Hybrid economical telecloning of equatorial qubits and generation of multipartite entanglement

Xin-Wen Wang and Guo-Jian Yang*
Department of Physics, Beijing Normal University, Beijing 100875, People's Republic of China and Applied Optics Beijing Area Major Laboratory, Beijing 100875, People's Republic of China
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We propose a simple scheme for hybrid economical telecloning (HETC) of equatorial qubits with the well-known W-type entangled states. The so-called HETC is a synthesis of several types of quantum cloning and quantum teleportation, in which economical symmetric and asymmetric clonings and anticlonings can be simultaneously achieved by teleportation. We use the global fidelity and average-single-qubit fidelity to estimate, respectively, the collective copying quality and show that the two criteria lead to different results. We obtain interesting equalities and inequalities about the fidelities of clones or anticlones. We also introduce controlled HETC of equatorial qubits with recently proposed Greenberger-Horne-Zeilinger (GHZ)–W-type entangled states [L. Chen and Y. X. Chen, Phys. Rev. A **74**, 062310 (2006)], in which the achievement of phase-covariant telecloning between the sender (Alice) and the receivers (Bobs) is conditioned on the collaboration of all the supervisors (Charlies). This idea may open a perspective for the applications of such interesting type of entangled states. A method for generating the GHZ–W-type entangled states is also presented.

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I. INTRODUCTION

Differing from classical information encoded in binary numbers, quantum information is encoded in two-state quantum systems, i.e., qubits, on which the manipulations are based on the fundamental principles of quantum mechanics. Quantum information theory can exploit the unique properties of quantum mechanics to realize many information transferring and processing tasks that classical information theory cannot achieve. On the other hand, the inner structure of quantum mechanics imposes many restrictions on the manipulation of quantum information, such as the no-cloning theorem [1,2] and no-flipping theorem [3] resulting from the linearity and unitarity of quantum mechanics, respectively. Although the exact cloning and flipping operations on an arbitrary unknown quantum state are unrealizable, the approximate ones are achievable [4,5]. As for the input states chosen from a subset of linear independent states, exact copying and flipping can also be realized probabilistically [6–8]. When the fidelities of the clones or orthogonal complement of the input unknown state reach the maximum (quantum mechanics allowed) and are independent of the input arbitrary state, the related quantum devices are called universal optimal quantum cloning (UOQC) machines [9] or universal NOT gates [5]. Quantum cloning has recently attracted much interest [10,11] because of its use in connection with quantum computation, quantum communication, and quantum cryptography (see, e.g., [12–15]). The cloning and flipping transformations are deeply interconnected [5,16,17]: the two processes are always realized contextually and their optimal fidelities are directly related. Consequently, more and more attention is paid to the study of quantum anticloning (copying the orthogonal complement of the input state) and cloning-cum-flipping (copying the input state and its orthogonal complement simultaneously) [5,8,18–20]. It has been shown that the fidelity of each copy in $1\rightarrow n$ symmetric UOQC is $F_{1,n}=(2n+1)/3n$ [9], and the optimal fidelity of universal quantum flipping or anticloning transformation is 2/3 which is equal to that of the measure-based flipping strategy (classical way) [3,5]. However, when partial information of the input state is known both the cloning and anticloning transformations can be dramatically improved. The typical examples are the phase-covariant cloning (PCC) and phase-covariant anticloning (PCAC), in which the initial state of the input qubit is on the equator of the Bloch sphere [21]. The PCC and PCAC for equatorial qubits has been extensively studied (see, e.g., [22–27]), and many PCC and PCAC machines or maps were presented, including a very interesting type of cloning machine in which no ancilla is needed, i.e., the so-called economical phase-covariant cloning machine [28–30].

In quantum approximate copying, the information on the input qubit is not degraded but only distributed on a larger quantum system. According to whether or not the initial quantum information is equally distributed, quantum cloning can be divided into two types, i.e., the symmetric quantum cloning (SQC) [4] and asymmetric quantum cloning (AQC) [31]. In SQC the fidelities of all outputs are equal, while in AQC they are usually different. AQC may be more efficient than its counterpart of SQC in applications such as quantum cryptography attacks. As a consequence, much attention has been paid to the AQC, especially the asymmetric phase-covariant cloning [28,32,33].

As mentioned above, quantum cloning process can be regarded as the distribution of quantum information from initial system to final ones. Thus quantum cloning combining with other quantum information processing tasks may have potential applications in many-party quantum communication and distributed quantum computation [34–36]. This leads to the advent of the concept of telecloning [37] that is the combination of quantum cloning and quantum teleportation [38]. Telecloning functions as transmitting many copies of an unknown state of the input qubit to many distant qubits, i.e., realizing one-to-many cloning at distant sites, via

*yanggj@bnu.edu.cn

previously shared multipartite entangled states. It can be regarded as a generalization of quantum teleportation to many-recipient case. In the past few years, telecloning was extensively studied and developed [39,40], involving symmetric telecloning [41] and asymmetric telecloning [42]. Recently, it has been shown that the quantum information previously distributed by telecloning procedure from an input qubit can be remotely concentrated back to a qubit via suitable entanglement channel [34]. Then the telecloning and concentrating processes can be regarded as, respectively, information depositing and withdrawing, or information encoding and decoding.

In this paper, we propose a simple scheme for hybrid economical telecloning (HETC) of the phase-covariant state

$$|\phi^{\delta}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\delta}|1\rangle), \tag{1}$$

where $\delta \in [0,2\pi]$ and $\{|0\rangle,|1\rangle\}$ represents the computational basis for a qubit. The qubits of this form are called "equatorial qubits" because the z component of their Bloch vector is zero, i.e., the Bloch vector is restricted to the intersection of the x-y plane with the sphere [21]. The parameter δ is the angle between the Bloch vector and the x axis. The so-called HETC is a synthesis of aforementioned several types of quantum cloning and quantum teleportation, in which economical symmetric and asymmetric clonings and anticlonings can be simultaneously achieved by teleportation. We use the global fidelity and average-single-qubit fidelity to estimate, respectively, the collective copying quality and show that the two criteria lead to different results. We obtain interesting equalities and inequalities about the fidelities of clones or anticlones. Our scheme may be very interesting from the point of view of quantum information distribution and have potential applications in quantum information depositing or encoding. The quantum information channel in our scheme is the well-known W-type entangled states [43]. The generation of W-type entangled states have been experimentally realized in different physical systems [44–46]. This implies that our HETC scheme can be directly demonstrated in these systems. We also introduce controlled HETC of equatorial qubits with recently presented Greenberger-Horne-Zeilinger (GHZ)–W-type entangled states [47], in which phase-covariant telecloning can be realized if and only if (iff) the supervisors cooperate. This idea may open a perspective for the applications of such interesting type of entangled states. A method for generating the GHZ–W-type entangled states is also presented.

The paper is organized as follows. In Sec. II, we describe the HETC protocol and use the global fidelity and average-single-qubit fidelity to estimate the collective copying quality, respectively. It will be shown that when the superposition coefficients of the W state satisfying a suitable condition, the suboptimal phase-covariant telecloning can be realized. In Sec. III, we demonstrate probabilistic suboptimal economical phase-covariant telecloning with a special configuration of asymmetric W state. In Sec. IV, we introduce the controlled HETC protocol. A generation scheme for GHZ–W-type entangled states is presented in Sec. V. Concluding remarks appear in Sec. VI.

II. HETC OF EQUATORIAL QUBITS VIA W-TYPE ENTANGLED STATES

The task of HETC is: Alice wishes to send the copies of the phase-covariant state $|\phi^{\delta}\rangle$ (δ is unknown) on the qubit T or its orthogonal complementary state $|\phi^{\delta}\rangle^{\perp}$ to distant associates Bob1, Bob2,…, Bob$n$. In this task, some Bobs receive the clones (copies of $|\phi^{\delta}\rangle$), while the others obtain the anticlones [copies of $|\phi^{\delta}\rangle^{\perp} = (|0\rangle - e^{i\delta}|1\rangle)/\sqrt{2}$]. Note that the fidelities of these clones or anticlones are not necessarily equal. To this end, we distribute in advance among Alice and n Bobs the quantum information channel

$$|\mathrm{W}_{n+1}\rangle = x_0|1_A\rangle\prod_{j=1}^{n}|0_{B_j}\rangle + |0_A\rangle\sum_{j=1}^{n}\left(x_j|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right), \tag{2}$$

where $x_0 = |x_0|e^{i\theta_0}$ and $x_j = |x_j|e^{i\theta_j}$ are complex coefficients satisfying the normalized condition $|x_0|^2 + \Sigma_j^n |x_j|^2 = 1$. The channel $|\mathrm{W}_{n+1}\rangle$ is a generalized $(n+1)$-qubit W-type entangled state which has been recently realized with linear optical elements [45]. Here qubit A is on Alice's hand and qubit B_j is held by the jth Bob. For clarity and convenience, the qubit T will be called input qubit, qubit A will be called port qubit, and the others will be called output qubits.

The state of the total system is

$$\begin{aligned}
|\psi\rangle_{\mathrm{total}} &= |\phi^{\delta}\rangle_T \otimes |\mathrm{W}_{n+1}\rangle \\
&= \frac{1}{2}\Bigg\{|\Psi^+\rangle_{TA}\left[x_0\prod_{j=1}^{n}|0_{B_j}\rangle + e^{i\delta}\sum_{j=1}^{n}\left(x_j|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right)\right] + |\Psi^-\rangle_{TA}\left[x_0\prod_{j=1}^{n}|0_{B_j}\rangle - e^{i\delta}\sum_{j=1}^{n}\left(x_j|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right)\right] \\
&\quad + |\Phi^+\rangle_{TA}\left[\sum_{j=1}^{n}\left(x_j|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right) + x_0e^{i\delta}\prod_{j=1}^{n}|0_{B_j}\rangle\right] + |\Phi^-\rangle_{TA}\left[\sum_{j=1}^{n}\left(x_j|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right) - x_0e^{i\delta}\prod_{j=1}^{n}|0_{B_j}\rangle\right]\Bigg\},
\end{aligned} \tag{3}$$

where $|\Psi^{\pm}\rangle_{TA}$ and $|\Phi^{\pm}\rangle_{TA}$ are the conventional four orthonormal Bell states given by

$$|\Psi^{\pm}\rangle_{TA}=\frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{TA},$$

$$|\Phi^{\pm}\rangle_{TA}=\frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{TA}. \quad (4)$$

The HETC of $|\phi^{\delta}\rangle$ can be accomplished by the following simple procedure. (i) Alice performs a Bell-basis measurement on the input qubit T and port qubit A, obtaining one of the four outcomes $\{|\Psi^{\pm}\rangle_{TA},|\Phi^{\pm}\rangle_{TA}\}$. Thereafter, Alice broadcasts her measurement outcome to Bobs through the classical channel. (ii) After receiving Alice's Bell-basis measurement outcome, Bobs perform, respectively, corresponding local operations on their output qubits to get the desired clones or anticlones.

We now discuss the second step in detail. As an example, we assume that Alice's measurement outcome is $|\Psi^{+}\rangle_{TA}$. Then the state of the output qubits collapses into

$$|\phi\rangle_{\text{out}}=x_0\prod_{j=1}^{n}|0_{B_j}\rangle+e^{i\delta}\sum_{j=1}^{n}\left(x_j|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right). \quad (5)$$

The state-density operator of the jth output qubit is

$$\rho_j=\left(\sum_{k=0,k\neq j}^{n}|x_k|^2\right)|0_{B_j}\rangle\langle 0_{B_j}|+|x_j|^2|1_{B_j}\rangle\langle 1_{B_j}|+e^{i\delta}x_jx_0^{*}|1_{B_j}\rangle \times\langle 0_{B_j}|+e^{-i\delta}x_0x_j^{*}|0_{B_j}\rangle\langle 1_{B_j}|. \quad (6)$$

Obviously, ρ_j only depends on the superposition coefficients x_0 and x_j of the quantum information channel $|\mathrm{W}_{n+1}\rangle$. The fidelities of the state of qubit B_j relative to the states $|\phi^{\delta}\rangle_j$ and $|\phi^{\delta}\rangle_j^{\perp}$ are, respectively,

$$F_j={}_j\langle\phi^{\delta}|\rho_j|\phi^{\delta}\rangle_j=\frac{1}{2}+|x_0x_j|\cos(\theta_0-\theta_j),$$

$$F_j^{\perp}={}_j^{\perp}\langle\phi^{\delta}|\rho_j|\phi^{\delta}\rangle_j^{\perp}=\frac{1}{2}-|x_0x_j|\cos(\theta_0-\theta_j). \quad (7)$$

It can be easily verified that a local phase-rotating operation,

$$U_{\vartheta_j}=\exp\left(-i\frac{\vartheta_j}{2}\sigma_j^{z}\right), \quad (8)$$

on the qubit B_j performed by the jth Bob will result in

$$F_j=\frac{1}{2}+|x_0x_j|\cos(\theta_0-\theta_j-\vartheta_j),$$

$$F_j^{\perp}=\frac{1}{2}-|x_0x_j|\cos(\theta_0-\theta_j-\vartheta_j). \quad (9)$$

$\sigma^{x,y,z}$ denotes conventionally the Pauli matrix, and ϑ_j is the rotated angle about the z axis of Bloch sphere. Obviously, the summation of F_j and $F_j^{\perp}$ is always equal to one, i.e., $F_j+F_j^{\perp}=1$. This can be easily understood from that two orthogonal components ψ and $\psi^{\perp}$ of an arbitrary state vector $\vec{\psi}$ in a plane satisfy the geometric relationship $|\psi|^2+|\psi^{\perp}|^2=1$. In addition, the jth Bob can modulate the rotation angle ϑ_j to alter the fidelities F_j and $F_j^{\perp}$. In other words, when the jth Bob wants to obtain a clone of $|\phi^{\delta}\rangle$ on his qubit B_j, he can maximize the fidelity F_j by setting $\vartheta_j=\theta_0-\theta_j-2m\pi$, with m being an integer; when the jth Bob wishes to get an anticlone of $|\phi^{\delta}\rangle$, i.e., a clone of $|\phi^{\delta}\rangle^{\perp}$, he can maximize the fidelity $F_j^{\perp}$ by choosing $\vartheta_j=\theta_0-\theta_j-(2m+1)\pi$. The maximum of both F_j and $F_j^{\perp}$ is

$$\tilde{F}_j=\frac{1}{2}+|x_0x_j|. \quad (10)$$

It can be seen that $\tilde{F}_j$ cannot reach one as long as $n>1$, which implies that the initial information of the input qubit cannot be totally assigned to any one of many receivers and also indicates that the rotation on the state vector $|\phi_j\rangle$ ($|\phi_j\rangle\langle\phi_j|=\rho_j$) in the $|\phi^{\delta}\rangle-|\phi^{\delta}\rangle^{\perp}$ plane is constrained by the quantum channel (x_0,x_j) and thus it cannot be rotated to the directions parallel to $|\phi^{\delta}\rangle$ or $|\phi^{\delta}\rangle^{\perp}$. As a consequence, we obtain the equality

$$\tilde{F}_j-\tilde{F}_k=|x_0|(|x_j|-|x_k|). \quad (11)$$

Equation (11) indicates that the difference of the fidelities of any two clones or anticlones is linearly dependent of the difference of corresponding probability amplitudes of the W-type entanglement channel of Eq. (2).

If Alice's measurement outcome is one of the other three Bell states, Bobs can also obtain the desired clones or anticlones with the maximal fidelity $\tilde{F}$ by suitable local operations (see Table I).

TABLE I. The corresponding local operations the jth Bob performed on the qubit B_j for getting the desired clones or anticlones of $|\phi^{\delta}\rangle$ with the maximal fidelity $\tilde{F}_j$, according to Alice's four possible Bell-measurement outcomes $|\Psi^{\pm}\rangle_{TA}$ and $|\Phi^{\pm}\rangle_{TA}$.

Bell states	Clone	Anticlone
$\lvert\Psi^{+}\rangle_{TA}$	$U_{\theta_0-\theta_j-2m\pi}$	$U_{\theta_0-\theta_j-(2m+1)\pi}$
$\lvert\Psi^{-}\rangle_{TA}$	$U_{\theta_0-\theta_j-(2m+1)\pi}$	$U_{\theta_0-\theta_j-2m\pi}$
$\lvert\Phi^{+}\rangle_{TA}$	$U_{\theta_0-\theta_j-2m\pi}\otimes\sigma_j^{x}$	$U_{\theta_0-\theta_j-(2m+1)\pi}\otimes\sigma_j^{x}$
$\lvert\Phi^{-}\rangle_{TA}$	$U_{\theta_0-\theta_j-(2m+1)\pi}\otimes\sigma_j^{x}$	$U_{\theta_0-\theta_j-2m\pi}\otimes\sigma_j^{x}$

According to the discussion above, we can safely conclude that the presented telecl
oning scheme may simultaneously achieve the cloning and anticloning of $|\phi^{\delta}\rangle$, with some Bobs obtaining the clones and the other Bobs getting the anticlones. It is worth pointing out that whether getting the clones or anticlones are determined by themselves, which is different from previous schemes. In addition, some clone's or anticlone's fidelities may be the same, while the others' may be different. For instance, if $|x_k|=|x_l|\neq|x_j|$ ($j=1,2,\ldots,n,l\neq j\neq k$) and all the $\{|x_j|\}$ are different, the fidelities of the clones or anticlones of the kth Bob and lth Bob are equal, and that of the other Bobs are unequal. In this sense, our scheme may simultaneously implement the symmetric and asymmetric clonings and anticlonings of $|\phi^{\delta}\rangle$. Our scheme does not require the ancilla and thus is economi-

cal. In a nutshell, our telecloning scheme can simultaneously achieve economical symmetric and asymmetric clonings and anticlonings of the phase-covariant state $|\phi^{\delta}\rangle$ by teleportation. Note that all the telecloning schemes in this paper involve both cloning and anticloning tasks. In addition, because all the telecloning schemes in our paper are economical, we will drop the word "economical" in description.

Next, we study the collective copying quality by considering the global and the average-single-qubit fidelities following the foregoing discussion.

First, we consider the global fidelity with definition [22,37],

$$F_g = \mathrm{Tr}[(U_{\mathcal{M}}\mathcal{M}|\psi\rangle_{\text{total}}\langle\psi|\mathcal{M}^{+}U_{\mathcal{M}}^{+})|\phi^{\delta}\rangle^{\perp}\langle\phi^{\delta}|^{\otimes m}|\phi^{\delta}\rangle \times\langle\phi^{\delta}|^{\otimes(n-m)}], \tag{12}$$

where $\mathcal{M}$ is the Bell-basis measurement operator given by

$$\mathcal{M} = |\Psi^{+}\rangle_{TA}\langle\Psi^{+}| + |\Psi^{-}\rangle_{TA}\langle\Psi^{-}| + |\Phi^{+}\rangle_{TA}\langle\Psi^{+}| + |\Phi^{-}\rangle_{TA}\langle\Psi^{-}|, \tag{13}$$

$U_{\mathcal{M}}$ is the proper local operations performed by Bobs for obtaining the desired clones or anticlones with the maximal fidelity $\widetilde{F}$ (see Table I), and $|\phi^{\delta}\rangle^{\perp}\langle\phi^{\delta}|^{\otimes m}|\phi^{\delta}\rangle\langle\phi^{\delta}|^{\otimes(n-m)}$ denotes m Bobs getting the anticlones and the other $n-m$ Bobs obtaining the clones. By some simple calculations, we obtain

$$F_g = \frac{1}{2^n}\left[1 + 2\sum_{k=0}^{n-1}\left(|x_k|\sum_{j=k+1}^{n}|x_j|\right)\right], \tag{14}$$

with the constraint $\Sigma_{k=0}^{n}|x_k|^2=1$. Evidently, F_g is the function of all the superposition coefficients of the entanglement channel $|\mathrm{W}_{n+1}\rangle$ and should have a maximum with suitable values of these parameters. By the Lagrange multipliers, we find the suitable values

$$|x_0| = |x_1| = \cdots = |x_n| = \frac{1}{\sqrt{n+1}} \tag{15}$$

corresponding to the maximal global fidelity $F_g^{\max}=(n+1)/2^n$. Thus when the quantum channel $|\mathrm{W}_{n+1}\rangle$ is fully symmetric, the global fidelity hits to the maximum. Then each clone or anticlone has the same fidelity, i.e., $\widetilde{F}_1=\widetilde{F}_2=\cdots=\widetilde{F}_n=1/2+1/(n+1)$.

Now let us move on to the average-single-qubit fidelity. It is defined as

$$\bar{F} = \frac{1}{n}\sum_{j=1}^{n}\widetilde{F}_j. \tag{16}$$

From Eq. (10), we obtain

$$\bar{F} = \frac{1}{2} + \frac{|x_0|}{n}\sum_{j=1}^{n}|x_j|, \tag{17}$$

with the constraint $\Sigma_{k=0}^{n}|x_k|^2=1$. With the help of Lagrange multipliers, we find the optimal solution that maximizes $\bar{F}$ given by

$$|x_0| = \frac{1}{\sqrt{2}},$$

$$|x_1| = |x_2| = \cdots = |x_n| = \frac{1}{\sqrt{2n}}. \tag{18}$$

The maximal average-single-qubit fidelity is

$$\bar{F}^{\max} = \frac{1}{2}\left(1 + \frac{1}{\sqrt{n}}\right). \tag{19}$$

In fact, each clone or anticlone has the same fidelity, i.e., $\widetilde{F}_1=\widetilde{F}_2=\cdots=\widetilde{F}_n=1/2(1+1/\sqrt{n})$. In the case $n=2$, the fidelity is equal to the optimal one of $1\rightarrow2$ phase-covariant cloning [21,22]. Thus the telecloning scheme is suboptimal with this choice of parameters. Then the entanglement channel of Eq. (2) reduces to

$$|\mathrm{W}'_{n+1}\rangle = \frac{1}{\sqrt{2}}\left[e^{i\theta_0}|1_A\rangle\sum_{j=1}^{n}|0_{B_j}\rangle + \frac{1}{\sqrt{n}}|0_A\rangle\sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right)\right]. \tag{20}$$

It can be easily verified that $\rho_A=\mathrm{tr}_{B_1B_2\cdots B_n}(|\mathrm{W}'_{n+1}\rangle\langle\mathrm{W}'_{n+1}|)=I/2$ and its von Neumann entropy is just one, which implies that there is one ebit of entanglement between the subsystem of Alice and the subsystem of Bobs. From Eq. (19), we can also obtain the inequality

$$\sum_{j=1}^{n}F_j \leq \frac{1}{2}(n+\sqrt{n}), \tag{21}$$

which indicates that the summation of the fidelities of all clones or anticlones has the upper bound $(n+\sqrt{n})/2$.

As discussed above, maximizing the global fidelity and the average-single-qubit fidelity generally leads to different results, i.e., different configuration of the W state [Eq. (2)] are expected, respectively, for the two criteria. In other words, the expected entanglement channel depends on the criterion adopted to assess the collective copying quality. However, both the two criteria indicate that the symmetric telecloning is optimal from the point of view of collective copying quality. In the special case $n=1$, the maximum of global fidelity $F_g^{\max}$ is equal to that of the average-single-qubit fidelity $\bar{F}^{\max}$. This can be understood from that the telecloning scheme reduces to the standard teleportation protocol with only one receiver.

III. PROBABILISTIC SUBOPTIMAL PHASE-COVARIANT TELECLONING

In Sec. II, we have shown that the W-type entangled state $|\mathrm{W}'_{n+1}\rangle$ [see Eq. (20)] can be used to achieve suboptimal (economical) telecloning of the phase-covariant state $|\phi^{\delta}\rangle$. As explained above, the "suboptimal" means that when $n=2$ the fidelity of clones or anticlones is equal to the optimal one of $1\rightarrow2$ phase-covariant cloning. The quantum channel

$|W'_{n+1}\rangle$ implies that there is one ebit of entanglement shared between the sender, Alice, and the receivers, Bobs. $|W'_{n+1}\rangle$ is obtained from $|W_{n+1}\rangle$ [see Eq. (2)] by setting the parametric values of Eq. (18). If part conditions of Eq. (18) are not satisfied, i.e., $|x_0| \neq 1/\sqrt{2}$ or $|x_1|=|x_2|=\cdots=|x_n|\neq 1/\sqrt{2n}$, the state $|W'_{n+1}\rangle$ will degrade to $|W''_{n+1}\rangle$ [see Eq. (22)] which has less than one ebit of entanglement between the subsystem of Alice (qubit A) and the subsystem of Bobs (qubits B_j, $j=1,2,\ldots,n$). The degradation of $|W'_{n+1}\rangle$ to $|W''_{n+1}\rangle$ may be induced by a kind of decoherence on the port-qubit A [48]. In this section, we demonstrate that the suboptimal telecloning can also be achieved via $|W''_{n+1}\rangle$ with a certain probability. It is easy to verify that the state $|W''_{n+1}\rangle$ can be written as

$$|W''_{n+1}\rangle = \frac{1}{Q}\left[q e^{i\theta_0}|1_A\rangle \sum_{j=1}^{n} |0_{B_j}\rangle + |0_A\rangle \sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle \prod_{k=1,k\neq j}^{n} |0_{B_k}\rangle \right)\right], \tag{22}$$

where $q=|x_0|/|x_1|=|x_0|/|x_2|=\cdots=|x_0|/|x_n|$ and $Q=\sqrt{n+q^2}$. Before describing the suboptimal telecloning procedure, we define a set of two-qubit orthonormal basis states as follows:

$$|\Psi_h^+\rangle = \frac{1}{H}(|01\rangle + h|10\rangle),$$

$$|\Psi_h^-\rangle = \frac{1}{H}(h|01\rangle - |10\rangle),$$

$$|\Phi_h^+\rangle = \frac{1}{H}(|00\rangle + h|11\rangle),$$

$$|\Phi_h^-\rangle = \frac{1}{H}(h|00\rangle - |11\rangle), \tag{23}$$

where $H=\sqrt{1+h^2}$, with h being real number. Here h is a free parameter manipulated by Alice. With this basis, the state of the whole system can be expanded as

$$\begin{aligned}|\psi\rangle_{\text{total}} = |\phi^{\delta}\rangle_T \otimes |W''_{n+1}\rangle = \frac{1}{QH}\Bigg\{ & |\Psi_h^+\rangle_{TA}\frac{1}{\sqrt{2}}\left[q e^{i\theta_0}\prod_{j=1}^{n}|0_{B_j}\rangle + h e^{i\delta}\sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right)\right] + |\Psi_h^-\rangle_{TA}\frac{1}{\sqrt{2}}\left[q h e^{i\theta_0}\prod_{j=1}^{n}|0_{B_j}\rangle \right.\\ & \left. - e^{i\delta}\sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right)\right] + |\Phi_h^+\rangle_{TA}\frac{1}{\sqrt{2}}\left[\sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right) + q h e^{i\delta}e^{i\theta_0}\prod_{j=1}^{n}|0_{B_j}\rangle\right]\\ & + |\Phi_h^-\rangle_{TA}\frac{1}{\sqrt{2}}\left[h\sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle\prod_{k=1,k\neq j}^{n}|0_{B_k}\rangle\right) - q e^{i\delta}e^{i\theta_0}\prod_{j=1}^{n}|0_{B_j}\rangle\right]\Bigg\}. \end{aligned}\tag{24}$$

In order to realize the suboptimal telecloning task, Alice first performs a complete projective measurement jointly on qubits T and A in the basis $\{|\Psi_h^{\pm}\rangle_{TA}, |\Phi_h^{\pm}\rangle_{TA}\}$ and informs Bobs her outcome. It can be seen that there are two different choices of parameter h with which suboptimal telecloning can be successfully realized for two out of four possible outcomes of Alice's measurement. (i) Choosing $h=q/\sqrt{n}$. If Alice's outcome is $|\Psi_h^+\rangle_{TA}$ or $|\Phi_h^-\rangle_{TA}$, suboptimal telecloning succeeds and Bobs can obtain the desired clones or anticlones with fidelity $\bar{F}^{\max}$ by making suitable local operations given in Table I with $|\Psi^+\rangle_{TA}$ and $|\Phi^-\rangle_{TA}$ replaced by $|\Psi_h^+\rangle_{TA}$ and $|\Phi_h^-\rangle_{TA}$, respectively. For the outcome $|\Psi_h^-\rangle_{TA}$ or $|\Phi_h^+\rangle_{TA}$, suboptimal telecloning fails. (ii) Choosing $h=\sqrt{n}/q$. When Alice's outcome is $|\Psi_h^-\rangle_{TA}$ or $|\Phi_h^+\rangle_{TA}$, suboptimal telecloning succeeds and Bobs can get the desired clones or anticlones with fidelity $\bar{F}^{\max}$ by making suitable local operations given in Table I with $|\Psi^-\rangle_{TA}$ and $|\Phi^+\rangle_{TA}$ replaced by $|\Psi_h^-\rangle_{TA}$ and $|\Phi_h^+\rangle_{TA}$, respectively. For the outcome $|\Psi_h^+\rangle_{TA}$ or $|\Phi_h^-\rangle_{TA}$, suboptimal telecloning fails. For each case, the total probability of successful suboptimal telecloning is

$$P = \frac{2nq^2}{(n+q^2)^2}. \tag{25}$$

We notice that all the four outcomes will lead to successful suboptimal telecloning if and only if $h=q/\sqrt{n}=1$, and then state $|W''_{n+1}\rangle$ reduces to $|W'_{n+1}\rangle$. This implies that only with the choice of the parameters of Eq. (18) the W-type entangled state $|W_{n+1}\rangle$ can realize deterministic suboptimal telecloning of the phase-covariant state $|\phi^{\delta}\rangle$. We can also conclude that when the quantum information channel is $|W'_{n+1}\rangle$, only the Bell-basis measurement can lead to suboptimal phase-covariant telecloning.

IV. CONTROLLED PHASE-COVARIANT TELECLONING WITH GHZ–W-TYPE ENTANGLED STATES

In this section, we introduce controlled telecloning of the state $|\phi^{\delta}\rangle_T$ with the following multiqubit entangled state:

$$|\Omega_m\rangle = x_0\left(\prod_{k=1}^{m}|0_{C_k}\rangle\right)|1_A\rangle\left(\prod_{j=1}^{n}|0_{B_j}\rangle\right) + \left(\prod_{k=1}^{m}|1_{C_k}\rangle\right)\times|0_A\rangle\sum_{j=1}^{n}\left(x_j|1_{B_j}\rangle\prod_{l=1,l\neq j}^{n}|0_{B_l}\rangle\right). \tag{26}$$

The state $|\Omega_m\rangle$ belongs to the GHZ–W-type entangled states recently proposed by Chen [47]. Here qubit C_k is held by the kth supervisor Charlie k, and the distribution of the other qubits is the same as that of Sec. II. It will be shown that for getting the desired clones or anticlones Bobs need not only the help of Alice's joint measurement but also the assistance of Charlies' one-qubit measurements. Generally, Charlies' one-qubit measurement basis can be described by

$$|+_{C_k}\rangle = \frac{1}{R}(|0\rangle + r_k|1\rangle)_{C_k},$$

$$|-_{C_k}\rangle = \frac{1}{R}(r_k|0\rangle - |1\rangle)_{C_k}, \tag{27}$$

where $R=\sqrt{1+r_k^2}$, with r_k being real numbers ($k=1,2,\ldots,m$) freely manipulated by Charlies. In a nutshell, the achievement of telecloning between the sender, Alice, and the receivers, Bobs, is conditioned on the collaboration of all the supervisors, Charlies. In other words, if any one of the Charlies does not cooperate, the desired telecloning will fails. We need pointing out that the qubits $\{C_k,k=1,2,\ldots,m\}$ are different from the ancilla of previous cloning machines in which the ancilla just be traced out but not be measured.

For clarity, we first consider the case $m=1$, i.e., there is only one supervisor. Then with the two-qubit basis $\{|\Psi_h^{\pm}\rangle_{TA},|\Phi_h^{\pm}\rangle_{TA}\}$ and one-qubit basis $\{|\pm_{C_1}\rangle\}$, the state of the whole system can be written as

$$|\widetilde{\psi}\rangle_{\text{total}} = |\phi^{\delta}\rangle_T|\Omega_1\rangle = \frac{1}{\sqrt{2}H}\Big\{|\Psi_h^{+}\rangle_{TA}\frac{1}{R}[|+_{C_1}\rangle(|\varphi_0\rangle + e^{i\delta}hr_1|\varphi_1\rangle) + |-_{C_1}\rangle(r_1|\varphi_0\rangle - e^{i\delta}h|\varphi_1\rangle)] + |\Psi_h^{-}\rangle_{TA}\frac{1}{R}[|+_{C_1}\rangle(h|\varphi_0\rangle - e^{i\delta}r_1|\varphi_1\rangle) + |-_{C_1}\rangle(hr_1|\varphi_0\rangle + e^{i\delta}|\varphi_1\rangle)] + |\Phi_h^{+}\rangle_{TA}\frac{1}{R}[|+_{C_1}\rangle(e^{i\delta}h|\varphi_0\rangle + r_1|\varphi_1\rangle) + |-_{C_1}\rangle(e^{i\delta}hr_1|\varphi_0\rangle - |\varphi_1\rangle)] + |\Phi_h^{-}\rangle_{TA}\frac{1}{R}[|+_{C_1}\rangle(- e^{i\delta}|\varphi_0\rangle + hr_1|\varphi_1\rangle) - |-_{C_1}\rangle(e^{i\delta}r_1|\varphi_0\rangle + h|\varphi_1\rangle)]\Big\}, \tag{28}$$

where $|\varphi_0\rangle$ and $|\varphi_1\rangle$ are given by

$$|\varphi_0\rangle = x_0\prod_{j=1}^{n}|0_{B_j}\rangle,$$

$$|\varphi_1\rangle = \sum_{j=1}^{n}\left(x_j|1_{B_j}\rangle\prod_{l=1,l\neq j}^{n}|0_{B_l}\rangle\right). \tag{29}$$

The telecloning procedure is as follows. (i) Alice performs a joint measurement on the qubits T and A with the basis $\{|\Psi_h^{\pm}\rangle_{TA},|\Phi_h^{\pm}\rangle_{TA}\}$, and Charlie1 performs an one-qubit measurement on the qubit C_1, and informs their outcomes to Bobs. (ii) Bobs make proper local unitary operations on their own qubits to get the desired clones or anticlones. The detailed discussion of (ii) is the same as that of Sec. II. But the fidelities of clones or anticlones are related to not only the channel ($|x_j|$) but also Alice's and Charlie1's measurement basis (h,r_1). It can be seen that if Charlie1 does not cooperate, i.e., broadcast faithfully his measurement outcome, Bobs will not know choosing what local operations and none of them can obtain the desired clones or anticlones. In this sense, we say that the communication (telecloning) between Alice and Bobs is supervised and controlled by Charlie1.

Next, we pay attention to the suboptimal telecloning with the choice of the parameters of Eq. (18). Then the quantum information channel is

$$|\Omega_1'\rangle = \frac{e^{i\theta_0}}{\sqrt{2}}\left(\prod_{k=1}^{m}|0_{C_k}\rangle\right)|1_A\rangle\left(\prod_{j=1}^{n}|0_{B_j}\rangle\right) + \frac{1}{\sqrt{2n}}\left(\prod_{k=1}^{m}|1_{C_k}\rangle\right)\times|0_A\rangle\sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle\prod_{l=1,l\neq j}^{n}|0_{B_l}\rangle\right). \tag{30}$$

The expansion of the total state is similar to Eq. (28) with x_0 replaced by $e^{i\theta_0}/\sqrt{2}$ and x_j $(j=1,2,\ldots,n)$ replaced by $e^{i\theta_j}/\sqrt{2n}$. According to the analysis of Sec. II, for suboptimal telecloning the output state of qubits $\{B_j,j=1,2,\ldots,n\}$ should be in the form

$$|\phi\rangle_{\text{out}}' = \frac{1}{\sqrt{2}}e^{i\theta_0}\prod_{j=1}^{n}|0_{B_j}\rangle \pm \frac{e^{i\delta}}{\sqrt{2n}}\sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle\prod_{l=1,l\neq j}^{n}|0_{B_l}\rangle\right) \tag{31}$$

or

$$|\phi\rangle'_{\text{out}} = \frac{1}{\sqrt{2n}}\sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle\prod_{l=1,l\neq j}^{n}|0_{B_l}\rangle\right) \pm \frac{e^{i\delta}e^{i\theta_0}}{\sqrt{2}}\prod_{j=1}^{n}|0_{B_j}\rangle. \tag{32}$$

It can be seen from Eq. (28) that when $h=r_1=1$ each pair of measurement outcomes can lead to suboptimal telecloning, and thus the success probability is one, provided Charlie1 cooperates.

For more general case that $|x_0|\neq 1/\sqrt{2}$ or $|x_1|=|x_2|=\cdots=|x_n|\neq 1/\sqrt{2n}$, the conclusions will be different, and an interesting phenomenon will be observed. Then the quantum channel $|\Omega_1'\rangle$ degrades to $|\Omega_1''\rangle$ [see Eq. (33)]. Such degradation may be induced by a kind of decoherence on the port-qubit A [48]. $|\Omega_1''\rangle$ can be written as

$$|\Omega_1''\rangle = \frac{1}{Q}\left[qe^{i\theta_0}\left(\prod_{k=1}^{m}|0_{C_k}\rangle\right)|1_A\rangle\left(\prod_{j=1}^{n}|0_{B_j}\rangle\right) + \left(\prod_{k=1}^{m}|1_{C_k}\rangle\right)\times|0_A\rangle\sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle\prod_{l=1,l\neq j}^{n}|0_{B_l}\rangle\right)\right], \tag{33}$$

where $q=|x_0|/|x_1|=|x_0|/|x_2|=\cdots=|x_0|/|x_n|$ and $Q=\sqrt{n+q^2}$. The state of the whole system can be expanded as

$$\begin{aligned}|\tilde{\psi}'\rangle_{\text{total}} = |\phi^{\delta}\rangle_T|\Omega_1''\rangle = \frac{1}{HQ}\Bigg\{&|\Psi_h^+\rangle_{TA}\frac{1}{R}\left[|+_{C_1}\rangle\frac{1}{\sqrt{2}}(q|\varphi_0'\rangle + e^{i\delta}hr_1|\varphi_1'\rangle) + |-_{C_1}\rangle\frac{1}{\sqrt{2}}(r_1q|\varphi_0'\rangle - e^{i\delta}h|\varphi_1'\rangle)\right] + |\Psi_h^-\rangle_{TA}\frac{1}{R}\Bigg[|+_{C_1}\rangle\frac{1}{\sqrt{2}}(hq|\varphi_0'\rangle\\ &- e^{i\delta}r_1|\varphi_1'\rangle) + |-_{C_1}\rangle\frac{1}{\sqrt{2}}(hr_1q|\varphi_0'\rangle + e^{i\delta}|\varphi_1'\rangle)\Bigg] + |\Phi_h^+\rangle_{TA}\frac{1}{R}\left[|+_{C_1}\rangle\frac{1}{\sqrt{2}}(e^{i\delta}hq|\varphi_0'\rangle + r_1|\varphi_1'\rangle) + |-_{C_1}\rangle\frac{1}{\sqrt{2}}(e^{i\delta}hr_1q|\varphi_0'\rangle - |\varphi_1'\rangle)\right]\\ &+ |\Phi_h^-\rangle_{TA}\frac{1}{R}\left[|+_{C_1}\rangle\frac{1}{\sqrt{2}}(-e^{i\delta}q|\varphi_0'\rangle + hr_1|\varphi_1'\rangle) - |-_{C_1}\rangle\frac{1}{\sqrt{2}}(e^{i\delta}r_1q|\varphi_0'\rangle + h|\varphi_1'\rangle)\right]\Bigg\},\end{aligned} \tag{34}$$

where $|\varphi_0'\rangle$ and $|\varphi_1'\rangle$ are given by

$$|\varphi_0'\rangle = e^{i\theta_0}\prod_{j=1}^{n}|0_{B_j}\rangle,$$

$$|\varphi_1'\rangle = \sum_{j=1}^{n}\left(e^{i\theta_j}|1_{B_j}\rangle\prod_{l=1,l\neq j}^{n}|0_{B_l}\rangle\right). \tag{35}$$

In order to realize the suboptimal telecloning, the parameters h and r_1 need conforming to a certain relationship. In other words, Alice's and Charlie1's measurements need satisfying a condition of "measurement matching." It can be seen from Eq. (34) that h and r_1 require satisfying one of the following conditions. (i) $hr_1=q/\sqrt{n}$. Then the measurement outcomes $|\Psi_h^+\rangle_{TA}|+_{C_1}\rangle$ and $|\Phi_h^-\rangle_{TA}|+_{C_1}\rangle$ can lead to suboptimal telecloning. (ii) $h/r_1=q/\sqrt{n}$. Then the measurement outcomes $|\Psi_h^+\rangle_{TA}|-_{C_1}\rangle$ and $|\Phi_h^-\rangle_{TA}|-_{C_1}\rangle$ can lead to suboptimal telecloning. (iii) $r_1/h=q/\sqrt{n}$. Then the measurement outcomes $|\Psi_h^-\rangle_{TA}|+_{C_1}\rangle$ and $|\Phi_h^+\rangle_{TA}|+_{C_1}\rangle$ can lead to suboptimal telecloning. (iv) $hr_1=\sqrt{n}/q$. Then the measurement outcomes $|\Psi_h^-\rangle_{TA}|-_{C_1}\rangle$ and $|\Phi_h^+\rangle_{TA}|-_{C_1}\rangle$ can lead to suboptimal telecloning. We notice that if $r_1=1$ or $h=1$, the conditions (i) and (ii) or (i) and (iii] are equivalent. For each of the two cases, the success probability is described by Eq. (25). We also notice that all eight pairs of measurement outcomes will lead to successful suboptimal telecloning iff $h=r_1=q/\sqrt{n}=1$. Then the state $|\Omega_1''\rangle$ reduces to $|\Omega_1'\rangle$.

Now a natural question arises: how to achieve the measurement matching. There may be two ways. (i) Alice and Charlie1 come to an agreement on the choice of h and r_1 in advance. (ii) One of them performs the measurement before the other one, and the latter one performs a corresponding measurement matching with that of the former one's. No matter which way, however, the classical communication between Alice and Charlie1 is required.

The conclusions above can be generalized to case where $m>1$. For the channel $|\Omega_m'\rangle$, the choice $h=r_1=1$ will lead to suboptimal telecloning with one probability provided all Charlies cooperate. As to the channel $|\Omega_m''\rangle$, the choice $\{q=\sqrt{n}h, r_1=r_2=\cdots=r_m=1\}$ will lead to suboptimal telecloning for the outcomes $\{|\Psi_h^+\rangle_{TA}\Pi_{k=1}^m|\pm_{C_k}\rangle, |\Phi_h^-\rangle_{TA}\Pi_{k=1}^m|\pm_{C_k}\rangle\}$, and the success probability can also reach P given in Eq. (25). Note that the above choice is not necessary condition. In other words, with some other choices of the parametric values satisfying the measurement matching the suboptimal telecloning can also be implemented with certain probabilities. But these probabilities are less than P.

V. GENERATION OF GHZ–W-TYPE ENTANGLED STATES

As discussed above, we need the GHZ–W-type entangled state $|\Omega_m\rangle$ acting as the quantum information channel to realize controlled phase-covariant telecloning. However, the generation of GHZ–W-type entangled states has not been reported yet. Here we propose a simple scheme for generating such states in the ion-trap system.

We consider that $m+n+1$ identical ions are confined in a linear Paul trap. Each of them has the ground state $|0\rangle$ and the excited state $|1\rangle$. We drive the former $m+2$ ions with two classical homogeneous lasers of frequencies $\omega_0+\nu+\delta$ and $\omega_0-\nu-\delta$. Here ω_0 is the frequency of the transition $|1\rangle\leftrightarrow|0\rangle$,

ν is the frequency of the center-of-mass mode of the collective motion of the ions, and δ is the detuning. Assuming $\delta \ll \nu$, then the excitation of the stretch modes is far off-resonant and is negligible. We consider the resolved sideband regime, where the vibrational frequency ν is much larger than other characteristic frequencies. In the Lamb-Dicke regime, i.e., $\eta\sqrt{N+1} \ll 1$, with η being the Lamb-Dicke parameter and N being the average phonon number of the center-of-mass mode, the Hamiltonian in the interaction picture is

$$H_I = i\eta\lambda e^{-i\varphi}\sum_{j=1}^{m+2} \sigma_j^+(a^\dagger e^{-i\delta t} + ae^{i\delta t}) + \text{H.c.}, \tag{36}$$

where $a^\dagger(a)$ denotes the creation (annihilation) operator for the vibrational mode, $\sigma^+=|1\rangle\langle 0|$ and $\sigma^-=|0\rangle\langle 1|$ are the spin flip operators, and λ and φ are the Rabi frequency and phase of the laser fields. In the case $\delta \gg \eta\lambda$, the effective Hamiltonian can be described by [49,50]

$$H_E = \gamma\sum_{j=1}^{m+2} (|1_j\rangle\langle 1_j| + |0_j\rangle\langle 0_j|) + 2\gamma \sum_{j,k=1,j\neq k}^{m+2} (\sigma_j^+\sigma_k^+ + \sigma_j^+\sigma_k^- + \text{H.c.}), \tag{37}$$

where $\gamma=(\lambda\eta)^2/\delta$. It can be seen that the Hamiltonian H_E is independent of the vibrational quantum number. This implies that the vibrational quantum number conserves during the interaction. Then if the vibrational mode is initially in the vacuum state $|0_v\rangle$, it will remain in this state.

We define an atomic basis

$$|\pm_j\rangle = \frac{1}{\sqrt{2}}(|0_j\rangle \pm |1_j\rangle). \tag{38}$$

Then H_E can be rewritten as

$$H_E = \gamma\left[\sum_{j=1}^{m+2} (|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j|)\right]^2. \tag{39}$$

It can be easily verified that the Hamiltonian H_E has the eigenvalue $(m+1-2l\pm 1)^2\gamma$ for the eigenstate $|\pm_1\rangle[|-\rangle^{\otimes l}|+\rangle^{\otimes(m+1-l)}]_{2,3,\ldots,m+2}$ $(l \leq m+1)$. Here $[|-\rangle^{\otimes l}|+\rangle^{\otimes(m+1-l)}]$ denotes that l ions are in the state $|-\rangle$ and $(m+1-l)$ ions are in the state $|+\rangle$.

Assume that these ions are initially in the state

$$|\Omega(0)\rangle = |1_1\rangle \prod_{j=2}^{m+n+1} |0_j\rangle = \frac{1}{\sqrt{2^{m+2}}}\left\{|+_1\rangle\left[\sum_{l=0}^{m+1} C_{m+1}^l|-\rangle^{\otimes l}|+\rangle^{\otimes(m+1-l)}\right]_{2,3,\ldots,m+2} - |-_1\rangle\left[\sum_{l=0}^{m+1} C_{m+1}^l|-\rangle^{\otimes l}|+\rangle^{\otimes(m+1-l)}\right]_{2,3,\ldots,m+2}\right\}\prod_{k=m+3}^{m+n+1}|0_k\rangle, \tag{40}$$

where C_{m+1}^l is combinational coefficient and defined as $(m+1)!/[l!(m+1-l)!]$. After an interaction time τ, the ionic state evolves into

$$|\Omega(\tau)\rangle = \frac{1}{\sqrt{2^{m+2}}}\left\{|+_1\rangle\left[\sum_{l=0}^{m+1} \exp[-i(m+2-2l)^2\gamma\tau]C_{m+1}^l|-\rangle^{\otimes l}|+\rangle^{\otimes(m+1-l)}\right]_{2,3,\ldots,m+2} - |-_1\rangle \times\left[\sum_{l=0}^{m+1} \exp[-i(m-2l)^2\gamma\tau]C_{m+1}^l|-\rangle^{\otimes l}|+\rangle^{\otimes(m+1-l)}\right]_{2,3,\ldots,m+2}\right\}\prod_{k=m+3}^{m+n+1}|0_k\rangle. \tag{41}$$

Choosing $\tau=\pi/(8\gamma)$, we obtain

$$|\tilde{\Omega}\rangle = \frac{1}{\sqrt{2}}\left[|1_1\rangle\prod_{j=2}^{m+2}|0_j\rangle + f(m)|0_1\rangle\prod_{j=2}^{m+2}|1_j\rangle\right]\prod_{k=m+3}^{m+n+1}|0_k\rangle, \tag{42}$$

where $f(m)$ is equal to $i(-1)^{m/2+1}$ (m is even number) or $i(-1)^{(m+3)/2}$ (m is odd number), and a common phase factor $\exp(-i\pi/4)$ is discarded.

Then we simultaneously excite each of the latter n atoms using, respectively, a red sideband laser of frequency $\omega_0-\nu$. The Hamiltonian of the system is (let $\hbar=1$)

$$H = H_0 + V,$$

$$H_0 = \nu a^\dagger a + \frac{1}{2}\omega_0\sum_{k=m+2}^{m+n+1}\sigma_k^z,$$

$$V = \sum_{k=m+2}^{m+n+1} \exp(-i\varphi_{k-m-1})\lambda_{k-m-1}\sigma_k^+ \exp\{i[\eta(a^\dagger + a) - (\omega_0 - \nu)]\} + \text{H.c.}, \tag{43}$$

where $\sigma^z = |0\rangle\langle 0| - |1\rangle\langle 1|$ is the usual Pauli matrix and λ_s and φ_s $(s=1,2,\ldots,n)$ are the Rabi frequency and phase of corresponding laser field, respectively. We here assume that the phases of laser fields are equal to $\pi/2$. In the interaction picture, the Hamiltonian reads

$$H_I' = -i\exp\left(-\frac{\eta^2}{2}\right)\sum_{k=m+2}^{m+n+1}\lambda_{k-m-1}\sigma_k^+\sum_{l=0}^{\infty}\frac{(i\eta)^{2l+1}}{l!\,(l+1)!}(a^\dagger)^l a^{l+1} + \text{H.c.}, \tag{44}$$

In the Lamb-Dicke regime, the interaction Hamiltonian can be approximated by

$$H_E' = \sum_{k=m+2}^{m+n+1}\gamma_{k-m-1}\sigma_k^+ a + \text{H.c}, \tag{45}$$

where $\gamma_s = \eta\lambda_s$ $(s=1,2,\ldots,n)$. Since $[H_E', \hat{M}]=0$, where $\hat{M} = a^\dagger a + \sum_{k=m+2}^{m+n+1}\sigma_k^+\sigma_k^-$ is the excitation number operator, the dynamics is separable into subspaces having a prescribed eigenvalue M of $\hat{M}$. In the subspace with $M=1$, there are the following $n+1$ basis states:

$$|b^{(1)}\rangle = |1_{m+2}\rangle\sum_{k=m+3}^{m+n+1}|0_k\rangle|0_v\rangle,$$

$$|b^{(2)}\rangle = |1_{m+3}\rangle\sum_{k=m+2,k\neq m+3}^{m+n+1}|0_k\rangle|0_v\rangle,$$

$$\vdots$$

$$|b^{(n)}\rangle = |1_{m+n+1}\rangle\sum_{k=m+2}^{m+n}|0_k\rangle|0_v\rangle,$$

$$|b^{(v)}\rangle = \sum_{k=m+2}^{m+n+1}|0_k\rangle|1_v\rangle. \tag{46}$$

After an interaction time τ', the state of the total system is

$$|\Omega(\tau')\rangle = \frac{1}{\sqrt{2}}|1_1\rangle\left(\prod_{j=2}^{m+n+1}|0_j\rangle\right)|0_v\rangle + \frac{f(m)}{\sqrt{2}}|0_1\rangle\left(\prod_{j=2}^{m+1}|1_j\rangle\right) \times\left\{[1-2\gamma_1^2\beta(\tau')]|b^{(1)}\rangle - 2\gamma_1\sum_{s=2}^{n}\gamma_s\beta(\tau')|b^{(s)}\rangle - i\frac{\gamma_1\sin(\Gamma\tau')}{\Gamma}|b^{(v)}\rangle\right\}, \tag{47}$$

where $\Gamma^2 = \sum_{s=1}^{n}\gamma_s^2$ and $\beta(\tau') = \sin^2(\Gamma\tau'/2)/\Gamma^2$. By choosing $\tau' = p\pi/\Gamma$ with p being odd number, we obtain the GHZ–W-type entangled state $|\Omega_m\rangle$ of Eq. (26), with $x_0 = 1/\sqrt{2}$, $x_1 = (1-2\gamma_1^2/\Gamma^2)f(m)/\sqrt{2}$, and $x_j = -\sqrt{2}\gamma_1\gamma_j f(m)/\Gamma^2$ $(j=2,3,\ldots,n)$. Setting $\gamma_2=\gamma_3=\cdots=\gamma_n=\gamma_1/(\sqrt{n}\pm 1)$, we can obtain the state $|\Omega_m'\rangle$ [see Eq. (30)]. All the facilities in the above scheme are well within the present ion-trap techniques [46,51,52].

VI. CONCLUDING REMARKS

In summary, we have studied the HETC of equatorial qubits, in which the symmetric and asymmetric clonings and anticlonings can be simultaneously achieved by teleportation. This may be very interesting from the point of view of quantum information distribution and have potential applications in quantum information depositing or encoding. Imagining that someone possesses the secret information carried by a quantum system about an important thing, but he or she has not enough ability to prevent the potential bad men from thieving or robbing of the information. Then he or she can divide the information into many parts and send them to many distant associates by telecloning protocol. In order to know about the important thing, all parts of information need being gathered together, i.e., a recovering process is required [34]. However, if any one of the associates betrays the collectivity, the content of the thing can only be approximately estimated by the other parts of information. Of course, the more information that is lost, the more difficult it is to estimate the thing. Thus he or she should distribute different amount of information to different associates in view of the reliability of them.

In Sec. II, we discussed the case where the quantum information channel is the well-known W-type entangled states. Interesting equalities and inequalities about the fidelities of clones or anticlones were obtained. We used the global fidelity and average-single-qubit fidelity to estimate the collective copying quality. The two criteria lead to different results. In order to obtain the maximal global fidelity we need using a fully symmetric W state as the quantum channel, while to maximize the average-single-qubit fidelity a special configuration of asymmetric W state $|\mathrm{W}_{n+1}'\rangle$ [see Eq. (20)] is required. But both the two criteria indicate that the fully symmetric telecloning is better than the other cases in the point of view of collective copying quality. The state $|\mathrm{W}_{n+1}'\rangle$ has one ebit of entanglement between the subsystem of Alice (qubit A) and the subsystem of Bobs (qubits B_j, $j=1,2,\ldots,n$), with which the telecloning is suboptimal. Our scheme is more close to practice than previous telecloning schemes because the well-known W-type entangled states have been successfully prepared in several physical systems [44–46]. In Sec. III, we demonstrated that when Alice and Bobs share less than one ebit of entanglement the suboptimal telecloning can also be implemented with a certain probability. In Sec. IV, we introduced the controlled HETC protocol with recently proposed GHZ–W-type entangled states [47], in which the achievement of phase-covariant telecloning is conditioned on the cooperation of all the supervisors. It has been shown that in order to realize the desired telecloning Alice's (the sender) and Charlies' (the supervisors) measurements need to satisfy the condition of measurement matching. This idea may open a perspective for the applications of such interesting type of entangled states. We anticipate that the GHZ–W-type entangled states, which have the properties of both GHZ- and W-type entangled states, may have other applications in quantum information science and the fundamental tests of quantum mechanics. In Sec. V, we presented a scheme for generating the GHZ–W-type entangled states with the ion-trap setup. The method can also be generalized to other systems.

As mentioned above, there have been many research works about telecloning (see e.g., [39–42]). Comparing with them, our schemes have many advantages and features. (1) The receivers can freely choose to getting the clones or anticlones, and one-to-many symmetric and asymmetric clonings and anticlonings can be simultaneously achieved. (2) In the schemes of Secs. II and III, the quantum channels are the well-known W-type of entangled states which have been experimentally realized in different systems [44–46]. (3) In the scheme of Sec. III, the quantum channel is a partially entangled state. This case has not been discussed. (4) The controlled telecloning introduced in Sec. IV is an idea in which the achievement of telecloning between the sender and the receivers is conditioned on the collaboration of all the supervisors.

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