Interference-induced enhancement of field entanglement from an intracavity three-level V-type atom

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We investigate the generation of two-mode-entangled light from a laser-driven three-level **V**-type atom inside a cavity by taking into account spontaneously generated quantum interference between two atomic decay channels. We show that under some conditions, the system can reduce to a nondegenerate parametric amplifier which is responsible for the entanglement between the two cavity modes. Compared to the case without the quantum interference, it is found that the entanglement of the cavity field can be significantly enhanced by the interference when the relative phase of the two pumping lasers $\delta\phi=\pi$.

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I. INTRODUCTION

Continuous-variable (CV) Gaussian-entangled light has become a valuable resource for quantum communications and quantum information processing [1]. Therefore, how to generate the highly entangled light has become one of the energetic research fields in quantum optics. Nondegenerate parametric down conversion (NPDC) in a crystal [2] and injecting single-mode-squeezed light to a beam splitter [3,4] have proven to be efficient ways for generating the CV entangled beams. Besides, the preparation of the highly entangled light based on atomic coherence has also been investigated extensively. For example, by the use of nondegenerate four-wave mixing (NFWM), the generation of strong entanglement of low-frequency photons which have potential applications in quantum memory [5] and long-distance quantum communication [6] has been investigated theoretically [7–10] and experimentally [11–14]. The schemes for generating the CV highly entangled light by engineering NPDC Hamiltonian with a single atom in high-Q cavity have been put forward [15,16]. In addition, the enhanced CV entanglement from the intracavity two-level NFWM has been discussed [17]. Apart from the entanglement generation, we also note that the experimentally measurable CV entanglement criteria have recently been proposed in Refs. [4,18].

On the other hand, since the initial work of Agarwal [19] predicting the modification of the spontaneous emission of a three-level **V**-type by quantum interference between two atomic decay channels, the spontaneously generated interference has been extensively investigated [20] and is responsible for many novel effects such as the cancellation of spontaneous emission [21], ultrasharp spectral lines of the resonant fluorescence [22–25], and enhanced quantum squeezing [26] and photon correlations [27] of the fluorescence light emitted from a driven three-level **V**-type atom. Also, recent studies show that the decay-induced interference has strong effects on the coherent population trapping of a moving atom [28] and coherent population transfer in a double lambda atom [29]. Moreover, it is very recently found that the spontaneously generated interference in the system of two three-level **V**-type atoms can slow down the disentanglement between the two atoms [30]. Therefore, motivated by this and considering of the generation of the high CV entanglement, we would ask that whether quantum interference can enhance the entanglement of the radiation field emitted by a three-level atom in Vee configuration.

In this paper, we thus investigate the effects of the spontaneously generated quantum interference on the two-mode entangled light generated from a laser-driven **V**-type atom inside a cavity. After deriving the master equation of the cavity field in the dressed-state picture of the driven atom, the influences of the interference on the field entanglement are discussed in detail. It shows that the system can reduce to an effective nondegenerate parametric amplifier (NPA) when the couplings of the dressed atom and the two cavity mode are far from resonance, which is responsible for the field entanglement in the system. Compared to the case without the quantum interference, it is found that the entanglement of the cavity field can be significantly enhanced by the interference when the relative phase of the two pumping laser $\delta\phi=\pi$. This paper is arranged as follows. In Sec. II, the model is introduced and the master equation of the cavity field is derived. In Sec. III, the establishment and the properties of the cavity-field entanglement are discussed in detail. In Sec. IV, we give our main summary.

II. MODEL AND CAVITY-FIELD MASTER EQUATION

We consider a three-level **V**-type atom inside a two-mode cavity. The two near-degenerate-excited levels $|1\rangle$ and $|2\rangle$ of the atom are coupled to the atomic ground state $|3\rangle$ by two coherent lasers with the same frequency ω_L (see Fig. 1). At the same time, the laser-driven transitions $|1\rangle\leftrightarrow|3\rangle$ and $|2\rangle\leftrightarrow|3\rangle$ are also coupled to two cavity modes a_1 and a_2 through the NFWM process. The interaction Hamiltonian of the system can be expressed as

$$V_{int}=V_{ac}+V_{al}, \tag{1}$$

where

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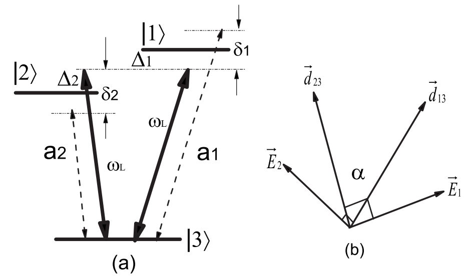

FIG. 1. (a) Schematic diagram of the atomic levels. (b) The arrangement of field polarization required for a single field driving one transition for nonorthogonal dipole moments.

$$V_{ac} = \sum_{j=1}^{2} [(-1)^{j+1}\delta_j a_j^\dagger a_j/2 + g_j a_j S_{j3}] + \text{H.c.}, \tag{2}$$

$$V_{al} = \sum_{j=1}^{2} (\Delta_j S_{jj}/2 + \Omega_j e^{i\phi_j} S_{j3}) + \text{H.c.} \tag{3}$$

Here $\Delta_j = \omega_{j3} - \omega_L$, $\delta_1 = \omega_1 - \omega_L$, and $\delta_2 = \omega_L - \omega_2$; ω_j are frequencies of the two cavity modes, ω_{j3} are atomic transition frequencies related to the transition operators S_{j3}, and S_{jj} are the atomic population operators. g_j denote the cavity-atom couplings and the Rabi frequencies of the laser fields $\Omega_j = \vec{E}_j \cdot \vec{d}_{j3}$, where $\vec{E}_j$ are the amplitudes of the laser fields with initial phases ϕ_j and $\vec{d}_{j3}$ are the dipole moments of the atomic transitions from $|j\rangle$ to $|3\rangle$. Taking into account the vacuum damping of the atom and the cavity modes, the density operator ρ of the coupled system is governed by the following master equation:

$$\frac{d}{dt}\rho = -i[V_{int}, \rho] + \mathcal{L}_f \rho + \mathcal{L}_a \rho, \tag{4}$$

$$\mathcal{L}_f \rho = \sum_{j=1}^{2} \kappa_j [a_j, \rho a_j^\dagger] + \text{H.c.}, \tag{5}$$

$$\mathcal{L}_a \rho = \gamma_1 [S_{31}, \rho S_{13}] + \gamma_2 [S_{32}, \rho S_{23}] + \gamma_{12}([S_{32}, \rho S_{13}] + [S_{31}, \rho S_{23}]) + \text{H.c.}, \tag{6}$$

where κ_j and γ_j are, respectively, the damping rates of the cavity modes and the atom. Here, the terms related to $\gamma_{12} = p\sqrt{\gamma_1 \gamma_2}$ represent the quantum interference resulted from the cross coupling between the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$. The quantum interference is dependent on the orientations of the atomic dipole polarizations and characterized by the parameter $p = (\vec{d}_{13} \cdot \vec{d}_{23})/(|\vec{d}_{13}||\vec{d}_{23}|) = \cos\alpha$, with α being the angle between the two dipole moments. The interference effect is maximal ($p=1$) for the parallel dipole moments, while it disappears ($p=0$) if the two dipole moments are perpendicular to each other. It should be noted that instead of nonorthogonal dipole moments, this kind of the decay-induce interference in three-level **V**-type emitter can be engineered in semiconductors such as quantum wells via Fano interference [31,32].

Using the master Eq. (4), the reduced density operator ρ_f of the cavity modes can be obtained simply by tracing out the atomic variables. Here, we will derive it in the dressed-state picture of the laser-driven atom to obtain physical insights into the establishment of the field entanglement. By applying the transformations $S_{j3} e^{i\phi_j} = \tilde{S}_{j3}$ to Eq. (4) [25], the dressed states which are the eigenstates of the transformed laser-atom interaction Hamiltonian $V_{al} = \Sigma_j (\frac{\Delta_j}{2} S_{jj} + \Omega_j S_{j3} + \text{H.c.})$ (the tilde is dropped, similarly hereinafter) in Eq. (3) can be obtained as

$$|\xi_k\rangle = M_{k1}|1\rangle + M_{k2}|2\rangle + M_{k1}|3\rangle (k=1,2,3), \tag{7}$$

where

$$M_{k1} = -\frac{\Omega_1 \Omega_2}{D_k}, \quad M_{k2} = \frac{\lambda_k \tilde{\Delta}_1 + \Omega_1^2}{D_k}, \quad M_{k3} = \frac{\Omega_2 \tilde{\Delta}_1}{D_k}, \tag{8}$$

$$D_k = \sqrt{(\lambda_k \tilde{\Delta}_1 + \Omega_1^2)^2 + \Omega_2^2 \tilde{\Delta}_1^2 + \Omega_1^2 \Omega_2^2}. \tag{9}$$

Here λ_k denote the eigenvalues (the dressed energies) corresponding to the eigenstates $|\xi_k\rangle$ and can be obtained by solving the following equation:

$$(\Delta_1 - \lambda)[\lambda(\Delta_2 - \lambda) + \Omega_2^2] + \Omega_1^2(\Delta_2 - \lambda) = 0. \tag{10}$$

Meanwhile, by defining the dressed-state operators $R_{kk'} = |\xi_k\rangle\langle\xi_{k'}|$, the transformed Hamiltonian V_{int} in Eq. (1) in the rotating frame with respect to the Hamiltonian V_{al} in the dressed-state picture and the free cavity Hamiltonian in Eq. (2) are changed into

$$V_{int}^d = \sum_{k' \geq k=1}^{3} A_{kk'}(t) R_{kk'} e^{i\Omega_{kk'} t} + \text{H.c.}, \tag{11}$$

where

$$A_{kk}(t) = (\tilde{g}_1 \alpha_{kk} a_1 e^{-i\delta_1 t} + \tilde{g}_2^* \beta_{kk} a_2^\dagger e^{-i\delta_2 t})/2 + \text{H.c.},$$

$$A_{kk'}(t) = \tilde{g}_1 \alpha_{kk'} a_1 e^{-i\delta_1 t} + \tilde{g}_1^* \alpha_{k'k} a_1^\dagger e^{i\delta_1 t} + \tilde{g}_2 \beta_{kk'} a_2 e^{i\delta_2 t} + \tilde{g}_2^* \beta_{k'k} a_2^\dagger e^{-i\delta_2 t} (k < k'), \tag{12}$$

$\Omega_{kk'} = \lambda_k - \lambda_{k'}$, $\tilde{g}_j = g_j e^{-i\phi_j} (j=1,2)$, and the expressions of $\alpha_{kk'}$ and $\beta_{kk'}$ are given in Appendix A. In addition, for the strong laser fields such that $|\Omega_{kk'}| \gg \gamma_{1,2}$ to neglect the fast oscillating terms within the secular approximation, the atomic damping in Eq. (6) becomes into

$$\mathcal{L}_a^d \rho = \sum_{k,k'=1}^{3} \Gamma_{kk'} [R_{kk'}, \rho R_{k'k}] + \sum_{k \neq k'=1}^{3} \bar{\Gamma}_{kk'} [R_{kk}, \rho R_{k'k'}] + \text{H.c.}, \tag{13}$$

where

$$\Gamma_{kk'} = \gamma_1 \alpha_{kk'}^2 + \gamma_2 \beta_{kk'}^2 + 2\gamma_{12} \alpha_{kk'} \beta_{kk'} \cos(\delta\phi),$$

$$\bar{\Gamma}_{kk'} = \gamma_1 \alpha_{kk} \alpha_{k'k'} + \gamma_2 \beta_{kk} \beta_{k'k'} + \gamma_{12}[\alpha_{kk} \beta_{k'k'} e^{i\delta\phi} + \alpha_{k'k'} \beta_{kk} e^{-i\delta\phi}], \tag{14}$$

with $\delta\phi=\phi_1-\phi_2$ being the relative phase of the two pumping lasers. From the above equation, we see that in the absence of the quantum interference, the effect of the relative phase of the two lasers will disappear.

In the dressed-state picture, the master Eq. (4) of the whole system can be written as

$$\frac{d\rho}{dt}=-i[V_{int}^{d},\rho]+\mathcal{L}_f\rho+\mathcal{L}_a^{d}\rho. \tag{15}$$

Assuming a high-quality cavity (i.e., $\gamma_j \gg \kappa_j$), the atom achieves its steady states on a time scale much faster than the cavity field. Since we are only interested in the times after which the atomic transients have died away, the atom can thus be described by a stationary process and the atomic variables can be eliminated. To the second order in perturbation theory in the cavity-atom couplings, the reduced master equation of the cavity field can be obtained as [17,33]

$$\begin{aligned}\frac{d}{dt}\rho_f+i\delta_{12}\sum_{j=1}^{2}[a_j^{\dagger}a_j,\rho_f]=\sum_{j=1}^{2}&[\bar{A}_j(a_j^{\dagger}\rho_f a_j-\rho_f a_j a_j^{\dagger})\\&+(\bar{B}_j+\kappa_j)(a_j\rho_f a_j^{\dagger}-a_j^{\dagger}a_j\rho_f)]\\&+\sum_{j\neq j'=1}^{2}[\bar{C}_j(a_j^{\dagger}a_{j'}^{\dagger}\rho_f-a_{j'}^{\dagger}\rho_f a_j^{\dagger})\\&+\bar{D}_j(\rho_f a_{j'}^{\dagger}a_j^{\dagger}-a_j^{\dagger}\rho_f a_{j'}^{\dagger})]+\text{H.c.},\end{aligned} \tag{16}$$

where $\delta_{12}=(\delta_2-\delta_1)/2$, and $\bar{O}_j=O_j^{s}+O_j^{c}(O=A,B,C,D)$. Here, the coefficients $\bar{A}_j$ and $\bar{B}_j$, respectively, describe the gain and absorption of the relevant cavity modes, and the coefficients $\bar{C}_j$ and $\bar{D}_j$ characterize the correlations between the cavity modes which are responsible for the cavity-field entanglement. The terms related to the coefficients O_j^{s} in Eq. (16) are contributed from the sideband transitions of the dressed atom and given by

$$A_1^{s}=g_1^2\sum_{k'\neq k=1}^{3}P_{kk}^{d}\alpha_{kk'}^{2}F_{kk'}(\delta_1),$$

$$B_1^{s}=g_1^2\sum_{k'\neq k=1}^{3}P_{kk}^{d}\alpha_{k'k}^{2}F_{kk'}(\delta_1),$$

$$A_2^{s}=g_2^2\sum_{k'\neq k=1}^{3}P_{kk}^{d}\beta_{kk'}^{2}F_{kk'}(-\delta_2),$$

$$B_2^{s}=g_2^2\sum_{k'\neq k=1}^{3}P_{kk}^{d}\beta_{k'k}^{2}F_{kk'}(-\delta_2),$$

$$C_1^{s}=-g_1g_2e^{i\phi_s}\sum_{k=1}^{3}P_{kk}^{d}\sum_{k'\neq k}\alpha_{k'k}\beta_{kk'}F_{kk'}^{*}(-\delta_2),$$

$$C_2^{s}=-g_1g_2e^{i\phi_s}\sum_{k=1}^{3}P_{kk}^{d}\sum_{k'\neq k}\alpha_{kk'}\beta_{k'k}F_{kk'}^{*}(\delta_1),$$

$$D_1^{s}=-g_1g_2e^{i\phi_s}\sum_{k=1}^{3}P_{kk}^{d}\sum_{k'\neq k}\alpha_{kk'}\beta_{k'k}F_{kk'}(\delta_2),$$

$$D_2^{s}=-g_1g_2e^{i\phi_s}\sum_{k=1}^{3}P_{kk}^{d}\sum_{k'\neq k}\alpha_{k'k}\beta_{kk'}F_{kk'}(-\delta_1),$$

$$F_{kk'}(x)=\frac{1}{\bar{\gamma}_{kk'}+i\Omega_{kk'}-ix},$$

$$\bar{\gamma}_{12}=\Gamma_{11}+\Gamma_{22}+\Gamma_{13}+\Gamma_{23}+\Gamma_{12}+\Gamma_{21}-2\bar{\Gamma}_{12},$$

$$\bar{\gamma}_{13}=\Gamma_{11}+\Gamma_{33}+\Gamma_{12}+\Gamma_{32}+\Gamma_{13}+\Gamma_{31}-2\bar{\Gamma}_{13},$$

$$\bar{\gamma}_{23}=\Gamma_{22}+\Gamma_{33}+\Gamma_{21}+\Gamma_{31}+\Gamma_{23}+\Gamma_{32}-2\bar{\Gamma}_{23}, \tag{17}$$

where P_{kk}^{d} are the atomic populations in the dressed states $|\xi_k\rangle$, $\phi_s=\phi_1+\phi_2$, and $\bar{\gamma}_{kk'}=\bar{\gamma}_{k'k}^{*}$. Meanwhile, the terms proportional to the coefficients O_j^{c} resulted from the central-peak transitions of the dressed atom which contribute little to the entanglement of the cavity field for the case $|\delta_j|\gg\gamma_j$ and their explicit expressions are given in Appendix B.

III. ENTANGLEMENT PROPERTIES OF THE CAVITY FIELD

Assuming the cavity modes initially in vacuum, the cavity-field state governed by the master Eq. (16) should be a two-mode Gaussian state (TMSS) because the master equation only contains the quadratic terms of the bosonic operators a_j and $a_j^{\dagger}(j=1,2)$. The quantum statistical properties of a TMSS are completely determined by its correlation matrix (CM) [1] which takes the following standard form:

$$V=\begin{pmatrix} n_1 & 0 & c_1 & 0\\ 0 & n_2 & 0 & c_2\\ c_1 & 0 & m_1 & 0\\ 0 & c_2 & 0 & m_2\end{pmatrix}, \tag{18}$$

for any TMSS after local operations [34]. Here, for the cavity field governed by Eq. (16) we can easily find from our previous investigations in Refs. [17,35] that $n_j=n=\langle a_1^{\dagger}a_1\rangle+1/2$, $m_j=m=\langle a_2^{\dagger}a_2\rangle+1/2$, and $c_1=-c_2=c=|\langle a_1a_2\rangle|$, where we have

$$\frac{d}{dt}\langle a_j^{\dagger}a_j\rangle=\frac{1}{2}(x_j+x_j^{*})\langle a_j^{\dagger}a_j\rangle+y_j\langle a_1^{\dagger}a_2^{\dagger}\rangle+e_j/2+\text{c.c.},$$

$$\frac{d}{dt}\langle a_1a_2\rangle=y_2\langle a_1^{\dagger}a_1\rangle+y_1\langle a_2^{\dagger}a_2\rangle+(x_1+x_2)\langle a_1a_2\rangle+e_3, \tag{19}$$

with $x_j=\bar{A}_j-\bar{B}_j-\kappa_j-i\delta_{12}$, $y_j=\bar{C}_j-\bar{D}_j$, $e_j=\bar{A}_j+\bar{A}_j^{*}(j=1,2)$, and $e_3=\bar{C}_1+\bar{C}_2$.

According to Duan's criterion for the separability of a TMSS [34], the two cavity modes are entangled if and only if the sum of the variances of the operators $\hat{u}=a\hat{X}_1-\frac{1}{a}\hat{X}_2$ and $\hat{v}=a\hat{Y}_1+\frac{1}{a}\hat{Y}_2$ satisfies the following inequality:

$$\Upsilon=\langle(\Delta\hat{u})^2\rangle+\langle(\Delta\hat{v})^2\rangle-a^2-\frac{1}{a^2}<0, \tag{20}$$

where the pair quadrature operators are defined as $\hat{X}_j=(a_j+a_j^\dagger)/\sqrt{2}$ and $\hat{Y}_j=-i(a_j-a_j^\dagger)/\sqrt{2}$ and a is a state-dependent real number. Accordingly, for our case we have [34]

$$\Upsilon=2na^2+2m/a^2-4c-a^2-\frac{1}{a^2}, \tag{21}$$

and $a^2=\sqrt{(2m-1)/(2n-1)}$. It should be noted that $\Upsilon=-2$ corresponds to the original EPR entanglement [36]. From Eq. (21), one can find that the entanglement condition $\Upsilon<0$ is equivalent to the inequality

$$\sqrt{\langle a_1^\dagger a_1\rangle\langle a_2^\dagger a_2\rangle}<|\langle a_1a_2\rangle|, \tag{22}$$

which indicates that the nonclassical correlation between the two cavity modes is necessary for the entanglement establishment of the cavity field. In addition, the entanglement degree of the two-mode cavity field can be quantified with logarithmic negativity E_N. For a TMSS with the CM cast in the 2×2 block form $V=\begin{pmatrix}N_1 & N_{12}\\ N_{12}^T & N_2\end{pmatrix}$, the logarithmic negativity E_N is defined as [37]

$$E_N=\max[0,-\ln 2d], \tag{23}$$

where d is given by

$$d=2^{-1/2}\sqrt{\Sigma(V)-\sqrt{\Sigma(V)^2-4\det V}}, \tag{24}$$

with $\Sigma(V)=\det N_1+\det N_2-2\det N_{12}$. Therefore, a TMSS is entangled if and only if $d<1/2$, which in our case—from Eq. (18)—is equivalent to

$$[c^2-(n+1/2)(m+1/2)][c^2-(n-1/2)(m-1/2)]<0. \tag{25}$$

Since $(n+1/2)(m+1/2)-c^2>0$ for the positivity of the cavity-field state [38], we can find that the entanglement condition of Eq. (25) is the same as that in Eq. (22) obtained with Duan's inseparability criterion. Therefore, based on Eqs. (18)–(20) and (23), the steady entanglement of the cavity field can be analyzed with the steady-state condition

$$\mathrm{Re}[x_1+x_2+\sqrt{(x_1-x_2^*)^2+4y_1y_2^*}]<0. \tag{26}$$

First, let us discuss in detail the establishment of the entanglement between the two cavity modes in the present system. Under the situation that $\{|\delta_j|,|\Omega_{kk'}\pm\delta_j|\}\gg\{\gamma_1,\gamma_2\}$, i.e., the coupling of each cavity mode to the dressed atom is far from resonance, the expression of $F_{kk'}(x)$ in Eq. (17) can thus become into $F_{kk'}(x)\approx -i/(\Omega_{kk'}-x)$, which means that the effect of the atomic spontaneous emission on the behavior of the cavity field can be negligible in this situation. By considering the case that $\delta_1\approx\delta_2=\delta$ (i.e., two-photon resonant coupling between the dressed atom and the cavity

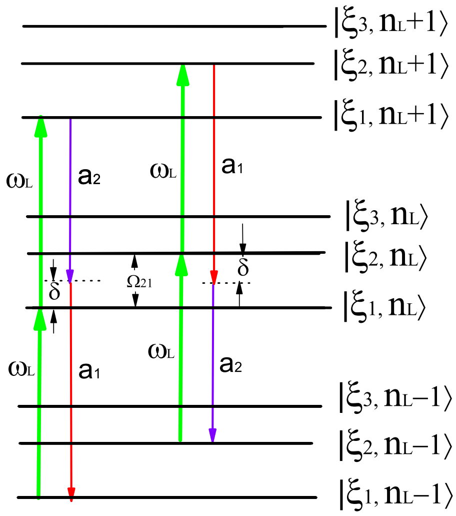

FIG. 2. (Color online) Two possible two-photon cascade transition channels of the dressed atom [characterized by the first term in Eq. (28)] which account for the entanglement between the two cavity modes a_1 and a_2.

field), it is not difficult to find that the master Eq. (16) becomes into

$$\frac{d}{dt}\rho_f=-ig_1g_2C_e[a_1a_2e^{-i\phi_s}+a_1^\dagger a_2^\dagger e^{i\phi_s},\rho_f]+\mathcal{L}_f\rho_f, \tag{27}$$

where

$$C_e=\left(\frac{\alpha_{12}\beta_{21}}{\Omega_{12}-\delta}+\frac{\alpha_{21}\beta_{12}}{\Omega_{12}+\delta}\right)(P_{11}^d-P_{22}^d)+\left(\frac{\alpha_{13}\beta_{31}}{\Omega_{13}-\delta}+\frac{\alpha_{31}\beta_{13}}{\Omega_{13}+\delta}\right)\times(P_{11}^d-P_{33}^d)+\left(\frac{\alpha_{23}\beta_{32}}{\Omega_{23}-\delta}+\frac{\alpha_{32}\beta_{23}}{\Omega_{23}+\delta}\right)(P_{22}^d-P_{33}^d). \tag{28}$$

Note that in deriving the above equation, the detunings of the cavity modes from ac Stark shift have been canceled by choosing the appropriate frequency difference δ_{12} [15,17]. We can see from Eq. (27) that the system now reduces to a NPA with its Hamiltonian

$$V_{eff}=-g_1g_2C_e(a_1a_2e^{-i\phi_s}+a_1^\dagger a_2^\dagger e^{i\phi_s}), \tag{29}$$

which is responsible for the entanglement between the two cavity modes. Physically, the cavity-field entanglement in the effective NPA process originates from the two-photon couplings between the cavity modes and the dressed atom. For example, the first term in Eq. (28) characterizes two possible two-photon channels for realizing the NPA process (see Fig. 2). By absorbing two pumping photons at frequency ω_L, the dressed atom at first jumps from the dressed level $|\xi_1\rangle$ (or $|\xi_2\rangle$) to $|\xi_2\rangle$ (or $|\xi_1\rangle$) [with the weighting factor P_{11}^d (or P_{22}^d)] through emitting a photon in the mode a_2 (or a_1); then, because such one-photon processes are far-off resonance, the

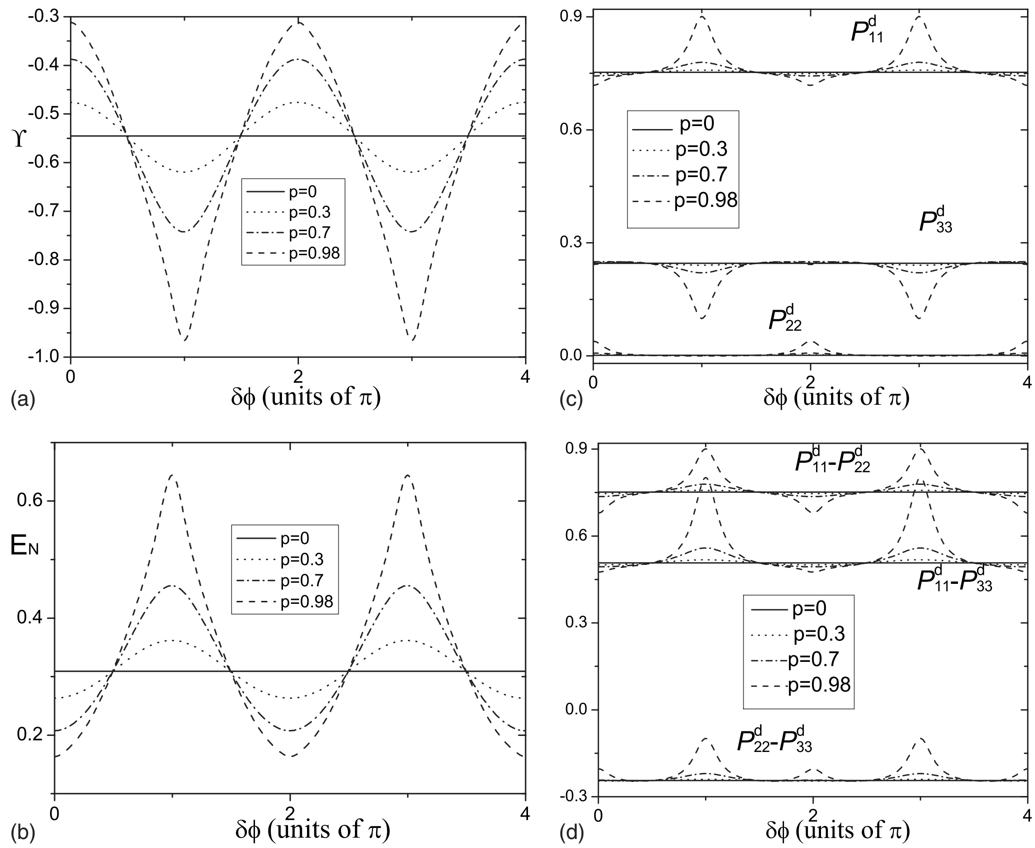

FIG. 3. The dependence of the steady-state entanglement in (a) and (b), the steady dressed-atomic populations in (c), and the population differences in (d) on the relative phase of the two pumping lasers for the different interference strength p. The other parameters are chosen as $\gamma=2\ \mathrm{g}$, $\kappa=3.5\times10^{-3}\ \mathrm{g}$, $\Omega_1=10\ \mathrm{g}$, $\Omega_2=13\ \mathrm{g}$, $\omega_{12}=2\ \mathrm{g}$, $\Delta=10\ \mathrm{g}$, and $\delta=50\ \mathrm{g}$.

dressed atom immediately emits another photon in the mode a_1 (or a_2) to $|\xi_1\rangle$ (or $|\xi_2\rangle$). In these two-photon cascade emissions, because the conditions of the two-photon resonance and one-photon far-off resonance are obeyed, the emitted two photons are highly correlated and the entanglement can thus be formed between the two cavity modes. Further, since these two channels of the two-photon emission interfere destructively with each other [17,39], the NPA strength contributed from the two possible channels is thus proportional to the population difference $P^d_{11}-P^d_{22}$ of the two dressed states $|\xi_1\rangle$ and $|\xi_2\rangle$. Evidently, the other similar two-photon channels may exist and they are, respectively, characterized by the second and third terms in Eq. (28), which—combined with the first one—lead to the effective NPA process in the system. In addition, from Eq. (29) the field entanglement is independent of the phase sum $\phi_s=\phi_1+\phi_2$ of the two laser fields since one can perform local rotations of $\phi_i(i=1,2)$ on the mode a_i to eliminate the phase dependence of Eq. (29) without changing the entanglement degree. However, due to the dependence of the dressed-state populations P^d_{kk} on the relative phase $\delta\phi$ of the two lasers [indicated in Eq. (14)], the phase-dependent CV entanglement can be achieved.

Now we proceed to investigate the effects of the quantum interference on the field entanglement which is determined by the master Eq. (16). By setting $\Delta_2=\Delta$ and $\Delta_1=\Delta-\omega_{12}$ and assuming $\gamma_1=\gamma_2=\gamma$ and $\kappa_1=\kappa_2=\kappa$, we plot the dependence of the field entanglement, which is, respectively, measured by the negativity Υ and the logarithmic negativity E_N in Figs. 3(a) and 3(b), on the relative phase $\delta\phi$ of the two lasers for the different interference strength characterized by the parameter p. One can see that the same entanglement properties are demonstrated in Figs. 3(a) and 3(b), respectively, which means that the negativity Υ can be used to characterize the entanglement properties of the cavity field. It shows that the entanglement is dependent on the relative phase $\delta\phi$ when the quantum interference between the two decay channels of $|1\rangle\leftrightarrow|3\rangle$ and $|2\rangle\leftrightarrow|3\rangle$ takes place ($p\neq0$). Moreover, compared to the case without the interference ($p=0$), the entanglement of the cavity field can be considerably increased by the interference and it is enhanced optimally when the relative phase $\delta\phi=(2s+1)\pi(s=0,1,2,\ldots)$. In the vicinity of $\delta\phi=2s\pi$, the entanglement is degraded by the interference. To understand these results, we plot in Fig. 3(c) the dependence of the dressed-state populations P^d_{kk} on the relative phase $\delta\phi$. From it we see that the population $P^d_{22}\approx0$ and the atom is thus mainly populated in the dressed states $|\xi_1\rangle$ and $|\xi_3\rangle$, which implies that the entanglement of the cavity field revealed in Figs. 3(a) and 3(b) is mainly contributed from the two-photon transitions $|\xi_1\rangle\rightarrow|\xi_{2(3)}\rangle\rightarrow|\xi_1\rangle$ and $|\xi_3\rangle\rightarrow|\xi_{2(1)}\rangle\rightarrow|\xi_3\rangle$. When $\delta\phi=(2s+1)\pi$, the population P^d_{11} obtains its maximal values, whereas P^d_{33} becomes minimal. So, as shown in Fig. 3(d), the population differences $P^d_{ii}-P^d_{jj}$ proportional to the effective NPA strength in Eq. (28) are increased significantly at $\delta\phi=(2s+1\pi)$ by the quantum interference, in comparison to the case in the absence of the interference. As a result, the strength of the effective NPA process in the present system can be increased by the quantum interference, which leads to the considerable enhancement of the entanglement of the cavity field. Here, as shown in Fig. 3(a), we see that the entanglement Υ in the presence of the interference with $p=0.98$ can approach $\Upsilon \approx -1$ which corresponds to the maximal intracavity entanglement achieved with a nondegenerate optical parametric oscillator [35]. Therefore, with the quantum interference, we can obtain the highly entangled light which is certainly beneficial to the field of quantum information processing. In addition, we plot in Fig. 4 the dependence of the steady entanglement and dressed population differences $P_{ii}^d - P_{jj}^d$ on the relative strength the laser-atom detuning Δ under the steady-state condition in Eq. (26). It shows that the entanglement enhancement by the interference is reduced as the detuning increases since the enhancement of the population differences by the interference decreases with the increase in the detuning Δ.

IV. CONCLUSIONS

Before concluding, let us briefly discuss the generation of the spontaneously generated interference. Beside natural atoms with nonorthogonal dipole moments as discussed extensively in Refs. [19–24,26,27], the decay-induced interference in **V**-type emitter can be achievable in coupled semiconductor quantum wells via Fano interference by phonon tunneling [31,32] instead of nonorthogonal dipole moments, which thus provides an effective alternative way to observe the interference-related effects revealed above. Additionally, the spontaneously induced interference can also be simulated through dc or microwave field in the atom with orthogonal dipole moments, which was initially shown by Ficek and Swain [27].

In conclusion, the generation of the two-mode-entangled light from an intracavity three-level **V**-type atom is investigated by taking into account the effects of the quantum interference resulted from the two decay pathways. It shows that under the condition that the couplings between the cavity modes and the dressed atom are far from resonance, the system can reduce to an effective NPA which is responsible for the field entanglement of the system. It is found that the decay-induced interference can greatly enhance the entanglement degree of the cavity field when the relative phase of the two lasers $\delta\phi=\pi$, in comparison to the case in the absence of the quantum interference.

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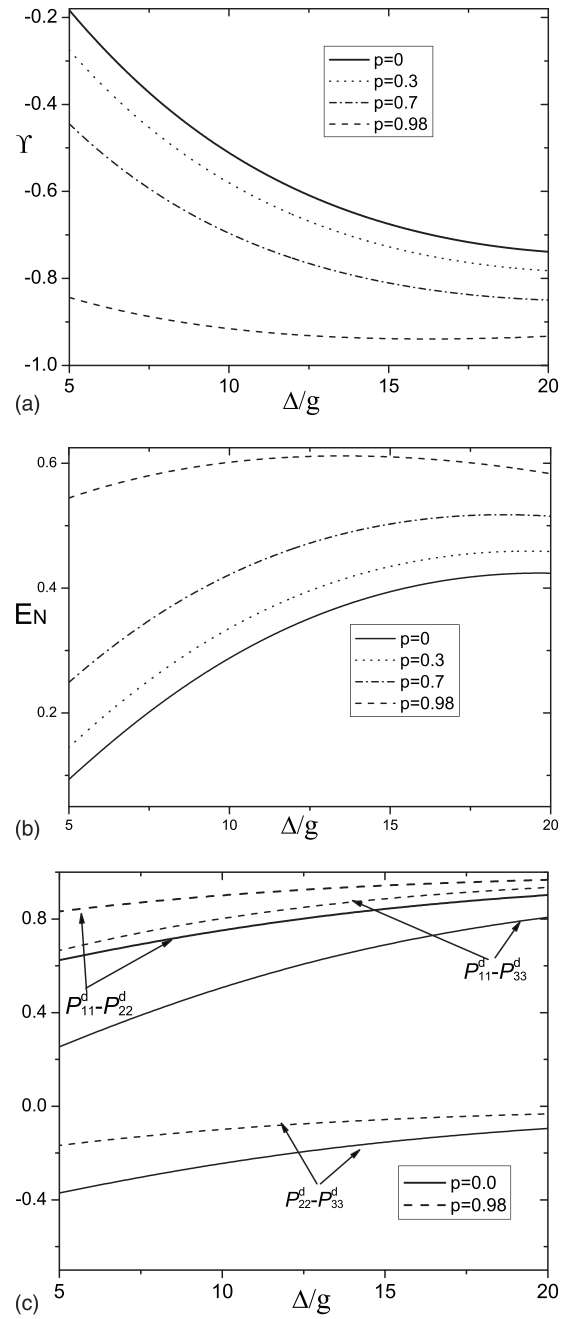

FIG. 4. The steady-state entanglement in (a) and (b) and the population differences of the dressed atom in (c) versus the laser-atom detuning Δ for $\kappa=0.004$ g and $\delta\phi=\pi$. The other parameters are the same as that in Fig. 2.

APPENDIX A

The expressions of $\alpha_{kk'}$ and $\beta_{kk'}$ in Eq. (12) are

$$\{\alpha_{kk'}\} = \begin{pmatrix} c_{11}c_{31} & c_{11}c_{32} & c_{11}c_{33} \\ c_{12}c_{31} & c_{12}c_{32} & c_{12}c_{33} \\ c_{13}c_{31} & c_{13}c_{32} & c_{13}c_{33} \end{pmatrix},$$

$$\{\beta_{kk'}\} = \begin{pmatrix} c_{21}c_{31} & c_{21}c_{32} & c_{21}c_{33} \\ c_{22}c_{31} & c_{22}c_{32} & c_{22}c_{33} \\ c_{23}c_{31} & c_{23}c_{32} & c_{23}c_{33} \end{pmatrix},$$

$$c_{11} = -\frac{D_1[(\Delta_1-\lambda_2)(\Delta_1-\lambda_3)+\Omega_1^2]}{\Omega_{12}\Omega_{13}\Omega_1\Omega_2},$$

$$c_{12} = \frac{D_2[\tilde{\Delta}_1(\Delta_1-\lambda_3)+\Omega_1^2]}{\Omega_{12}\Omega_{23}\Omega_1\Omega_2}, \quad c_{21} = -\frac{D_1}{\Omega_{12}\Omega_{13}},$$

$$c_{13} = -\frac{D_3[\tilde{\Delta}_1(\Delta_1-\lambda_2)+\Omega_1^2]}{\Omega_{13}\Omega_{23}\Omega_1\Omega_2}, \quad c_{22} = \frac{D_2}{\Omega_{12}\Omega_{23}},$$

$$c_{23} = -\frac{D_3}{\Omega_{13}\Omega_{23}}, \quad c_{31} = -\frac{D_1(\Delta_1-\lambda_2-\lambda_3)}{\Omega_{12}\Omega_{13}\Omega_2},$$

$$c_{32} = \frac{D_2(\tilde{\Delta}_1-\lambda_3)}{\Omega_{12}\Omega_{23}\Omega_2}, \quad c_{33} = -\frac{D_3(\tilde{\Delta}_1-\lambda_2)}{\Omega_{13}\Omega_{23}\Omega_2}. \tag{A1}$$

APPENDIX B

The expressions of the coefficients O_j^c in Eq. (16) are

$$A_1^c = g_1^2 \sum_{k,k'=1}^{3} \alpha_{kk}\alpha_{k'k'}x_{kk'}(i\delta_1),$$

$$A_2^c = g_2^2 \sum_{k,k'=1}^{3} \beta_{kk}\beta_{k'k'}x_{kk'}(-i\delta_2), \tag{B1}$$

$$C_1^c = -g_1g_2e^{i(\phi_1+\phi_2)} \sum_{k,k'=1}^{3} \alpha_{kk}\beta_{k'k'}x_{kk'}(i\delta_2), \tag{B2}$$

$$C_2^c = -g_1g_2e^{i(\phi_1+\phi_2)} \sum_{k,k'=1}^{3} \beta_{kk}\alpha_{k'k'}x_{kk'}(-i\delta_1), \tag{B3}$$

$$D_1^c = -g_1g_2e^{i(\phi_1+\phi_2)} \sum_{k,k'=1}^{3} \beta_{kk}\alpha_{k'k'}x_{kk'}(i\delta_2), \tag{B4}$$

$$D_2^c = -g_1g_2e^{i(\phi_1+\phi_2)} \sum_{k,k'=1}^{3} \alpha_{kk}\beta_{k'k'}x_{kk'}(-i\delta_1),$$

$$B_1^c = A_1^{c*}, \quad B_2^c = A_2^{c*}.$$

$$x_{11} = \frac{P_{11}^d(1-P_{11}^d)(2d_1+s) - 2P_{11}^dP_{22}^d(\Gamma_{21}-\Gamma_{31})}{F(s)},$$

$$x_{12} = \frac{-P_{11}^dP_{22}^d(2d_1+s) + 2P_{22}^d(1-P_{22}^d)(\Gamma_{21}-\Gamma_{31})}{F(s)},$$

$$x_{13} = \frac{-P_{11}^dP_{33}^d(2d_1+s) - 2P_{22}^dP_{33}^d(\Gamma_{21}-\Gamma_{31})}{F(s)},$$

$$x_{21} = \frac{-P_{11}^dP_{22}^d(2d_2+s) + 2P_{11}^d(1-P_{11}^d)(\Gamma_{12}-\Gamma_{32})}{F(s)},$$

$$x_{22} = \frac{P_{22}^d(1-P_{22}^d)(2d_2+s) - 2P_{11}^dP_{22}^d(\Gamma_{12}-\Gamma_{32})}{F(s)},$$

$$x_{23} = \frac{-P_{22}^dP_{33}^d(2d_2+s) - 2P_{11}^dP_{33}^d(\Gamma_{21}-\Gamma_{31})}{F(s)},$$

$$x_{31} = \frac{-P_{11}^dP_{33}^d(2d_3+s) + 2P_{11}^d(1-P_{11}^d)(\Gamma_{13}-\Gamma_{23})}{F(s)},$$

$$x_{32} = \frac{-P_{22}^dP_{33}^d(2d_3+s) - 2P_{11}^dP_{22}^d(\Gamma_{13}-\Gamma_{23})}{F(s)},$$

$$x_{33} = \frac{P_{33}^d(1-P_{33}^d)(2d_3+s) - 2P_{11}^dP_{33}^d(\Gamma_{13}-\Gamma_{23})}{F(s)},$$

$$F(s) = s^2 + 2ds + d_0,$$

$$d = \Gamma_{12}+\Gamma_{21}+\Gamma_{13}+\Gamma_{31}+\Gamma_{23}+\Gamma_{32}, d_1 = \Gamma_{21}+\Gamma_{23}+\Gamma_{32},$$

$$d_2 = \Gamma_{12}+\Gamma_{13}+\Gamma_{31}, d_3 = \Gamma_{13}+\Gamma_{12}+\Gamma_{21},$$

$$d_0 = (\Gamma_{12}+\Gamma_{21}+\Gamma_{13})(\Gamma_{23}+\Gamma_{32}+\Gamma_{31}) - (\Gamma_{31}-\Gamma_{21})(\Gamma_{13}-\Gamma_{23}). \tag{B5}$$

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