)/2s, \quad \theta_{\min} < \theta < \theta_{\max}, \\ \kappa_w(\theta) &= 1, \quad \theta \geq \theta_{\max},\end{aligned}\quad (8)$$

where

$$\theta_{\min} = [(w - s)/\alpha L]^{1/2} \quad \text{and} \quad \theta_{\max} = [(w + s)/\alpha L]^{1/2}.$$

A similar procedure gives the efficiency κ_h when the scattering is out of the top or bottom of the detector slit. If H is the height of the detector and h is the height of the atom beam in the interaction region, then for angles $\chi < (H - h)/2L$, $\kappa_h = 0$ and for angles $\chi > (H + h)/2L$, $\kappa_h = 1$. In between, κ_h is taken to be linear in χ . The expression of this result as a function of the electron-polar-scattering angle is complicated since χ depends on both θ and φ [Eq. (6)]. For a given value of θ , we ascertain what fraction of the scattered atoms correspond to φ values which give rise to efficiencies of 1, 0, or in between, respectively, in connection with scattering out of the top or bottom of the detector slit. The result is

$$\begin{aligned}\kappa_h(\theta) &= 1 - \frac{2}{\pi} \sin^{-1} \frac{\chi_{\max}}{\alpha\theta} \\ &+ \frac{2}{\pi} \int_{\chi_{\min}}^{\chi_{\max}} \frac{\chi - \chi_{\min}}{\chi_{\max} - \chi_{\min}} \sin^{-1} \frac{\chi}{\alpha\theta} d\chi,\end{aligned}\quad (9)$$

where $\chi_{\min} = (H - h)/2L$ and $\chi_{\max} = (H + h)/2L$. The first term alone would imply that all scattering events are observed. The second term subtracts the fraction of scattered atoms which correspond to φ values less than $\varphi_{\max} = \sin^{-1}(\chi_{\max}/\alpha\theta)$. The third term adds back on the fraction of atoms scattered with angles such that they may or may not be detected depending on their position in the beam at the interaction region. In this term an average of $\varphi = \sin^{-1}(\chi/\alpha\theta)$ is taken over the detector height using the atom angle χ as the variable. The second and third terms are multiplied by 4 to account for the fact that the momentum sphere⁴ has a front, back, top, and bottom. That is, not only values of $\varphi < \varphi_{\max}$ are affected by the detector height but also $\pi - \varphi_{\max} < \varphi < \pi + \varphi_{\max}$ and $2\pi - \varphi_{\max} < \varphi < 2\pi$.

The analysis for $\kappa_w(\theta)$ assumes an infinitely high detector slit. The analysis for $\kappa_h(\theta)$ assumes an infinitely wide detector. The over-all efficiency $\kappa(\theta)$ is a combination of $\kappa_w(\theta)$ and $\kappa_h(\theta)$:

$$\kappa(\theta) = \kappa_w(\theta) + \kappa_h(\theta) - \kappa_w(\theta)\kappa_h(\theta),\quad (10)$$

in accordance with the rule for adding probabilities.

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