Exactly controllable transmission of nonautonomous optical solitons

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We investigate nonautonomous optical soliton dynamics accurately governed by the nonlinear Schrödinger equation with varying fiber loss and/or Raman gain. Based on the Painlevé analysis a Painlevé integrability condition of this equation, which means a balance between the dispersion, nonlinearity, and the fiber loss and/or gain, has been obtained. Under this condition the optical soliton transmission in fibers can be exactly controlled by proper dispersion and nonlinearity managements and the Raman gain. The existing experiments confirm the validity of our result, which provides a general guidance to design a fiber in information transmission using optical solitons.

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The concept of solitons proposed by Zabusky and Kruskal [1] in 1965 describes a nonlinear wave with a finite localized energy; its shape and velocity keep unchanged during propagation and after collisions just like an elementary particle. The robustness of a soliton is due to the balance between dispersion and nonlinearity in a medium. Soon after that Hasegawa and Tappert found that a soliton can also be formed in fiber systems, i.e., the optical soliton [2–5]. Due to the particlelike property of the soliton, they proposed that the optical soliton could be an ideal subject to transmit optical signals. The realization of the optical soliton transmission was first reported by Mollenauer *et al.* [6]. Since then the fundamental properties of optical solitons and their applications in optical communication have been extensively investigated. For a review one refers to Refs. [3–5,7–11].

However, an ideal optical soliton communication is very difficult to obtain since the fiber loss is inevitable in the propagation of the optical soliton. The dissipation would weaken the nonlinearity and finally the optical soliton would broaden and lose its signal due to dispersion. To overcome the fiber loss it is possible to compensate for the fiber loss by optical gain via Raman amplification. The first optical soliton transmission experiment using the Raman gain showed that the optical soliton can transmit more than 4000 km [12]. Another way is to use a technique of dispersion management [13], which has been intensively investigated in recent years [10]. It should be pointed out that in both schemes the optical solitons are modified because the balance between dispersion and nonlinearity cannot be kept constant during propagation of the optical solitons. In a sense of average [14] the modified soliton is known as a quasisoliton [15], but the overall features are far from an ideal soliton. The serious modification of the solitons would inevitably increase the bit error rate and degrade the reliability of the soliton communication. Therefore it is desired to obtain an ideal optical soliton transmission in fibers but the question arises: how to properly manage the dispersion and nonlinearity in fibers even in the presence of dissipation and/or gain?

A clear answer for this question highly depends on our knowledge of the exact dynamical behavior of the optical solitons during propagation along the fibers. The dynamics of optical solitons are governed by a nonautonomous nonlinear Schrödinger (NLS) equation [16] (see below) due to the managements of dispersion and nonlinearity and the presence of dissipation and/or gain [17]. In this paper we present optical soliton solutions of the nonautonomous NLS equation in the presence of dissipation and/or gain. Through the Painlevé analysis we find a generalized integrability condition of the nonautonomous NLS equation. This condition shows how to balance dispersion, nonlinearity, and dissipation and/or gain in the transmission of optical solitons. Meanwhile under this condition the nonautonomous NLS equation can be reduced to the standard one as discussed early in [18]. The canonical soliton solution of the standard NLS equation can be recast into the corresponding solitonlike of the nonautonomous NLS equation, which is an excellent object to realize an ideal optical soliton communication in fibers when dispersion, nonlinearity, and fiber loss and/or gain can be accurately balanced by modern optical technology.

An ideal optical soliton obeys a normalized standard NLS equation [2,3]

$$i\frac{\partial}{\partial Z}Q(Z,T) + \varepsilon\frac{\partial^2}{\partial T^2}Q(Z,T) + \delta|Q(Z,T)|^2 Q(Z,T) = 0. \quad (1)$$

Here $Q(Z,T)$ is the normalized solitary envelope of optical pules and Z and T denote the normalized distance and time duration of the temporal propagation of the optical soliton [3]. $\varepsilon = \pm 1/2$ denote the anomalous and normal dispersion regions and $\delta = \pm 1$ represent the defocusing and focusing interactions, respectively. Usually, when $\varepsilon\delta > 0$, Eq. (1) has bright soliton solutions, otherwise Eq. (1) has dark soliton solutions. In optical soliton communication the fundamental bright soliton, which is

$$Q(Z,T) = \text{sech}(T)e^{iZ/2}, \quad (2)$$

plays a key role. The fundamental bright soliton (2) is ideal in transmission of optical soliton in fibers. In reality the fiber loss is inevitable. A long-distance transmission of the optical solitons requires one to compensate for the fiber loss by the managements of dispersion, nonlinearity, and Raman gain. Thus the dynamics of the transmission of optical solitons in fibers should be governed by a nonautonomous normalized NLS equation [16]

$$i\frac{\partial u(z,t)}{\partial z}+\varepsilon f(z,t)\frac{\partial^2 u(z,t)}{\partial t^2}+\delta g(z,t)|u(z,t)|^2 u(z,t) = i\frac{\gamma(z)}{2}u(z,t), \tag{3}$$

where $f(z,t)$ and $g(z,t)$ denote the time- and space-dependent managements of dispersion and nonlinearity, respectively. Here $\gamma(z)=\gamma_{\rm loss}+\gamma_R$, where $\gamma_{\rm loss}$ means the fiber loss ($\gamma_{\rm loss}<0$) and γ_R the Raman gain ($\gamma_R>0$). They are also assumed to be distance dependent. Generally these coefficients are assumed to be real. For convenience we use normalized model and thus all quantities in Eq. (3) are dimensionless and one can refer to, e.g., Ref. [3] for their physical meanings.

To explore the dynamics of the optical soliton obeying Eq. (3) it is useful first to perform the Painlevé analysis [19,20] since there is a close connection between complete integrability and the Painlevé property of partial differential equations. To proceed, one first expands $u(z,t)$, $f(z,t)$, and $g(z,t)$ on a noncharacteristic singularity manifold and then a recursion relation of the expanded coefficients of $u(z,t)$ can be written down. These coefficients can be uniquely determined by such a relation except for at some resonance points of $j=-1$, 0, 3, and 4. For details one can refer to Refs. [20,21]. These resonance points put forward compatibility conditions under which Eq. (3) can pass through the Painlevé test. While the first two give trivial results, the latter two lead to (i) $f(z,t)=f(z)$ and $g(z,t)=g(z)$ and (ii) a constraint condition on $f(z)$, $g(z)$ and $\gamma(z)$,

$$0=\frac{1}{f(z)}\frac{d^2}{dz^2}f(z)-\frac{1}{f^2(z)}\left[\frac{d}{dz}f(z)\right]^2-\frac{1}{g(z)}\frac{d^2}{dz^2}g(z)+\frac{2}{g^2(z)}\left[\frac{d}{dz}g(z)\right]^2-\frac{1}{f(z)}\left[\frac{d}{dz}f(z)\right]\frac{1}{g(z)}\frac{d}{dz}g(z)-\left[\frac{1}{f(z)}\frac{d}{dz}f(z)-\frac{2}{g(z)}\frac{d}{dz}g(z)\right]\gamma(z)-\frac{d}{dz}\gamma(z)+\gamma^2(z). \tag{4}$$

In the absence of $\gamma(z)$ [21], this condition is completely consistent with the integrability condition given in [16] using the Lax pair method [22,23]. However, to our knowledge condition (4) has not yet been given by the Lax pair method. Thus Eq. (4) is a generalized integrability condition of Eq. (3) in a sense of the Painlevé analysis. Before discussing physical implication of Eq. (4) to soliton dynamics, we first explore explicit solutions of Eq. (3) under the condition.

To obtain an analytic soliton solution of Eq. (3) it is useful to find a transformation that can reduce Eq. (3) to the standard NLS Eq. (1). Such a transformation can have the general form [21]

$$u(z,t)=Q(Z(z),T(z,t))e^{ia(z,t)+c(z)}. \tag{5}$$

Note that $T(z,t)$, $Z(z)$, $a(z,t)$, and $c(z)$ are only assumed to be real functions but no *a priori* forms, which is in sharp contrast to the special forms used in the literature [24–26]. The explicit forms of these transformation functions can be uniquely determined by the requirement that $u(z,t)$ and $Q(Z,T)$ satisfy Eqs. (3) and (1), respectively. Inserting Eq. (5) into Eq. (3) and comparing with Eq. (1), we obtain a set of differential equations of $T(z,t)$, $Z(z)$, $a(z,t)$, and $c(z)$, which have solutions under the condition Eq. (4),

$$a(z,t)=\frac{1}{4\varepsilon f(z)}\left\{\frac{d}{dz}\ln\left[\frac{f(z)}{g(z)}\right]-\gamma(z)\right\}t^2+C_1\frac{g(z)}{f(z)}e^{\Gamma(z)}t-\varepsilon C_1^2\int dz'\frac{g(z')^2}{f(z')}e^{2\Gamma(z')}+C_2, \tag{6}$$

$$T(z,t)=\frac{g(z)}{f(z)}e^{\Gamma(z)}t-2\varepsilon C_1\int dz'\frac{g(z')^2}{f(z')}e^{2\Gamma(z')}. \tag{7}$$

$$Z(z)=\int dz'\frac{g^2(z')}{f(z')}e^{2\Gamma(z')}+C_3, \tag{8}$$

$$c(z)=\frac{1}{2}\ln\frac{g(z)}{f(z)}+\Gamma(z), \tag{9}$$

where $\Gamma(z)=\int_0^z\gamma(z')dz'$ and $C_2=C_3=0$ if one takes initial condition $u(0,t)=Q(0,t)$. The constant C_1 is related to the central position of soliton and without loss of generality we take $C_1=0$. This means that the moving solutions in time domain and the phase contributed by the related terms in $a(z,t)$ are not considered. It is also interesting to point out that the results obtained are in agreement with the perturbation solutions of the nonautonomous NLS equation without the dispersion and nonlinearity managements but the fiber loss was taken into account [3]. However, the present results are exact and not limited to the weak dissipation cases.

Importantly these equations provide a systematic way to find the selected form solutions of the nonautonomous NLS Eq. (3) from the solutions of the standard NLS Eq. (1). For a given nonautonomous NLS equation, if its coefficients satisfy the Painlevé integrability condition (4), then the nonautonomous NLS equation can be reduced to the standard NLS equation by the transformation proposed. Thus all solutions, including the canonical solitons, of the standard NLS equation can be converted into the corresponding solutions of the nonautonomous NLS Eq. (3). The result can be applied to any modulation and gain/loss functions if they satisfy Eq. (4). This evidently provides a possibility to control exactly transmission of optical soliton even in the presence of the fiber loss and/or the Raman gain, as discussed below.

Before showing how to control exactly the soliton dynamics, let us first demonstrate the physical meaning of Eq. (4) by experiments [27,28]. These two experiments transmitted optical soliton in a lossy fiber with a nearly exponential group-velocity dispersion (GVD) profile. It was found that solitons preserved their width and shape in spite of energy losses of more than 8 dB/km in a 40 km fiber [27] and a 6.5 ps soliton train at 10 Gps could be transmitted over 300 km [28]. To fit the experiments, we set $g(z)=1$ and solve for $f(z)$ from Eq. (4) $f(z)=f_0\exp\{\int_1^z dz' e^{\tilde{\gamma}(z')}[f_1+\int_1^{z'}dz'' e^{-\tilde{\gamma}(z'')}h(z'')]\}$, where $\tilde{\gamma}(z)=\int_1^z\gamma(z')dz'$ and $h(z'')=\frac{d}{dz''}\gamma(z'')-\gamma^2(z'')$. Here f_0

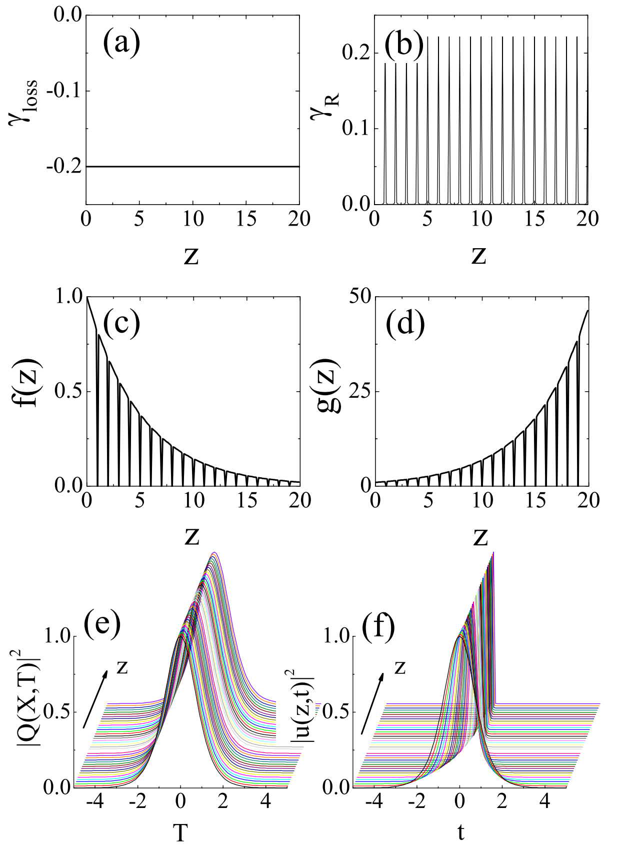

FIG. 1. (Color online) The dispersion and nonlinearity managed optical bright soliton with $c(z)=0$. (a) The fiber loss $\gamma_{loss}=-0.2$ dB$/L_D$, where L_D is the dispersion distance [29]. (b) The Raman gain γ_R with $Z_a=1$. [(c) and (d)] The management of dispersion $f(z)$ [Eq. (14)] and nonlinearity $g(z)$ [Eq. (12)], respectively. [(e) and (f)] The canonical optical soliton and the corresponding nonautonomous optical soliton, respectively.

and f_1 are constants determined by initial conditions. In a lossy fiber $\gamma(z)=\gamma=$const, $f(z)$ can be simplified to

$$f(z) = e^{\gamma z} \tag{10}$$

when $f_0=e^{\gamma}$ and $f_1=\gamma$ are chosen. Equation (10) is consistent with experimental GVD profile. In this case the fundamental bright soliton solution of Eq. (3) can be written as

$$u(z,t) = e^{1/2\gamma z}\,\mathrm{sech}(t)e^{i/2\gamma e^{\gamma z-1}}. \tag{11}$$

In comparison to the perturbed soliton behavior without the management of dispersion [3], the bright solitonlike shown in Eq. (11) maintains its width and shape, and has a much slow decay of its amplitude, which is qualitatively consistent with the experimental observations [27,28].

After having confirmed the validity of our results, we discuss how to control the soliton dynamics in a general case. Equation (9) provides an explicit way to control the shape of soliton, which can be written as

$$g(z) = f(z)e^{2\tilde{\Gamma}(z)}. \tag{12}$$

Here $\tilde{\Gamma}(z)=c(z)-\Gamma(z)$. According to ansatz (5), if $c(z)=0$, then the amplitude of optical soliton remains unchanged. If

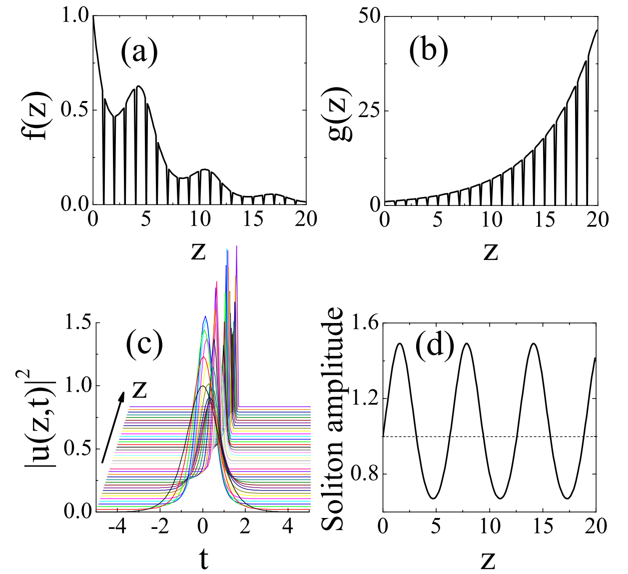

FIG. 2. (Color online) The dispersion and nonlinearity managed optical bright soliton with $c(z)=0.2\sin(z)$. [(a) and (b)] The management of dispersion $f(z)$ [Eq. (14)] and nonlinearity $g(z)$ [Eq. (12)], respectively. [(c) and (d)] The nonautonomous optical bright soliton and its amplitude as a function of the distance. The fiber loss and the Raman gain are the same as Fig. 1.

$c(z)>0$ (or <0) means that the soliton's amplitude increases (or decreases) exponentially. When $c(z)$ changes periodically its sign during the propagation of optical soliton, as a result, the optical soliton oscillates with the same period. From Eq. (12), Eq. (4) can be further simplified to

$$\alpha(z)\frac{d}{dz}f(z) - \beta(z)f(z) = 0, \tag{13}$$

where $\alpha(z)=\gamma(z)+2\frac{d}{dz}\tilde{\Gamma}(z)$ and $\beta(z)=\frac{d}{dz}\alpha(z)-\alpha^2(z)$. Equation (13) has a closed form solution $f(z)=\exp[\int_0^z dz'\beta(z')/\alpha(z')]$ with $f(0)=1$. When one takes into account the expression of $\beta(z)$, $f(z)$ is

$$f(z) = \frac{\gamma(z)}{\gamma(0)}e^{-\int_0^z \alpha(z')dz'}, \tag{14}$$

where $\gamma(z)=\gamma_{loss}(z)+\gamma_R(z)$. Here $\gamma_{loss}(z)$ denotes the fiber loss and $\gamma_R(z)$ represents the Raman gain, which can be modeled by $\gamma_R=\alpha_R\sum_{n=1}^{N}\delta(z-nZ_a)$ [29]. In this expression α_R is the Raman-gain coefficient and Z_a is the distance of two amplifiers. The fiber loss is usually a constant and here we set it to be -0.2 dB$/L_D$, where L_D is the dispersion distance [29]. The Raman-gain coefficient is taken as $\alpha_R=\exp(-\gamma_{loss}Z_a)-1$ [see Figs. 1(a) and 1(b) for $Z_a=1$]. Equations (12) and (14) provide an exact and explicit way to manage dispersion and nonlinearity in the presence of the fiber loss and/or the Raman gain. The management of dispersion and the nonlinearity in the case of $c(z)=0$ are shown in Figs. 1(c) and 1(d), respectively. As a result, the managed optical soliton shown in Fig. 1(f) has the same amplitude as that of the fundamental optical soliton presented in Fig. 1(e). However, the managed optical soliton is compressed rapidly due to rapid increase of the nonlinearity in compensating for the fiber loss in order to keep the amplitude of the optical soliton unchanged. Another interesting feature of the management of dispersion and nonlinearity has a direct response to the Raman gain at nZ_a, as shown in Figs. 1(c) and 1(d), respectively. This result has not been reported in the literature and is important for an optimal control of the transmission of optical soliton.

In the case of $c(z)\neq 0$ the amplitude of the optical soliton increases or decreases. Figure 2 shows that the amplitude of the optical soliton is modulated periodically as $c(z)=0.2\sin(z)$. Correspondingly the dispersion and the nonlinearity are also periodically modulated when the overall behaviors are similar to the case of $c(z)=0$.

Equation (13) can also be written as

$$\frac{d}{dz}\gamma(z)+\gamma^2(z)-\left[\frac{d}{dz}\ln f(z)\right]\gamma(z)=0 \tag{15}$$

when the amplitude of soliton is controlled to remain unchanged $[c(z)=0]$. A general solution of Eq. (15) is

$$\gamma(z)=f(z)/[A+\textstyle\int_1^z f(z')dz'], \tag{16}$$

where A is a constant related to $\gamma(0)$. This expression and Eq. (12) are useful to match properly the Raman gain and the nonlinearity modulation when the dispersion management is used in the transmission of optical soliton. The result can again build the balance between dispersion, nonlinearity, and the fiber loss and/or the Raman gain, which is helpful to decrease the bit error rate. This is evidently significant to the communication of realistic optical solitons.

In conclusion through the Painlevé analysis we obtained a Painlevé integrability condition of the nonautonomous NLS equation in the presence of the fiber loss. Under this condition the nonautonomous NLS equation can be reduced to the standard NLS equation by a general transformation. This result provides a possibility to control exactly the transmission of nonautonomous optical soliton in fibers, even in the presence of the fiber loss. The result has a significant contribution to the study of the dynamics of optical soliton and is expected to be useful to the technology of optical soliton communication.

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