Probabilistic ancilla-free phase-covariant telecloning of qudits with the optimal fidelity

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We study the ancilla-free $1\to 2$ phase-covariant telecloning for qudits. We show that the fidelity of the two clones can probabilistically reach that of the clones in the optimal $1\to 2$ phase-covariant cloning (involving an ancilla). More interestingly, that can realize the above nonlocal cloning tasks are suitable nonmaximally entangled states rather than the maximally entangled states.

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It is impossible to exactly copy (that is, clone) an arbitrary quantum state because of the linearity of quantum mechanics [1,2]. Nevertheless, the question of how well one can clone an unknown or partially unknown quantum state has been attracting much interest [3] since Bužek and Hillery [4] first introduced the concept of approximate quantum copying, because it is related to quantum computation, quantum communication, and quantum cryptography (see, e.g., [5–8]). If the input quantum state is chosen from a subset of linear independent states, exact copying can be realized probabilistically [9,10]. For the input state $|\psi\rangle=\sum_{j=0}^{d-1}\alpha_j e^{i\theta_j}|j\rangle$ ($d\geq 2$ is the dimension) with α_j being real numbers satisfying the normalization condition $\sum_{j=0}^{d-1}\alpha_j^2=1$ and $\theta_j\in[0,2\pi)$, three types of (approximate) quantum cloning have been intensively studied, i.e., universal quantum cloning with α_j and θ_j being completely unknown [11–13], real state cloning with $\theta_j=0$ and α_j being unknown [14–16], and phase-covariant cloning with $\alpha_j=1/\sqrt{d}$ and θ_j being unknown [14,17]. In general, the more the information about the input state is known, the better the state can be cloned. As a consequence, the optimal fidelities of clones (the fidelity limit that quantum mechanics allows) in the real state cloning and phase-covariant cloning are higher than that in the universal quantum cloning. Recently, more attention was paid to phase-covariant cloning because of its use in connection with quantum cryptography [18].

Quantum-cloning process can be regarded as distribution of quantum information from the initial system to a larger one. Thus quantum cloning combining with other quantum-information processing tasks may have potential applications in quantum communication, distributed quantum computation, and so on [19,20]. This leads to the advent of the concept of telecloning [21], which is the combination of quantum cloning and quantum teleportation [22]. Telecloning functions as transmitting multiple copies of an unknown (or partially unknown) quantum state to distant sites, i.e., realizing one-to-many nonlocal cloning, via previously shared multipartite entangled states. The entanglement channel for telecloning can be directly constructed by the corresponding cloning transformation [23].

In the aforementioned quantum cloning and telecloning, the ancillas (extra quantum systems besides the ones used to carry the cloned states) play an important role. Recently, quantum cloning without ancillas, i.e., the so-called ancilla-free (or economical) cloning [24–27], has attracted much interest, because it may be easier than the one with ancillas for experimental implementation [28]. Durt *et al.* [27] showed that an ancilla-free version of the $1\to 2$ universal cloning with the optimal fidelity (the fidelity limit that quantum mechanics allows) cannot be realized in any dimension, and ancilla-free versions of both the $1\to 2$ Fourier-covariant [29] and phase-covariant cloning with the optimal fidelity can be implemented only for qubits. They also presented an ancilla-free phase-covariant cloning machine for qudits, with the fidelity being lower than that of the optimal phase-covariant cloning machine involving an ancilla. Note that what they discussed is the deterministic cloning where cloning is realized with 100% probability. Because of the relationship between the cloning and the corresponding telecloning [23], their conclusions also imply that the ancilla-free $1\to 2$ phase-covariant telecloning with the optimal fidelity for qudits and universal telecloning with the optimal fidelity in any dimension cannot be realized in deterministic protocols. Now a question arises: whether the ancilla-free $1\to 2$ phase-covariant telecloning for qudits and universal telecloning in any dimension with the optimal fidelity can be implemented with a certain probability? This deserves our investigation.

In this Brief Report, we present a scheme for ancilla-free $1\to 2$ phase-covariant telecloning of qudits. We show that the fidelity can probabilistically reach that of the $1\to 2$ phase-covariant cloning machine of Ref. [17]. That is, the fidelity of the clones in our ancilla-free telecloning scheme can hit to the optimal fidelity with a certain probability. More interestingly, the suitable quantum channels for realizing the above telecloning tasks are nonmaximally entangled states rather than the maximally entangled states.

First, we briefly review Durt's ancilla-free $1\to 2$ phase-covariant (symmetric) cloning machine for a d-dimensional system. For the input state

$$|\psi^{in}\rangle_1=\frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}e^{i\theta_j}|j_1\rangle, \tag{1}$$

the cloning machine (transformation) functions as [27]

$$|j_1 0_2\rangle\to|\phi^{(j)}\rangle_{12} \tag{2}$$

where

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$$|\phi^{(0)}\rangle_{12} = |0_1 0_2\rangle,$$

$$|\phi^{(j)}\rangle_{12} = \frac{1}{\sqrt{2}}(|j_1 0_2\rangle + |0_1 j_2\rangle), \quad j \neq 0. \tag{3}$$

Here, we have assumed that the second quantum system (carrier) is initially in the state $|0_2\rangle$. The output state reads

$$|\psi^{out}\rangle_{12} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} e^{i\theta_j} |\phi^{(j)}\rangle_{12}. \tag{4}$$

The fidelity of each clone (copy) is

$$F_{econ}(d) = \langle \psi^{in}|_{1(2)} \mathrm{Tr}_{2(1)}(|\psi^{out}\rangle_{12}\langle \psi^{out}|)|\psi^{in}\rangle_{1(2)}$$
$$= \frac{1}{2d^2}[(d-1)^2 + (1+2\sqrt{2})(d-1) + 2]. \tag{5}$$

However, the optimal fidelity of $1 \to 2$ phase-covariant cloning (with an ancilla) is [17]

$$F_{opt}(d) = \frac{1}{4d}(d + 2 + \sqrt{d^2 + 4d - 4}). \tag{6}$$

It can be verified that for $d=2$, $F_{econ}(2) = F_{opt}(2)$, while for $d > 2$, $F_{econ}(d) < F_{opt}(d)$. Thus this type of ancilla-free phase-covariant cloning is "suboptimal."

We now describe our telecloning protocol. The task is: Alice wants to transmit one copy of the state $|\psi^{in}\rangle_{A_1}$ of particle A_1 to distant Bob and Charlie, respectively. Assume that the quantum channel among them is a three-particle entangled state as follows:

$$|\Psi\rangle_{A_2BC} = \sum_{j=0}^{d-1} x_j |j_{A_2}\rangle |\phi^{(j)}\rangle_{BC}, \tag{7}$$

where x_j are probability amplitudes satisfying normalization condition $\sum_{j=0}^{d-1} x_j^2 = 1$. For simplicity, we have assumed that x_j are real numbers. Here, particle A_2 is on Alice's hand and particles B and C are held by Bob and Charlie, respectively. The von Neumann entropy of $\rho_{A_2} = \mathrm{tr}_{BC}(|\Psi\rangle_{A_2BC}\langle \Psi|)$ is

$$S(\rho_{A_2}) = -\sum_{j=0}^{d-1} x_j^2 \log_2 x_j^2. \tag{8}$$

The state of the total system is

$$|\Psi\rangle_{total} = |\psi^{in}\rangle_{A_1} \otimes |\Psi\rangle_{A_2BC}$$
$$= \frac{1}{d} \sum_{l=0}^{d-1} \sum_{k=0}^{d-1} |\Phi\rangle^{lk}_{A_1A_2} \sum_{j=0}^{d-1} e^{-2\pi ijk/d} x_j e^{i\theta_j} |\phi^{(j\oplus l)}\rangle_{BC}, \tag{9}$$

where $j \oplus l$ denotes $j+l$ modulo d and $|\Phi\rangle^{lk}_{A_1A_2}$ are generalized Bell-basis states given by

$$|\Phi\rangle^{lk}_{A_1A_2} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \exp\left(\frac{2\pi ijk}{d}\right) |j\rangle |j \oplus l\rangle. \tag{10}$$

Alice performs a complete projective measurement jointly on particles A_1 and A_2 in the generalized Bell-basis $\{|\Phi\rangle^{lk}_{A_1A_2}, l,k = 0,1,2,\ldots,d-1\}$. If Alice gets the outcomes $|\Phi\rangle^{0k}_{A_1A_2}$ (with probability $1/d$), the state of particles B and C collapses into

$$|\widetilde{\psi}\rangle_{BC} = \sum_{j=0}^{d-1} e^{-2\pi ijk/d} x_j e^{i\theta_j} |\phi^{(j)}\rangle_{BC}. \tag{11}$$

After receiving the measurement outcome, Bob and Charlie perform, respectively, their particles the following local operation:

$$U_{A(B)} = \sum_{j=0}^{d-1} \exp\left(\frac{2\pi ijk}{d}\right) |j\rangle_{A(B)}\langle j|. \tag{12}$$

Then the state of Eq. (11) evolves into

$$|\psi^{out'}\rangle_{BC} = \sum_{j=0}^{d-1} x_j e^{i\theta_j} |\phi^{(j)}\rangle_{BC}. \tag{13}$$

The fidelity of clones that Bob and Charlie obtained is

$$F^t_{econ}(d) = \frac{1}{d}\left(1 + \sqrt{2}x_0 \sum_{j=1}^{d-1} x_j + \sum_{j=1}^{d-2} \sum_{k=j+1}^{d-1} x_j x_k\right). \tag{14}$$

Unlike Ref. [23], our protocol does not involve ancillas and thus it is ancilla-free.

If $x_j = 1/\sqrt{d}$, $S(\rho_{A_2}) = \log_2 d$ and the quantum channel is a maximally entangled state in terms of the subsystem of Alice (particle A_2) and the subsystem of Bob and Charlie (particles B and C). Then the state $|\psi^{out'}\rangle_{BC}$ reduces to $|\psi^{out}\rangle_{BC}$ given in Eq. (4) and $F^t_{econ}(d) = F_{econ}(d)$ less than $F_{opt}(d)$ for $d > 2$. In the following, we shall show that the fidelity $F^t_{econ}(d)$ can be equal to $F_{opt}(d)$ for any d with another choice of $\{x_j\}$.

We set

$$x_0 = X(d) = \sqrt{\frac{4(d-1)}{D(D+d-2)}},$$

$$x_j = Y(d) = \sqrt{\frac{d^2 + (d-2)D}{D(D+d-2)(d-1)}}, \quad j \neq 0, \tag{15}$$

where $D = \sqrt{d^2 + 4d - 4}$. Then it can be verified that $F^t_{econ}(d) = F_{opt}(d)$ for any d. In fact, the output state $(|\psi^{out'}\rangle_{BC}\langle \psi^{out'}|)$ of our telecloner is then equivalent to that (ρ^{out}_{opt}) of the optimal phase-covariant cloner after tracing out the ancilla [17,18]. Particularly, $\rho^{out}_{opt} = |\psi^{out'}\rangle_{BC}\langle \psi^{out'}| + \widetilde{\rho}$ with $\langle \psi^{in}|_{B(A)} \mathrm{tr}_{A(B)}(\widetilde{\rho}) |\psi^{in}\rangle_{B(A)} = 0$. In this case, the entanglement channel of Eq. (7) reduces to

$$|\Psi'\rangle_{A_2BC} = X(d)|0_{A_2}\rangle|\phi^{(0)}\rangle_{BC} + Y(d)\sum_{j=1}^{d-1} |j_{A_2}\rangle|\phi^{(j)}\rangle_{BC}. \tag{16}$$

If $d=2$, $S(\rho_{A_2}) = 1$ and $|\Psi'\rangle_{A_2BC}$ is a maximally entangled state. For $d > 2$, however, the amount of entanglement with von Neumann measure between particle A_2 and particles B and C is $E(|\Psi'\rangle_{A_2(BC)}) = -X^2 \log_2 X^2 - (d-1)Y^2 \log_2 Y^2 < \log_2 d$, which implies that the subsystem of Alice (sender) and the subsystem of Bob and Charlie (receivers) in the state of Eq. (16) are only partially entangled. Thus we can safely

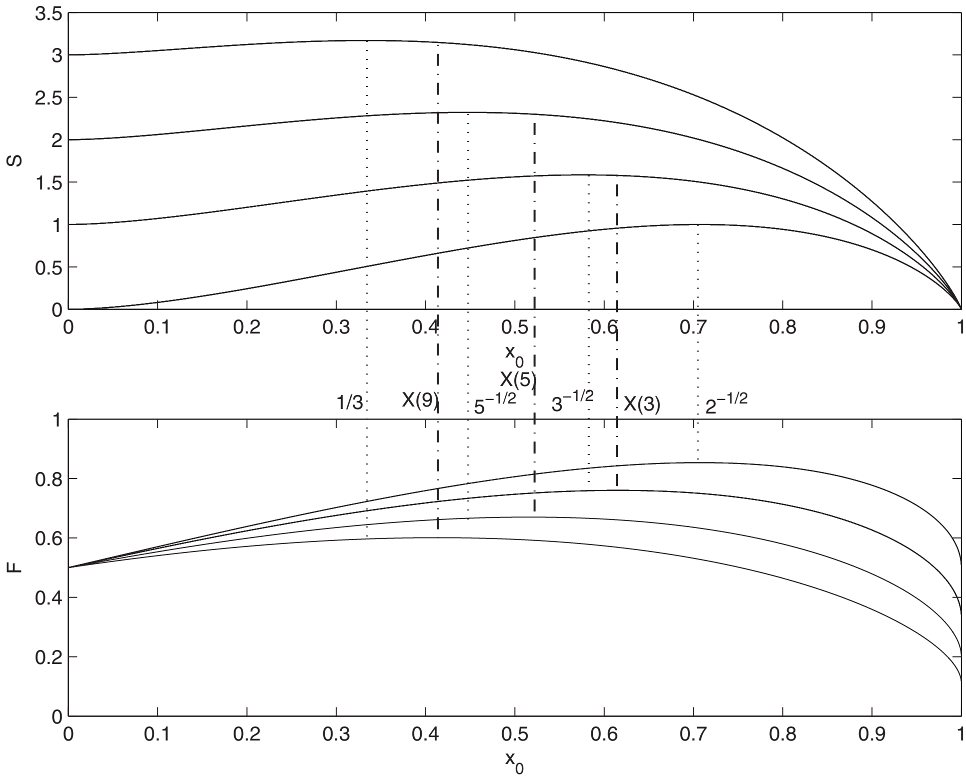

FIG. 1. The von Neumann entropy $S(\rho_{A_2})$ (upper graph) and the fidelity $F^t_{econ}(d)$ (lower graph) versus the probability amplitude x_0, where $x_1=x_2=\cdots=x_{d-1}$. From bottom (top) to top (bottom) in the upper (lower) graph, the curves correspond to $d=2$, 3, 5, and 9, respectively. The vertical dotted lines ending in the corresponding curves represent that S reaches the maximum when $x_0=1/\sqrt{d}$, and the dash-dotted lines denote that F hits to the maximum when $x_0=X(d)$.

conclude that the ancilla-free $1\rightarrow 2$ phase-covariant telecloning with the optimal fidelity can be realized with a certain probability $(1/d)$ via suitable nonmaximally entangled states acting as the quantum channel.

In order to reveal clearly the relationship between the fidelity of clones and the amount of entanglement of the quantum channel, we show how $F^t_{econ}(d)$ varies with the variation of von Neumann entropy $S(\rho_{A_2})$ in Fig. 1. For simplicity, we have assumed that $x_1=x_2=\cdots=x_{d-1}$. It can be seen that for $d=2$, the increase (decrease) in $S(\rho_{A_2})$ always leads to increase (decrease) in $F^t_{econ}(2)$. For $d>2$, however, a counterintuitive phenomenon appears: when $1/\sqrt{d}\leq x_0\leq X(d)$, $F^t_{econ}(d)$ increases (decreases) with the decrease (increase) in $S(\rho_{A_2})$.

Before ending this Brief Report, we should point out that if Alice's joint measurement outcome is not $|\Phi\rangle^{0k}_{A_1A_2}$ but $|\Phi\rangle^{jk}_{A_1A_2}(j\neq 0)$, Bob and Charlie can also obtain the clones of $|\psi^{in}\rangle_{A_1}$ with a certain fidelity $F^{t'}_{econ}(d)$ by performing appropriate local operations. With the quantum channel $|\Psi'\rangle_{A_2BC}$, $F^{t'}_{econ}(d)=\frac{1}{d}[1+(d-2+\sqrt{2})X(d)Y(d)+\frac{d-2}{2}(d-3+2\sqrt{2})Y^2(d)]$. It can be easily verified that $F^{t'}_{econ}(d)$ is less than $F_{econ}(d)$ and $F^t_{econ}(d)$. This case will not be discussed in detail because what we are interested in is to show how to obtain the optimal fidelity (the fidelity limit that quantum mechanics allows) of clones in the ancilla-free phase-covariant telecloning for qudits in this Brief Report.

In conclusion, we have studied the ancilla-free $1\rightarrow 2$ phase-covariant telecloning for qudits. We have shown that the fidelity can probabilistically reach that of the $1\rightarrow 2$ phase-covariant cloning machine of Ref. [17]. In other words, the fidelity of the clones in our ancilla-free telecloning scheme can hit to the optimal fidelity (the fidelity limit that quantum mechanics allows for phase-covariant cloning) with a certain probability. We have also shown that the increase (decrease) in amount of entanglement of the quantum channel may lead to the decrease (increase) in the fidelity of clones in the ancilla-free phase-covariant telecloning for qudits. This effect leads to another interesting phenomenon: the suitable quantum channels for realizing the ancilla-free $1\rightarrow 2$ phase-covariant telecloning of qudits are special configurations of nonmaximally entangled states rather than the maximally entangled states. Note that nonmaximally entangled states can be better than the maximally entangled states for several other quantum tasks has also been reported [30].

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