Effect of dephasing on quantum features of the cavity radiation of an externally pumped correlated emission laser

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Analysis of the effects of dephasing on quantum features of the cavity radiation of externally pumped correlated emission laser is presented. It turns out that entanglement and nonclassical photon number correlations are strongly reliant on the rate at which the atomic coherent superposition decays specially when the amplitude of the driving radiation is smaller. It is, particularly, found that almost perfectly entangled light can be generated while the rate of dephasing and the amplitude of the external driving radiation are kept low in case the atoms are initially prepared in possible maximum coherence. It is also found that the nonclassical behavior of the photon correlation associated with the violation of Cauchy-Schwarz inequality decreases with the rate of dephasing except for some values of the amplitude of the driving radiation.

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I. INTRODUCTION

Two-mode squeezing, entanglement, and violation of Cauchy-Schwarz inequality of the cavity radiation of the three-level cascade laser of various forms have received a great deal of attention in recent years [1–11]. It has been established that the atomic coherence that can be induced via various mechanisms is accountable for observing the quantum features. In a view that the coherent superposition creates population transfer pathway, which is a basis for the correlated two-photon emission, nondegenerate three-level cascade laser in general has shown to be a source of light characterized by strong correlation of radiation modes with two different frequencies. In connection to the strong correlation between the two modes, substantial degree of nonclassicality in the cavity radiation of various forms of nondegenerate three-level cascade laser has been predicted [1–7]. Particularly, externally pumped correlated emission laser is found to be a source of entangled light that can also exhibit quantum nonlocality [1,2]. It has been claimed that the degree by which the nonclassical features are exhibited in such a system is governed by the competition between the coherence induced by the initial preparation and external pumping (coherence beating). It has been also shown recently that the fluctuations in the environment modes specially in the form of decoherence affect quantum properties although the effect is not as such devastating in the nondegenerate three-level laser as in the other quantum optical systems [8,12].

In earlier communications, the atomic decay from each energy level to any lower energy level including those involved in lasing process is taken to be the same whereupon the coherent superposition is implicitly taken to decay by the same rate as the atomic damping. This assumption unfortunately overshadows the effects of dephasing on various quantum features of the radiation as recently discussed [13]. In the present work, therefore, the atomic coherent superposition is presumed to wear away due to the existing physical processes such as vacuum fluctuations with arbitrary decay rate. It is worth noting that the atomic decay rate corresponds directly to the spontaneous emission of the radiation whereas the coherent superposition decay rate is related to the process by which the quantum phenomenon is destroyed by the pertinent fluctuations in the environment. In other words, the spontaneous emission of the cavity radiation in the system under consideration is responsible for establishing the correlation which is unfortunately destroyed by dephasing. No doubt that the competition between these two processes is profoundly interesting in studying the degree with which the quantum properties of the radiation are observed. Similar assumption has been considered earlier, although the contribution of dephasing has not been thoroughly analyzed [13–15]. It is not basically difficult to comprehend that taking the two damping rates as different leads to considerable deviation in statistical and quantum properties of the generated radiation.

Even though dephasing is expected to impose serious challenge in the manifestation of quantum features, the system under consideration (externally pumped correlated emission laser [1]) is anticipated to be a viable scheme to study the effects of dephasing on quantum features of the radiation. On the basis that the quantum properties of the nondegenerate three-level laser significantly depend on the linear gain coefficient [9,16], which is found to be inversely proportional to the square of the rate of dephasing, it has been claimed very recently that the decaying process of coherent superposition substantially affects the achievable nonclassicality [13]. This is a motivation for thoroughly studying the influence of dephasing on the quantum features of the cavity radiation. To this end, the effect of dephasing on entanglement and violation of Cauchy-Schwarz inequality is investigated employing various correlations in terms of c-number variables associated with the normal ordering. Moreover, with the aid of the commutation property of the boson operators describing different modes, the photon number correlation is associated with the Cauchy-Schwarz inequality. In the same spirit, by making use of the criterion introduced by Hillery and Zubairy [17] for separability of the product states, the photon number correlation is related to entanglement. To achieve these goals, the equations of evolution re-

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cently derived for a general pumped three-level cascade laser are adapted for a case when the atoms are initially prepared in a maximum atomic coherence [13].

II. EQUATIONS OF EVOLUTION

Interaction of pumped nondegenerate three-level cascade atom with two-mode cavity radiation is describable in the rotating-wave approximation and interaction picture by Hamiltonian of the form

$$\hat{H} = ig[\hat{a}|a\rangle\langle b| - |b\rangle\langle a|\hat{a}^\dagger + \hat{b}|b\rangle\langle c| - |c\rangle\langle b|\hat{b}^\dagger] + i\frac{\Omega}{2}[|c\rangle\langle a| - |a\rangle\langle c|], \quad (1)$$

where Ω is a real-positive constant proportional to the amplitude of the coherent radiation and g is a coupling constant chosen to be the same for both transitions. $\hat{a}$ and $\hat{b}$ are the annihilation operators that represent the two cavity modes. In the cascade configuration, the transitions from upper energy level $|a\rangle$ to the intermediate energy level $|b\rangle$ and from level $|b\rangle$ to the lower energy level $|c\rangle$ are presumed to be resonant with the cavity radiation whereas the direct spontaneous transition from $|a\rangle$ to $|c\rangle$ is dipole forbidden. In addition, it is assumed that these atoms are initially prepared to be in a maximum coherent superposition of the upper and lower energy levels, that is, the initial state of the three-level atom can be taken as $|\Psi_A(0)\rangle = \frac{1}{\sqrt{2}}[|a\rangle + |c\rangle]$. Therefore, the corresponding initial density operator would be

$$\hat{\rho}_A(0) = \tfrac{1}{2}[|a\rangle\langle a| + |a\rangle\langle c| + |c\rangle\langle a| + |c\rangle\langle c|]. \quad (2)$$

For convenience, the case in which three-level atoms in a cascade configuration and initially prepared in a maximum coherent superposition of the upper and the lower energy levels are injected into the cavity at a constant rate r_a and removed after some time which is long enough for the atoms to spontaneously decay to levels other than the middle or the lower is considered. Moreover, the atomic spontaneous damping rate from each energy level involved in lasing process is taken as constant Γ whereas the atomic coherent superposition decay rate is γ. With this assumption, the master equation that describes the cavity radiation coupled to two-mode vacuum reservoir turns out to be [13]

$$\begin{aligned}\frac{d\hat{\rho}(t)}{dt} &= \frac{\kappa}{2}[2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}] + \frac{AC_+}{2B}[2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} \\ &\quad - \hat{\rho}\hat{a}\hat{a}^\dagger] + \frac{1}{2}\left(\frac{AC_-}{B} + \kappa\right)[2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger\hat{b}] \\ &\quad - \frac{AD_+}{2B}[\hat{a}^\dagger\hat{\rho}\hat{b}^\dagger - \hat{a}\hat{b}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{b}^\dagger + \hat{b}\hat{\rho}\hat{a}] \\ &\quad - \frac{AD_-}{2B}[\hat{a}^\dagger\hat{\rho}\hat{b}^\dagger - \hat{a}^\dagger\hat{b}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{b} + \hat{b}\hat{\rho}\hat{a}],\end{aligned} \quad (3)$$

where

$$A = \frac{2r_a g^2}{\gamma^2}, \quad (4)$$

$$B = (4 + \varepsilon^2)(1 + \varepsilon'\varepsilon), \quad (5)$$

$$C_\pm = 2\varepsilon'^2 + 2\varphi \mp (2\varepsilon' + \varepsilon), \quad (6)$$

$$D_\pm = 2 - \varepsilon'\varepsilon \mp \varepsilon'(1 + \varepsilon'\varepsilon), \quad (7)$$

with $\varepsilon = \frac{\Omega}{\gamma}$, $\varepsilon' = \frac{\Omega}{\Gamma}$, and $\varphi = \frac{\gamma}{\Gamma}$. In view of the form of this master equation, it is possible to infer that C_+ stands for the gain of mode a whereas C_- stands for the lose of mode b. On the other hand, $D_\pm$ are associated with the correlation between the two modes responsible for manifestation of nonclassical features.

Applying the pertinent master equation (3), the evolution of the cavity radiation in terms of c-number variables associated with the normal ordering is found to be

$$\alpha(t) = E_+(t)\alpha(0) + F_+(t)\beta^*(0) + G_+(t) + H_+(t), \quad (8)$$

$$\beta(t) = E_-(t)\beta(0) + F_-(t)\alpha^*(0) + G_-(t) + H_-(t), \quad (9)$$

where

$$E_\pm(t) = \frac{1}{2}[(1 \pm p)e^{-\mu_+ t} + (1 \mp p)e^{-\mu_- t}], \quad (10)$$

$$F_\pm(t) = \frac{q_\pm}{2}[e^{-\mu_- t} - e^{-\mu_+ t}], \quad (11)$$

$$G_+(t) = \frac{1}{2}\int_0^t [(1+p)e^{-\mu_+(t-t')} + (1-p)e^{-\mu_-(t-t')}]f_a(t')dt', \quad (12)$$

$$G_-(t) = \frac{1}{2}\int_0^t [(1-p)e^{-\mu_+(t-t')} + (1+p)e^{-\mu_-(t-t')}]f_b(t')dt', \quad (13)$$

$$H_+(t) = \frac{q_+}{2}\int_0^t [e^{-\mu_-(t-t')} - e^{-\mu_+(t-t')}]f_b^*(t')dt', \quad (14)$$

$$H_-(t) = \frac{q_-}{2}\int_0^t [e^{-\mu_-(t-t')} - e^{-\mu_+(t-t')}]f_a^*(t')dt', \quad (15)$$

with

$$\mu_\pm = \frac{\kappa}{2} + \frac{A}{2B}\{2\varepsilon' + \varepsilon \pm [\varepsilon'^2(1 + \varepsilon\varepsilon')^2 + 4(\varepsilon'^2 + \varphi)^2 - (2 - \varepsilon'\varepsilon)^2]^{1/2}\}, \quad (16)$$

$$p = \frac{2(\varepsilon'^2 + \varphi)}{[\varepsilon'^2(1 + \varepsilon\varepsilon')^2 + 4(\varepsilon'^2 + \varphi)^2 - (2 - \varepsilon'\varepsilon)^2]^{1/2}}, \quad (17)$$

$$q_\pm = \frac{-\varepsilon'(1 + \varepsilon'\varepsilon) \pm (2 - \varepsilon'\varepsilon)}{[\varepsilon'^2(1 + \varepsilon\varepsilon')^2 + 4(\varepsilon'^2 + \varphi)^2 - (2 - \varepsilon'\varepsilon)^2]^{1/2}}. \quad (18)$$

Furthermore, $f_a(t)$ and $f_b(t)$ are the noise forces which satisfy the correlation properties

$$\langle f_a(t')f_a^*(t)\rangle = \frac{AC_+}{2B}\delta(t-t'), \tag{19}$$

$$\langle f_b(t')f_a(t)\rangle = -\frac{AD_-}{4B}\delta(t-t'), \tag{20}$$

$$\langle f_b(t')f_b^*(t)\rangle = \langle f_b(t')f_a^*(t)\rangle = \langle f_b(t')f_b(t)\rangle = \langle f_a(t')f_a(t)\rangle = 0. \tag{21}$$

It is not difficult to notice that these solutions [Eqs. (8) and (9)] would be well-behaved functions at steady state provided that $\mu_\pm \geq 0$. As a result, the case for which $\mu_\pm = 0$ is designated as the threshold condition. Despite the similarity of the form of the solution with earlier works [7], the involved parameters $(\mu_\pm, p, q_\pm)$ are significantly reliant on the rate of dephasing.

III. CONTINUOUS VARIABLE ENTANGLEMENT

It is well known by now that one of the sufficient conditions for continuous variables entanglement is

$$\Delta u^2 + \Delta v^2 < 2, \tag{22}$$

where $\hat{u} = \hat{X}_a - \hat{X}_b$ and $\hat{v} = \hat{P}_a + \hat{P}_b$ with $\hat{X}_a = \frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{a})$, $\hat{X}_b = \frac{1}{\sqrt{2}}(\hat{b}^\dagger + \hat{b})$, $\hat{P}_a = \frac{i}{\sqrt{2}}(\hat{a}^\dagger - \hat{a})$, and $\hat{P}_b = \frac{i}{\sqrt{2}}(\hat{b}^\dagger - \hat{b})$ [9,18]. In view of this, the sum of the variance for EPR-type operators can be put in terms of c-number variables associated with the normal ordering for the system under consideration as

$$\Delta u^2 + \Delta v^2 = 2[1 + \langle \alpha^*(t)\alpha(t)\rangle + \langle \beta^*(t)\beta(t)\rangle - \langle \alpha(t)\beta(t)\rangle - \langle \alpha^*(t)\beta^*(t)\rangle]. \tag{23}$$

On the basis of the fact that the noise force at time t does not statistically correlated with the cavity mode variables at earlier times and applying Eqs. (8)–(15) and (19)–(21), it is possible to find at steady state

$$\langle \alpha^*(t)\alpha(t)\rangle = A\left[\frac{C_+(1-p)^2 + D_-q_+(1-p)}{8B\mu_+}\right] + A\left[\frac{C_+(1+p)^2 - D_-q_+(1+p)}{8B\mu_-}\right] + A\left[\frac{C_+(1-p^2) + D_-q_+p}{2B(\mu_+ + \mu_-)}\right], \tag{24}$$

$$\langle \beta^*(t)\beta(t)\rangle = A\left[\frac{C_+q_-^2 + D_-q_-(1+p)}{8B\mu_+}\right] + A\left[\frac{C_+q_-^2 - D_-q_-(1-p)}{8B\mu_-}\right] - A\left[\frac{C_+q_-^2 + D_-q_-p}{2B(\mu_+ + \mu_-)}\right], \tag{25}$$

$$\langle \alpha(t)\beta(t)\rangle = A\left[\frac{2C_+q_-(1-p) + D_-(1-p^2+q_-q_+)}{16B\mu_+}\right] - A\left[\frac{2C_+q_-(1+p) - D_-(1-p^2+q_-q_+)}{16B\mu_-}\right] + A\left[\frac{2C_+q_-p + D_-(1+p^2-q_-q_+)}{4B(\mu_+ + \mu_-)}\right], \tag{26}$$

$$\langle \alpha^2(t)\rangle = \langle \beta^2(t)\rangle = \langle \alpha^*(t)\beta(t)\rangle = 0. \tag{27}$$

Since the atoms are assumed to leave the cavity within short interval of time to avoid interaction among them, the autocorrelation $[\langle \alpha^2(t)\rangle$ and $\langle \beta^2(t)\rangle]$ due to emission absorption mechanism like in the trapped ensemble of atoms [19] is not expected to show up.

Now on account of Eqs. (23)–(27), the sum of the variance of EPR-type operators turns out to be

$$\begin{aligned}\Delta u^2 + \Delta v^2 = {} & 2 + \lambda[C_+ - D_-]\frac{(\mu_+ + \mu_-)^2 + 4\mu_+\mu_-}{4B\varphi^2\mu_+\mu_-(\mu_+ + \mu_-)} \\ & + \lambda[C_+(p^2 + q_-^2 + 2q_-p) - D_-(p(q_+ - q_-) \\ & - (p^2 - q_-q_+))]\frac{(\mu_+ + \mu_-)^2 - 4\mu_+\mu_-}{4B\varphi^2\mu_+\mu_-(\mu_+ + \mu_-)} \\ & + \lambda[2C_+(p + q_-) - D_-(q_+ + q_-)]\frac{\mu_+ - \mu_-}{4B\varphi^2\mu_+\mu_-},\end{aligned} \tag{28}$$

where $\lambda = \frac{2r_a g^2}{\Gamma^2}$. In view that g and Γ are constant, it is possible to interpret λ as the rate at which the atoms are injected into the cavity. In the following, the sum of the variance of the pair of EPR-type operators $(\Delta u^2 + \Delta v^2)$ is plotted against the parametrized amplitude of the driving radiation (Ω/Γ) and the rate of dephasing (γ/Γ). It is worth noting that the admissible value of γ depends on the range of the amplitude of the driving radiation that one seeks to consider due to the restriction imposed by the condition $\mu_\pm \geq 0$. In connection to the same restriction, the rates at which the atoms are injected into the cavity are taken to be significantly different in the weak $(\Omega < \Gamma)$ and the strong $(\Omega \gg \Gamma)$ driving limits.

It is not difficult to see from Figs. 1–4 that the cavity radiation produced by the externally pumped correlated emission laser exhibits entanglement for some values of the amplitude of the external driving radiation, the rate at which the atomic coherent superposition decays, and the rate at which the atoms are injected into the cavity. Particularly, it is clearly shown in Figs. 1 and 2 that the degree of entanglement significantly depends on the rate at which the coherent superposition decays. It has been established earlier that there is no entanglement if there is no driving radiation, dephasing is not taken into consideration (which corresponds to the assumption that $\gamma = \Gamma$), and the atoms are initially prepared in a maximum atomic coherence [9]. Hence the observed entanglement for $\Omega = 0$ is directly related to the assumption that the rate of dephasing is less than the atomic decay rate. It is noticeable that for $\Omega = 0$ and $\gamma = \Gamma$ there is no entanglement in agreement with earlier report [9]. As clearly

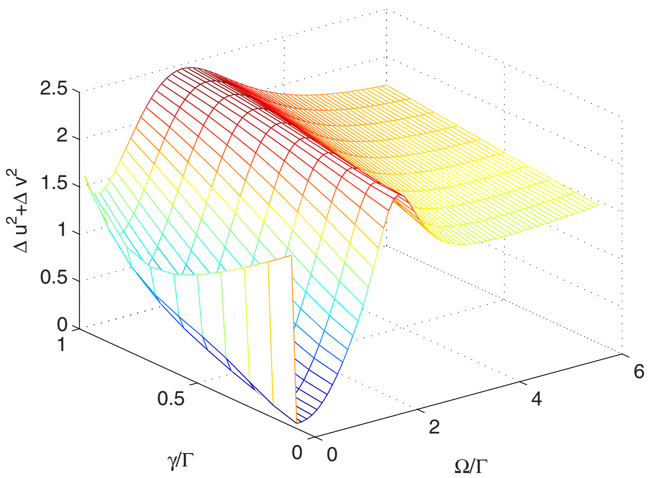

FIG. 1. (Color online) Plot of the sum of the variance of EPR-type operators $(\Delta v^2+\Delta u^2)$ of the cavity radiation at steady state for $\kappa=0.5$ and $\lambda=1.5$.

presented in Fig. 2 almost 50% degree of entanglement $(\Delta u^2+\Delta v^2\approx 1)$ occurs for different values of λ related to different rates at which coherent superposition decays. This indicates that the effects of dephasing on the degree of achievable entanglement is quite significant particularly when there is no external pumping. In this case, the value of the rate of dephasing for which maximum entanglement occurs increases with the number of the atoms available in the cavity at a given time.

On the other hand, the results shown in Figs. 3 and 4 reveal that the degree of entanglement can increase with the rate at which the atoms are injected into the cavity and can fairly decrease with the amplitude of the driving radiation in the strong driving limit. It is possible to see from Fig. 2 that $\Delta u^2+\Delta v^2\approx 1.8$ for $\Omega=0$ and $\gamma=\Gamma/10$ almost for all values of λ which is also found to be the same in Fig. 3. Due to the additional coherence induced by the external pumping, $\Delta u^2+\Delta v^2$ drops fast to zero. This shows that the effect of pumping process is significant specially when dephasing is assumed to be small. The falling of $\Delta u^2+\Delta v^2$ in strong driving limit (Fig. 4), on the other hand, is related to the fact that near critical point the entanglement vanishes in which increasing the amplitude of the driving radiation establishes the correlation responsible for entanglement. In this case, the pumping process is found to affect the entanglement significantly near threshold. In the same manner as recently predicted [7], a higher degree of entanglement is generally observed in the weak and the strong driving limits. In contrast to this, the entanglement vanishes for intermediate values of the amplitude of the driving radiation for larger values of λ. Similar conclusion has been drawn earlier without taking dephasing into consideration. Moreover, the dependence of the degree of entanglement on the amplitude of the external radiation can be related to the modification of the absorption emission mechanism of the atoms by pumping. It is hence

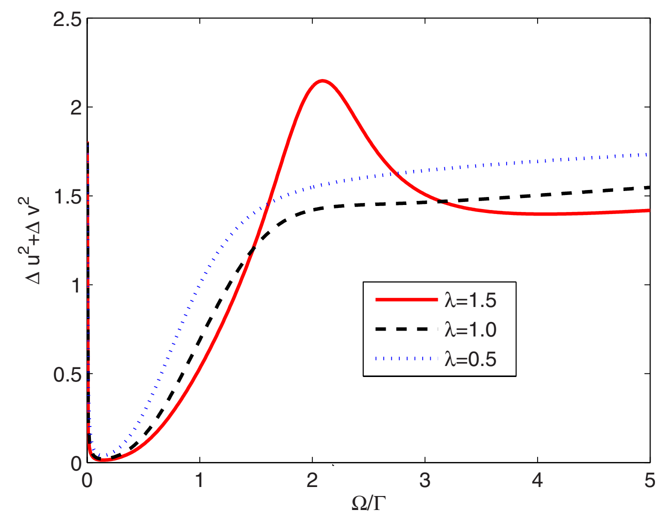

FIG. 3. (Color online) Plots of the sum of the variance of EPR-type operators $(\Delta v^2+\Delta u^2)$ of the cavity radiation at steady state for $\kappa=0.5$, $\gamma=\Gamma/10$, and different values of λ.

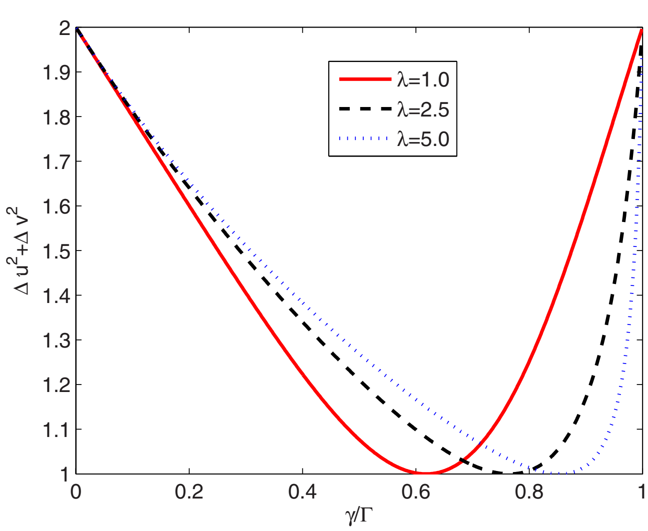

FIG. 2. (Color online) Plots of the sum of the variance of EPR-type operators $(\Delta v^2+\Delta u^2)$ of the cavity radiation at steady state for $\kappa=0.5$, $\Omega=0$, and different values of λ.

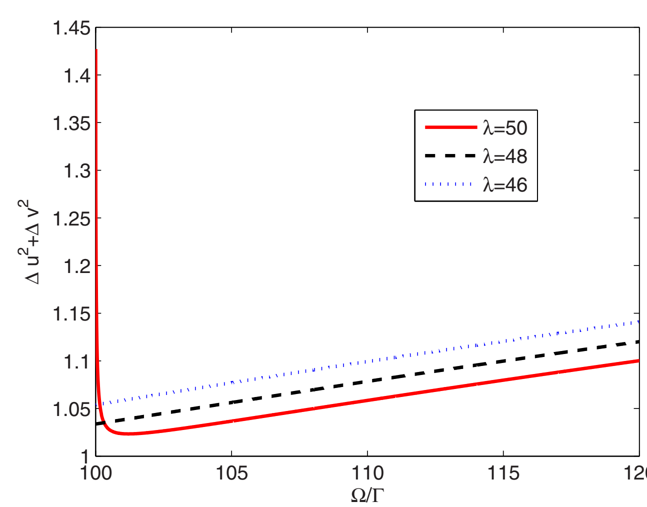

FIG. 4. (Color online) Plots of the sum of the variance of EPR-type operators $(\Delta v^2+\Delta u^2)$ of the cavity radiation at steady state for $\kappa=0.5$, $\gamma=\Gamma/10$, $\Omega\gg\Gamma$, and different values of λ.

possible to infer that the obtained degree of entanglement can be directly attributed to the interplay among the initially prepared atomic coherence, externally induced coherence, and the rate at which the available coherence decays due to the coupling of the fluctuations in the environment modes directly to the corresponding atomic transitions.

Furthermore, earlier studies have shown that a better degree of entanglement is observed when nearly 48% of the atoms are initially prepared to be in the upper energy level [9], that is, when the atoms are initially prepared in such a way that the atomic coherence is slightly less than the maximum possible value and for $\gamma = \Gamma$. Therefore, the absence of the entanglement when the atoms are initially prepared in the maximum coherent superposition and when there is no external driving radiation can be explained as the dephasing is too large to overcome the atomic coherence, so that the entanglement that could have been produced completely washed out by the dephasing in this case [9]. Fortunately, nearly perfect entanglement is witnessed for $\gamma = \Omega = \Gamma/10$ and different values of λ as clearly shown in Fig. 3. This outcome is possible provided that the atomic coherence destroyed by the dephasing is fairly compensated by the driving mechanism for optimum coherence required to produce maximum degree of entanglement to be established. Hence it would not be difficult to realize that, by properly adjusting the initially prepared atomic coherence, the strength of the external pumping, the rate at which the atoms are injected into the cavity, and the rate of dephasing, significantly entangled light can be generated in the pumped correlated emission laser coupled to two-mode vacuum reservoir.

IV. NONCLASSICAL PHOTON NUMBER CORRELATIONS

Nonclassical properties of the cavity radiation is usually studied using the correlations in the quadrature operators. In which case, experimental realization of the theoretical prediction of quantum features is found to be a formidable task due to the complications associated with the homodyne and phase measurements in many instances. Hence an alternative approach in investigating the quantum features of the radiation which involves simultaneous photon count measurement is advantageous.

A. Violation of Cauchy-Schwarz inequality

It has been argued recently that the nonclassical photon number correlation at equal time can be studied with the aid of [7]

$$C_{\alpha\beta} = \frac{1}{4}\left[1 + \frac{\langle\alpha(t)\beta(t)\rangle^2}{\langle\alpha^*(t)\alpha(t)\rangle\langle\beta^*(t)\beta(t)} \times \left(2 + \frac{\langle\alpha(t)\beta(t)\rangle^2}{\langle\alpha^*(t)\alpha(t)\rangle\langle\beta^*(t)\beta(t)\rangle}\right)\right]. \tag{29}$$

It can be claimed that nonclassical photon number correlation at equal time exits, which is also interpreted as the violation of Cauchy-Schwarz inequality [2,20], provided that

$$C_{\alpha\beta} > 1. \tag{30}$$

The dependence of the nonclassical photon number correlation on the initially prepared and externally induced atomic coherence in the nondegenerate three-level cascade scheme has been studied [2,7]. In addition to this, the comparison between the nonclassical photon number correlations and the contrast between the nonclassical photon number correlation and entanglement have been investigated. It is, in general, observed that the entanglement can be quantified via simultaneous two-photon count measurement. But, in this contribution, the main aim is shifted to studying the effect of dephasing on the violation of the Cauchy-Schwarz inequality. In order to achieve this goal, $C_{\alpha\beta}$ is plotted using Eqs. (24)–(26) against γ/Γ and Ω/Γ.

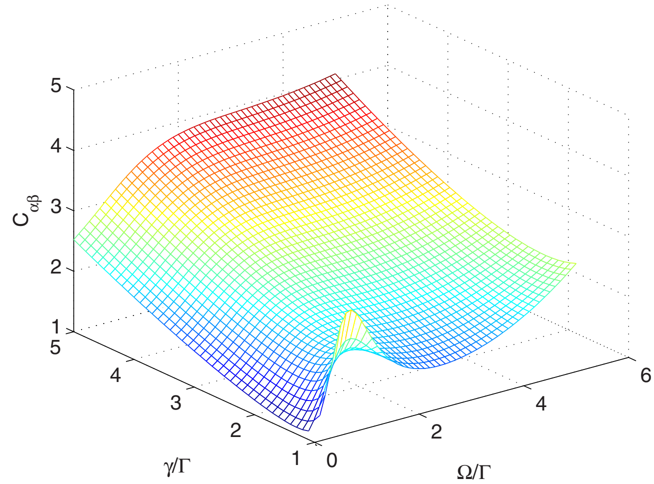

FIG. 5. (Color online) Plot of the nonclassical photon number correlation ($C_{\alpha\beta}$) of the cavity radiation at steady state for $\kappa = 0.5$ and $\lambda = 1.5$.

It is straightforward to see from Fig. 5 that the cavity radiation violates Cauchy-Schwarz inequality. On the basis that the photon number correlation is directly related to two-photon measurement at equal time, it is possible to learn about the nonclassicality in the photon number employing the usual two-photon coincidence counting. It appears natural to see that large $C_{\alpha\beta}$ does not necessarily imply that the nonclassicality in the photon number correlation is also large, since the correlation leading to nonclassical property is associated with cross correlation $[\langle\alpha(t)\beta(t)\rangle]$ compared to the photon numbers of the separate modes $[\ \alpha^*(t)\alpha(t)\rangle$ and $\beta^*(t)\beta(t)\rangle]$. In this respect, higher $C_{\alpha\beta}$ does not necessarily mean that the quantum correlation directly related with quantum features is strong. Although there is no sufficient evidence in this study, it has been argued earlier that the nonclassicality of the photon number correlation would be larger when $C_{\alpha\beta}$ is closer to 1 [7]. Consequently, it is possible to realize that the nonclassical correlation in the photon number decreases with the rate of dephasing except for some values of Ω/Γ.

B. Quantifying entanglement via photon number correlation

The photon number correlation of the two modes of the cavity radiation at equal time can also be investigated using

$$g(n_a, n_b) = \frac{\langle\hat{n}_a\hat{n}_b\rangle}{\langle\hat{n}_a\rangle\langle\hat{n}_b\rangle}, \tag{31}$$

which can be expressed, in view of the fact that $\alpha(t)$ and $\beta(t)$ are Gaussian variables of zero mean, in terms of c-number

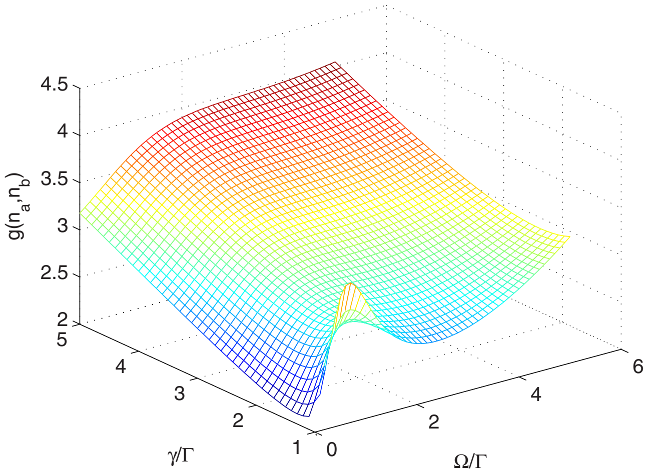

FIG. 6. (Color online) Plot of the photon number correlation $[g(n_a,n_b)]$ of the cavity radiation at steady state for $\kappa=0.5$ and $\lambda=1.5$.

variables associated with the normal ordering as

$$g(n_a,n_b) = 1 + \frac{\langle \alpha(t)\beta(t)\rangle^2}{\langle \alpha^*(t)\alpha(t)\rangle\langle \beta^*(t)\beta(t)\rangle}. \tag{32}$$

The dependence of the photon number correlation (32) on the induced atomic coherence has been discussed recently [1,9], but the main interest in this work is to study the effect of dephasing. On the other hand, according to the recent criterion set by Hillery and Zubairy [17], the product states are said to be entangled if $|\langle \hat{a}\hat{b}\rangle| > \sqrt{\langle \hat{N}_a\rangle\langle \hat{N}_b\rangle}$, where $\langle \hat{N}_a\rangle$ and $\langle \hat{N}_b\rangle$ are the corresponding mean photon numbers. It is not difficult to observe that for the product states to be entangled in this context $\langle \alpha(t)\beta(t)\rangle^2 > \langle \alpha^*(t)\alpha(t)\rangle\langle \beta^*(t)\beta(t)\rangle$ in the notation used in the present work. In this respect, it is possible to say that the two-mode cavity radiation would be entangled provided that

$$g(n_a,n_b) > 2. \tag{33}$$

In order to study the dependence of the photon number correlation related to entanglement on dephasing, $g(n_a,n_b)$ is plotted against γ/Γ and Ω/Γ in the following.

On account of the preceding discussion, it is not difficult to see from Fig. 6 that the cavity radiation exhibits entanglement. Since criterion (33) is open from above, it would not be generally appropriate and straightforward to relate the degree of entanglement with the extent at which this criterion is violated. Even then, this approach with no doubt connects the quantification of entanglement with simultaneous two-photon count measurement. When compared to the usual phase and homodyne measurements employed in quantifying entanglement, this appears to be more easier and viable approach. It is hence expected that such a procedure can mitigate the problem associated with quantifying entanglement as recently claimed [7,17]. Comparing Eqs. (29) and (32) reveals that the violation of Cauchy-Schwarz inequality and the entanglement describable by the criterion of Hillery and Zubairy [17] have close relation. Contrary to the criterion (22) which directly corresponds to EPR-type operators, violation of Cauchy-Schwarz inequality and criterion (33) are directly related to the correlation in the photon number. It is due to this underlying disparity that direct agreement between the conclusion drawn from the two criteria of entanglement is absent. In this line, critical scrutiny reveals that, even though the generated radiation can violate Cauchy-Schwarz and Hillery-Zubairy inequalities when $\gamma<\Gamma$, entanglement [the case for which condition (22) is satisfied] is observed only for specific ranges of Ω/Γ. It is, therefore, possible to argue based on this discussion that conditions (22) and (33) which meant to quantify entanglement are referring to different correlations and hence should not necessarily lead to the same conclusion. Similar disparity between the two criteria has been observed in a nondegenerate parametric oscillator recently [21].

V. CONCLUSION

In this contribution, thorough analysis of the effects of dephasing associated with the destruction of the coherent superposition by environment on quantum features of the cavity radiation is presented. Since dephasing substantially alters the coherent superposition responsible for observing quantum features, the obtainable quantum properties of the cavity radiation are found to be significantly reliant on the rate of dephasing. Particularly, the quantum features of the radiation including entanglement and violation of Cauchy-Schwarz inequality are found to be appreciably dependent on the strength of the external pumping, the rate of dephasing, and the rate at which the atoms are injected into the cavity. In general, the degree of manifestation of the quantum properties increases with the number of atoms that can be available in the cavity in a given time. In this line, since the external radiation significantly modifies the absorption emission phenomena in atoms, which predominantly affect the initially prepared atomic coherence, the pumping process is found to affect the quantum features significantly especially in the weak driving limit.

The assumption that the rate at which the atoms decays and the rate at which the available atomic coherent superposition decays are different leads to a considerable modification of the degree in which the quantum properties are exhibited. This should be clear from the outset since in the correlated emission laser the quantum features are directly attributed to the atomic coherence. In this connection, the effect of the rate of dephasing on entanglement is substantial in the weak driving limit. Furthermore, it is not difficult to realize that the actual realizable quantum property is the result of the interplay among the initially prepared atomic coherence, the externally induced coherence, and the rate at which the available coherence is destroyed by the environment. In view of this, it is possible to conclude that almost perfectly entangled light can be generated from pumped correlated emission laser if the rate of dephasing can be lowered provided that other parameters can be adjusted at will.

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