Lamb shift of a uniformly accelerated hydrogen atom in the presence of a conducting plate

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We investigate the effects of acceleration on the energy-level shifts of a hydrogen atom interacting with the electromagnetic field and in the presence of an infinite perfectly conducting plate. We consider the contributions of vacuum fluctuations and of the radiation reaction field to the Lamb shift, and we discuss their dependence from the acceleration of the atom. We show that, because of the presence of the boundary, both vacuum field fluctuations and radiation reaction field contributions are affected by atomic acceleration. In particular, the effect of the vacuum field fluctuations on the energy-level shifts is not equivalent to that of a thermal field. We also discuss the dependence of the Lamb shift from the orientations of the atomic dipole. We show that, for specific orientations of the atomic dipole, the atomic level shifts contain an extra contribution, as compared with the case of an atom at rest, suggesting the possibility to detect the effects of the acceleration for atoms with anisotropic polarizability.

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I. INTRODUCTION

One of the most remarkable consequences of quantum electrodynamics is the existence of zero-point fluctuations of electric and magnetic fields. These fluctuations are at the origin of many observable effects, such as the Lamb shift, i.e., the shift of the energy levels of an atom interacting with the radiation field, the Casimir effect, and the Casimir-Polder forces between neutral atoms (see [1] for a review). The Casimir effect [2] refers to the force between macroscopic objects, such as conducting or dielectric plates in vacuum; also, the Casimir-Polder forces [3] are long-range interactions between microscopic objects, such as atoms or molecules or between an atom and a conducting plate. These effects are of a purely quantum origin and they can be interpreted as a manifestation of quantum fluctuations of vacuum [4]. The Casimir effect, although it is a tiny force, has been recently measured with remarkable precision for various geometrical configurations of the objects [5]. Also the atom- or condensate-plate Casimir-Polder forces have been recently measured [6]. The increasing interest in this subject is also connected to possible application in nanotechnologies. For example, nanoelectromechanical and microelectromechanical systems can be activated by Casimir forces [7,8]. An intriguing feature of these effects is that they depend on the presence of external fields or boundaries [9]. For example, it is well known that the atomic level shifts change if the atom is located inside a cavity [1,10]. Also, the Casimir-Polder interaction between atoms changes qualitatively and quantitatively if the atoms are in the presence of a boundary or immersed in a magnetodielectric medium [11,12]. Generally speaking the presence of a boundary affects the structure of the modes of the vacuum and leads to a change in the energy-level shifts of an atom or of the Casimir-Polder potential of two atoms [13].

Recently, it has been also investigated how a dynamical change in the boundary conditions affects the field vacuum fluctuations, giving rise to observable phenomena such as the dynamical Casimir effect [14]. This effect consists of the emission of electromagnetic radiation when the neutral objects are put in an accelerated motion or when their dielectric constant is subjected to a change. Similar effects have been also studied in the case of dynamical Casimir-Polder forces [15,16]. Although less known than its static counterpart, the dynamical Casimir effect has attracted much attention, in particular in connection with the possibility to detect real photons from vacuum. Also, the dynamical Casimir effect is closely related to the Unruh effect according to which an atom (or a charge) that accelerates in a vacuum with uniform acceleration perceives the vacuum fluctuations as a thermal field with a temperature proportional to the acceleration a, $T_U=\hbar a/(2\pi c k_B)$, where k_B is the Boltzmann constant [17–21]. Unfortunately, this is a very small effect: in order to have Unruh radiation at 1 K, the atomic acceleration has to be on the order of 10^{22} cm/s^2. Actually, the question of the appearance of the vacuum in an accelerated frame is a widely controversial problem [22,23]. All these phenomena underline the highly nontrivial nature of quantum vacuum, whose dynamical properties critically depend on the topological properties and/or geometry of the system under scrutiny.

In this paper, we investigate the effects of acceleration on energy-level shifts of a hydrogen atom interacting with the electromagnetic field in the presence of an infinite perfectly conducting plate. Recently, the energy-level shifts of an accelerated two-level system interacting with a scalar field have been calculated [24,25]. The interest on this issue was motivated by the possibility of detecting the effects of the atomic acceleration on the energy-level shifts of the atom accelerated. Unfortunately, the results obtained show that the accelerations required to obtain an appreciable effect are very large to be experimentally realized ($a \sim 10^{25}$ cm/s^2). On the other hand, the model considered consists of a two-level system interacting with a scalar field. A more realistic calculation of the effect of the atomic acceleration on the Lamb shift would require considering a multilevel atom in-

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teracting with the electromagnetic field. Recently Passante [26] investigated the energy-level shifts of a hydrogen atom moving with uniform acceleration and interacting with the electromagnetic field in the multipolar coupling scheme. The main conclusion of his paper is that the effect of the vacuum fluctuations on the energy-level shifts of the accelerated atom is not totally equivalent to that of a thermal field with the Unruh temperature $T_U=\hbar a/(2\pi c k_B)$. An estimation on the order of magnitude of the *nonthermal* correction to the energy-level shifts shows that it is comparable with the thermal one and it should be in principle observable.

In this paper we shall extend this investigation to the case when a boundary, such as an infinite perfectly reflecting plate, is present. As it is well known the presence of a conducting plate changes the structure of field modes and it is interesting to investigate its effect on energy-level shifts of a uniformly accelerated atom. Our calculation is carried out using a generalization of the formalism of Dalibard *et al.* [27,28], according to which it is possible to separate the contributions of vacuum field fluctuations and of the radiation reaction field to the energy-level shifts of the atom.

In Sec. II we shall introduce the model and the general formalism developed in [27]. In Sec. III we shall generalize this approach to the case of an accelerated multilevel atom interacting with the electromagnetic field in the presence of a conducting plate and we shall investigate the effects of acceleration on the Lamb shift of the accelerated atom.

II. VACUUM FLUCTUATIONS AND RADIATION REACTION CONTRIBUTIONS TO THE RADIATIVE ENERGY SHIFT

Let us consider a uniformly accelerated atom interacting with the electromagnetic field in the vacuum state and in the presence of an infinite perfectly conducting plate. The atom accelerates in the x direction, parallel to the plate. The Hamiltonian that describes the atom-field interacting system in the instantaneous inertial frame of the atom and in the multipolar coupling scheme is ($\hbar=c=1$) [26,29]

$$H(\tau) = H_A(\tau) + H_F(\tau) + H_{AF}(\tau), \tag{1}$$

where

$$H_A(\tau) = \sum_n \omega_n \sigma_{nn}(\tau), \tag{2}$$

$$H_F(\tau) = \sum_{\mathbf{k}j} \omega_k a^\dagger_{\mathbf{k}j} a_{\mathbf{k}j} \frac{dt}{d\tau}, \tag{3}$$

$$H_{AF}(\tau) = -e\mathbf{r}(\tau)\cdot\mathbf{E}(x(\tau)) = -e\sum_{mn} \mathbf{r}_{mn}\cdot\mathbf{E}(x(\tau))\sigma_{mn}(\tau), \tag{4}$$

where $\sigma_{\ell m}=|\ell\rangle\langle m|$, $|n\rangle$ being a complete set of atomic states with energy ω_n and $\mu=e\mathbf{r}=e\Sigma_{\ell m}\mu_{\ell m}\sigma_{\ell m}$ is the atomic electric dipole moment. Also, $x=(t,\mathbf{x})$ is the space-time coordinate of the atom. Finally,

$$\mathbf{E}(\mathbf{x}) = i\sum_{\mathbf{k}j}\sqrt{\frac{2\pi\omega_k}{V}}\mathbf{f}(\mathbf{k}j,\mathbf{x})(a_{\mathbf{k}j}-a^\dagger_{\mathbf{k}j}), \tag{5}$$

where $\mathbf{f}(\mathbf{k}j,\mathbf{x})$ are the appropriate mode functions for the field operators taking into account the presence of the conducting plate at $z_0=0$. The mode functions satisfy the normalization condition

$$\frac{1}{V}\int d^3\mathbf{x} f(\mathbf{k}j,\mathbf{x}) f(\mathbf{k}'j',\mathbf{x}) = \delta_{\mathbf{k}\mathbf{k}'}\delta_{jj'}. \tag{6}$$

Hamiltonian $H_F(\tau)$ in Eq. (3) governs the evolution of the field in terms of the proper time τ in the instantaneous inertial frame of the atom. It reduces to the usual free-field Hamiltonian in the simple case of an atom at rest, where $dt/d\tau=1$.

We are interested in evaluating the contributions of vacuum field fluctuations and the radiation reaction field to the energy-level shifts of the accelerated atom in the presence of the reflecting plate. As discussed in [24,26,27], these two contributions can be obtained from an effective Hamiltonian, $H^{eff}(\tau)$, which can be expressed as a sum of two terms,

$$H^{eff}_{vf}(\tau) = i\frac{e^2}{2}\int_{\tau_0}^{\tau} d\tau'\, C^F_{\ell m}(x(\tau),x(\tau'))[\mathbf{r}_\ell(\tau'),\mathbf{r}_m(\tau)], \tag{7}$$

$$H^{eff}_{rr}(\tau) = -i\frac{e^2}{2}\int_{\tau_0}^{\tau} d\tau'\, \chi^F_{\ell m}(x(\tau),x(\tau'))\{\mathbf{r}_\ell(\tau'),\mathbf{r}_m(\tau)\}, \tag{8}$$

where we defined the symmetrical correlation function and the linear susceptibility of the field in the vacuum state,

$$C^F_{\ell m}(x(\tau),x(\tau')) = \tfrac{1}{2}\langle 0|\{E_\ell(x(\tau)),E_m(x(\tau'))\}|0\rangle, \tag{9}$$

$$\chi^F_{\ell m}(x(\tau),x(\tau')) = \tfrac{1}{2}\langle 0|[E_\ell(x(\tau)),E_m(x(\tau'))]|0\rangle. \tag{10}$$

[,] and {,} denote commutator and anticommutator, respectively. The expectation values of $H^{eff}_{vf}(\tau)$ and $H^{eff}_{rr}(\tau)$ on a generic atomic state $|a\rangle$ give the vacuum fluctuations and radiation reaction field contributions to the energy shifts of the atomic level a at the second order in the coupling constant,

$$(\delta E_a)_{vf} = -ie^2\int_{\tau_0}^{\tau} d\tau'\, C^F_{\ell m}(x(\tau),x(\tau'))(\chi^A_{\ell m})_a(\tau,\tau'), \tag{11}$$

$$(\delta E_a)_{rr} = -ie^2\int_{\tau_0}^{\tau} d\tau'\, \chi^F_{\ell m}(x(\tau),x(\tau'))(C^A_{\ell m})_a(\tau,\tau'), \tag{12}$$

where we introduced the atomic statistical functions,

$$\begin{aligned}(C^A_{\ell m})_a(\tau,\tau') &= \frac{1}{2}\langle a|[\mathbf{r}_\ell(\tau'),\mathbf{r}_m(\tau)]|a\rangle\\ &= \frac{1}{2}\sum_b [\langle a|\mathbf{r}_\ell(0)|b\rangle\langle b|\mathbf{r}_m(0)|b\rangle e^{i\omega_{ab}(\tau-\tau')}\\ &\quad + \langle a|\mathbf{r}_m(0)|b\rangle\langle b|\mathbf{r}_\ell(0)|b\rangle e^{-i\omega_{ab}(\tau-\tau')}],\end{aligned} \tag{13}$$

$$(\chi^A_{\ell m})_a(\tau,\tau') = \frac{1}{2}\langle a|[\mathbf{r}_\ell(\tau'),\mathbf{r}_m(\tau)]|a\rangle$$
$$= \frac{1}{2}\sum_b [\langle a|\mathbf{r}_\ell(0)|b\rangle\langle b|\mathbf{r}_m(0)|b\rangle e^{i\omega_{ab}(\tau-\tau')}$$
$$- \langle a|\mathbf{r}_m(0)|b\rangle\langle b|\mathbf{r}_\ell(0)|b\rangle e^{-i\omega_{ab}(\tau-\tau')}]. \quad (14)$$

In Sec. III we shall apply this formalism to investigate the effects of acceleration on the energy-level shift of the accelerated atom in the presence of the reflecting plate.

III. ENERGY LEVEL SHIFTS OF AN ACCELERATED HYDROGEN ATOM NEAR A CONDUCTING PLATE

We now calculate the energy-level shifts of an atom accelerating in a parallel direction to an infinite perfectly reflecting plate. The plate is located at $z=0$ and we suppose that the atom accelerates along the x direction. Our approach generalizes the method developed by Takagi [19] to the case when boundary conditions are present. In the laboratory frame, the trajectory of the atom is described, as a function of the proper time τ, by the equations

$$t(\tau) = \frac{1}{a}\sinh(a\tau), \quad x(\tau) = \frac{1}{a}\cosh(a\tau),$$
$$y(\tau) = 0, \quad z(\tau) = z_0, \quad (15)$$

where a is the proper acceleration.

In order to obtain the statistical functions of the radiation field [Eqs. (9) and (10)], we first calculate the Wightmann function for the electromagnetic field

$$G_{\alpha\beta\gamma\delta}(x,x') = \langle 0_M|F_{\alpha\beta}(x)F_{\gamma\delta}(x')|0_M\rangle,$$

where $F_{\alpha\beta}(x)=\partial_\alpha A_\beta-\partial_\beta A_\alpha$ is the electromagnetic tensor and $|0_M\rangle$ is the Minkowski vacuum. This function can be obtained from the two-point correlation function of the electromagnetic four-vector potential $A_\alpha(x)$ taking into account the condition that the transverse components of electric field vanish on the conducting plate. We have [we introduce $x \equiv x(\tau)$ and $x' \equiv x(\tau')$]

$$\langle 0_M|A_\alpha(x)A_\beta(x')|0_M\rangle$$
$$= \langle 0_M|A_\alpha(x)A_\beta(x')|0_M\rangle_o + \langle 0_M|A_\alpha(x)A_\beta(x')|0_M\rangle_{b.}$$
$$= \eta_{\alpha\beta}\mathcal{G}^{(o)}(x,x') + (\eta_{\alpha\beta}-2n_\alpha n_\beta)\mathcal{G}^{(b.)}(x,x'), \quad (16)$$

$\eta_{\alpha\beta}=\mathrm{diag}(-1,1,1,1)$, and $n_\alpha=(0,0,0,1)=(0,\mathbf{n})$, where $\mathbf{n}$ is the unitary normal vector. Also, $\mathcal{G}(x,x') = \langle 0_M|\phi(x)\phi(x')|0_M\rangle$ is the Wightman function of the scalar field. In the presence of a boundary, it consists of a sum of two terms—an empty-space contribution $\mathcal{G}^{(o)}(x,x')$ and a term $\mathcal{G}^{(b.)}(x,x')$—which is the correction induced by the presence of the plate with Dirichlet boundary conditions [25,30],

$$\mathcal{G}^{(o)}(x,x') = \frac{1}{4\pi^2}\frac{1}{(\Delta(\mathbf{x}))^2-(\Delta t-i\epsilon)^2}, \quad (17)$$

$$\mathcal{G}^{(b.)}(x,x') = -\frac{1}{4\pi^2}\frac{1}{(\overline{\Delta(\mathbf{x})})^2-(\Delta t-i\epsilon)^2}, \quad (18)$$

where $\epsilon\to 0^+$, $\Delta(\mathbf{x})=|\mathbf{x}(\tau)-\mathbf{x}(\tau')|$ [the difference between atomic coordinates $\mathbf{x}(\tau)$ taken at two different proper times], and $\overline{\Delta(\mathbf{x})}=|\mathbf{x}(\tau)-\sigma\mathbf{x}(\tau')|$ with

$$\sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (19)$$

Finally, $\Delta t=t(\tau)-t(\tau')$. As it has been shown in [30], the Wightman function obtained satisfies the boundary condition on the conducting plate.

We now focus our attention only on the boundary-dependent term, which is relevant for our calculation of Lamb shift of the accelerated atom in the presence of a reflecting plate. The Wightman function for the fields in the laboratory frame follows from Eq. (16). We get

$$G^{(b.)}_{\alpha\beta\gamma\delta}(x,x') = \langle 0_M|F_{\alpha\beta}(x)F_{\gamma\delta}(x')|0_M\rangle_{b.}$$
$$= [(\eta_{\beta\delta}\partial_\alpha\partial'_\gamma - \eta_{\beta\gamma}\partial_\alpha\partial'_\delta - \eta_{\alpha\delta}\partial_\beta\partial'_\gamma + \eta_{\alpha\gamma}\partial_\beta\partial'_\delta)$$
$$- 2(n_\beta n_\delta\partial_\alpha\partial'_\gamma - n_\beta n_\gamma\partial_\alpha\partial'_\delta - n_\alpha n_\delta\partial_\beta\partial'_\gamma$$
$$+ n_\alpha n_\gamma\partial_\beta\partial'_\delta)]G^{(b.)}(x,x'), \quad (20)$$

where $\partial' \equiv \partial/\partial x'$. From this expression we may easily obtain the two-point correlation function for the electric field,

$$G^{(b.)}_{0\ell 0m}(x,x') = \langle 0_M|E_\ell(x)E_m(x')|0_M\rangle_{b.}$$
$$= [(\delta_{\ell m}-2n_\ell n_m)\partial_0\partial'_0 - \partial_\ell\partial'_m]G^{(b.)}(x,x'). \quad (21)$$

We are now interested in calculating the two-point correlation function of the electric field in the reference frame comoving with the atom (that is the Rindler function of the electric field),

$$g_{\ell m}(\tau,\tau') = \langle 0_M|\hat{E}_\ell(\tau)\hat{E}_m(\tau')|0_M\rangle, \quad (22)$$

where we supposed the atom at the origin of the accelerating frame and where $\hat{E}_i(\tau)$ indicates a generic component of electric field operator at the point of the accelerating atom in the instantaneously inertial frame comoving with atom. The electric field $\hat{\mathbf{E}}(\tau)$ observed at proper time τ by the atom accelerating along the x direction (at a fixed distance z_0 from the plate) is related by a Lorentz transformation to the electromagnetic field in the laboratory frame by the relation

$$\hat{E}_i = \hat{F}_{i0} = (\Lambda_\tau)_i^{\alpha}(\Lambda_\tau)_0^{\beta}F_{\alpha\beta}, \quad (23)$$

where

$$\Lambda_\tau = \begin{pmatrix} \cosh(a\tau) & -\sinh(a\tau) & 0 & 0 \\ -\sinh(a\tau) & \cosh(a\tau) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{24}$$

is the boost relating the laboratory frame to the atomic proper reference frame. Using Lorentz transformation (23) in Eq. (20), after some algebra we obtain

$$\begin{aligned} g_{\ell m}^{(b.)}(\tau-\tau') &= \langle 0_M|\hat{E}_\ell(\tau)\hat{E}_m(\tau')|0_M\rangle_{b.} \\ &= \frac{a^4}{16\pi^2}\left[\delta_{\ell m}\left\{\sinh^2\left[\frac{a}{2}(\tau-\tau'-i\epsilon)\right]+a^2z_0^2\right\}-2n_\ell n_m\sinh^2\left[\frac{a}{2}(\tau-\tau'-i\epsilon)\right]\right]\frac{1}{\left\{\sinh^2\left[\frac{a}{2}(\tau-\tau'-i\epsilon)\right]+a^2z_0^2\right\}^3} \\ &\quad -\frac{a^4}{16\pi^2}[2a^2z_0^2(\delta_{\ell m}-k_\ell k_m)+2az_0(n_\ell k_m+k_\ell n_m)]\frac{\sinh^2\left[\frac{a}{2}(\tau-\tau'-i\epsilon)\right]}{\left\{\sinh^2\left[\frac{a}{2}(\tau-\tau'-i\epsilon)\right]+a^2z_0^2\right\}^3}, \end{aligned} \tag{25}$$

where $\mathbf{k}=(1,0,0)$, $\epsilon\to 0^+$, and z_0 is the atom-plate distance. From the above expression follows that the only nonzero components of $g_{\ell m}^{(b.)}(\tau,\tau')$ are the xx, yy, zz, and xz components. In particular, we have

$$g_{ii}^{(b.)}(\tau,\tau') \neq g_{jj}^{(b.)}(\tau,\tau') \quad \text{for any} \quad i\neq j\{i,j=x,y,z\}, \tag{26}$$

which shows that the two parallel directions to the plate, that is, the x direction (along the acceleration) and the y direction, are not equivalent since $g_{xx}^{(b.)}$ is different from $g_{yy}^{(b.)}$. Therefore, in contrast with the case of unbounded space where the correlation function is isotropic, in the present case the function $g_{\ell m}^{(b.)}(\tau-\tau')$ is not isotropic and displays nondiagonal components. This anisotropy is due to a *cooperative* effect of boundary and atomic acceleration.

We now calculate the correction induced by the presence of the plate on the atomic energy-level shifts of the accelerated atom. In order to do that, we express the correlation function $g_{lm}(\tau,\tau')$ in terms of frequency integration. The calculation is sketched in Appendix. From Eqs. (A8) and (A17) we have

$$\begin{aligned}(C_{\ell m}^F(\tau,\tau'))^{(b.)} &= -\frac{1}{8\pi^2}\frac{1}{(2z_0)^3}\int_0^\infty d\omega K_{\ell m}(\omega;z_0,a)\coth\left(\frac{\pi\omega}{a}\right) \\ &\quad \times(e^{i\omega u}+e^{-i\omega u})\end{aligned} \tag{27}$$

and

$$(\chi_{\ell m}^F(\tau,\tau'))^{(b.)} = \frac{1}{4\pi^2}\frac{1}{(2z_0)^3}\int_0^\infty d\omega K_{\ell m}(\omega;z_0,a)(e^{i\omega u}-e^{-i\omega u}), \tag{28}$$

where $u=\tau-\tau'$ and

$$\begin{aligned} K_{\ell m}(\omega;z_0,a) &= \frac{\sigma_{\ell n}}{(1+a^2z_0^2)^{1/2}}\left\{(\delta_{nm}-n_n n_m)\frac{(2\omega z_0)^2}{(1+a^2z_0^2)}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\right. \\ &\quad +(\delta_{nm}-3n_n n_m)\left[\frac{2\omega z_0}{(1+a^2z_0^2)^{3/2}}\cos\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)-\frac{1}{(1+a^2z_0^2)^2}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\right] \\ &\quad +a^2z_0^2\left[(\delta_{nm}+3\sigma_{nm})\frac{2\omega z_0}{(1+a^2z_0^2)^{3/2}}\cos\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)+(\delta_{nm}-2\sigma_{nm})\frac{2}{(1+a^2z_0^2)^2}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\right] \\ &\quad \left.-\sigma_{nm}a^4z_0^4\frac{4}{(1+a^2z_0^2)^2}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\right\}+az_0[(\delta_{lm}-k_lk_m)az_0+n_lk_m+k_ln_m] \\ &\quad \times\left\{\frac{(2\omega z_0)^2}{(1+a^2z_0^2)}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)-\frac{2\omega z_0}{(1+a^2z_0^2)^{3/2}}\cos\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)+\frac{1}{(1+a^2z_0^2)^2}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\right\} \\ &\quad +a^3z_0^3[(\delta_{\ell m}-k_\ell k_m)az_0+n_\ell k_m+k_\ell n_m]\left\{\frac{4\omega z_0}{(1+a^2z_0^2)^{3/2}}\cos\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)+\frac{4}{(1+a^2z_0^2)^2}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\right\}. \end{aligned} \tag{29}$$

Let us now calculate the vacuum fluctuations and radiation reaction contributions to the energy-level shifts of the accelerated atom. Putting Eqs. (13), (14), (27), and (28) in Eqs. (11) and (12), we obtain, after some algebraic manipulation,

$$(\delta E_a)_{vf}^{(b.)} = -\frac{1}{8\pi^2}\sum_b (\mu_\ell^{ab}\mu_m^{ba})\frac{1}{(2z_0)^3}\mathrm{P}\int_0^\infty d\omega K_{\ell m}(\omega;z_0,a) \times\coth\left(\frac{\pi\omega}{a}\right)\left(\frac{1}{\omega+\omega_{ab}}-\frac{1}{\omega-\omega_{ab}}\right) \tag{30}$$

and

$$(\delta E_a)_{rr}^{(b.)} = \frac{1}{8\pi^2}\sum_b (\mu_\ell^{ab}\mu_m^{ba})\frac{1}{(2z_0)^3}\mathrm{P}\int_0^\infty d\omega K_{\ell m}(\omega;z_0,a) \times\left(\frac{1}{\omega+\omega_{ab}}+\frac{1}{\omega-\omega_{ab}}\right). \tag{31}$$

Equations (30) and (31) reveal interesting features. Let us first discuss the contribution of vacuum field fluctuations. Expression (30) clearly shows that this contribution contains not only a *thermal* correction due to the factor $\coth(\pi\omega/a)$ but also an extra term proportional to the function $K_{\ell m}(\omega;z_0,a)$, which is induced by the presence of the conducting plate. This term does not have the form of a thermal correction. The presence of nonthermal corrections in the vacuum fluctuation part of the shift is not a new one. In fact, as discussed in [26], the equivalence between uniform acceleration and thermal field is lost even in the unbounded space when considering the case of an accelerated atom interacting with the electromagnetic field (in multipolar coupling scheme). In that case

$$(\delta E_a)_{vf}^{(o)} = \frac{1}{8\pi^2}\sum_b (\mu_\ell^{ab}\mu_m^{ba})\mathrm{P}\int_0^\infty d\omega\omega^3\left(1+\frac{a^2}{\omega^2}\right) \times\coth\left(\frac{\pi\omega}{a}\right)\left(\frac{1}{\omega+\omega_{ab}}-\frac{1}{\omega-\omega_{ab}}\right), \tag{32}$$

which clearly shows the presence of a nonthermal term proportional to a^2. We also note that the function $K_{\ell m}(\omega;z_0,a)$ modulates the energy shift as a function of the atom-plate distance z_0 and of the atomic acceleration a. This suggests that for some values of z_0 and a, the nonthermal contribution present in Eq. (30) can be enhanced or weakened. In particular, we may envisage situations in which the nonthermal correction present in the vacuum fluctuation part of the shift in the unbounded space [Eq. (32)] can be canceled by that induced by the boundary for specific values of z_0 and a. For example, for a dipole oriented along a specific direction (x, y, or z), it can be found some value of atom-plate distance z_0 for acceleration on the order of 10^{25} cm/s^2, such that $a^2\omega_{ba}-1/(2z_0)^3 K_{\ell m}(\omega_{ba},z_0,a)=0$. We have evaluated some value of atom-plate distance for the typical transition frequency of a hydrogen atom, $\omega_{ba}\sim 10^{15}$ s^{-1} and $a\sim 10^{25}$ cm/s^2, obtaining $z_0\sim 0.01$ μm. This means that for some specific trajectory of the atom, nonthermal effects could vanish.

Let us now discuss the contribution of radiation reaction field. Expression (31) reveals that this contribution is modified by the atomic acceleration. This behavior is expected; in fact, this term is essentially the field radiated by the atom. When a boundary is present, this field can act back on the atom after a reflection on the conducting plate. Since the atom accelerates, in the time-interval between the emission and the subsequent absorption of the reflected field, the atom has moved from its position for a stretch depending on its acceleration [see Eq. (15)]. This gives rise to a dependence of the radiation reaction part of the shift from the atomic acceleration. This is obviously in contrast with the case of unbounded space, where the contribution of radiation reaction does not depend on atomic acceleration, being

$$(\delta E_a)_{rr}^{(o)} = -\frac{1}{8\pi^2}\sum_b (\mu_\ell^{ab}\mu_m^{ba})\mathrm{P}\int_0^\infty d\omega\omega^3 \times\left(\frac{1}{\omega+\omega_{ab}}+\frac{1}{\omega-\omega_{ab}}\right). \tag{33}$$

Similar features have been already discussed in the case of an accelerated two-level system interacting with scalar field in the presence of a plate [25].

Summing the contributions of vacuum field fluctuations and of radiation reaction field to the Lamb shift in the presence of the reflecting plate [Eqs. (30) and (31)], we obtain the energy-level shifts of the accelerated atom in the presence of the plate. We get

$$(\delta E_a)^{b.} = (\delta E_a)_{vf}^{(b.)} + (\delta E_a)_{rr}^{(b.)}. \tag{34}$$

In Fig. 1 we have plotted the result obtained as a function of distance z_0 for a ground-state atom with isotropic polarizability and for different values of atomic acceleration. The distances considered are on the order of microns, which are the typical distances in experimental setup for measurements of Lamb shift in the presence of a conducting plate. A comparison between the different curves shows that, in order to reveal effects of acceleration on the energy-level shifts, it is necessary to consider accelerations on the order of $10^{24}-10^{25}$ cm/s^2. These are the typical accelerations necessary to observe the Unruh effect. In other words, as in the case of unbounded space, in the presence of a reflecting plate the effects of atomic acceleration on the Lamb shift become appreciable for accelerations on the order of 10^{24} cm/s^2. We have also considered the ratio between the energy-level shifts of an atom at rest in front of the plate and the energy-level shift of the accelerated atom $\delta E^{b.}(a=0)/\delta E^{b.}(a\neq 0)$ with $a=10^{25}$ cm/s^2. Figure 2 shows the behavior of this quantity as a function of the distance z_0. We observe that this quantity decreases, increasing z_0. This suggests that the effects of atomic acceleration on the energy-level shifts become more relevant for distances on the order of 10^{-4} cm.

We also stress that our calculation concerns with an accelerated atom in the ground state. Although we do not have performed explicit calculations, we expect that in the case of an atom in the excited state, resonant terms could appear due to the resonant atom-field interaction, in analogy with the case of an excited atom at rest near a conducting plate.

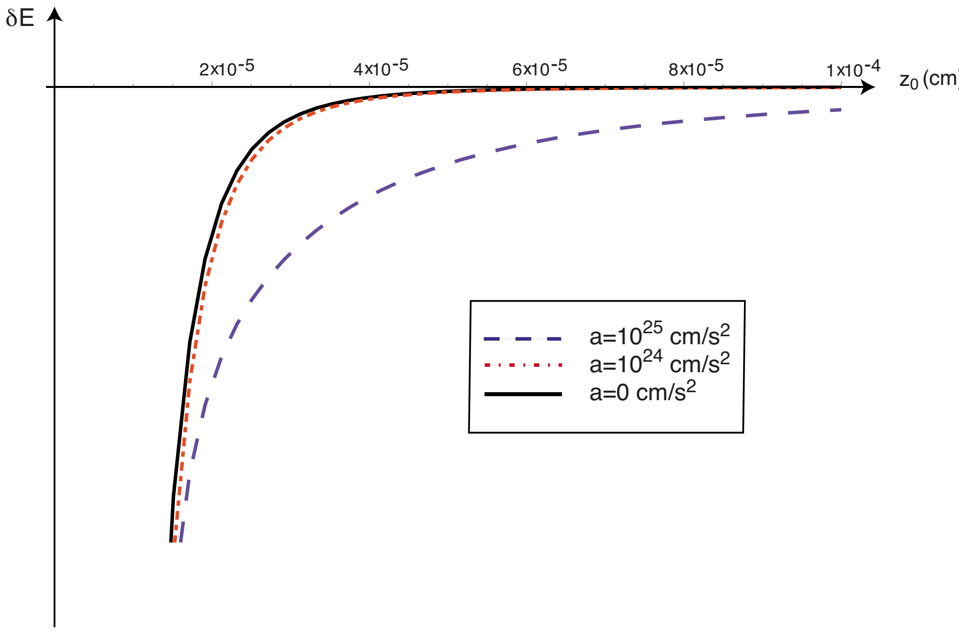

FIG. 1. (Color online) Energy level shifts as a function of the atom-plate distance z_0 and for different values of atomic acceleration. Blue, red, and black curves (long-dashed, dashed, and solid lines, respectively) represent the shifts for $a=10^{25}$ cm/s^2, $a=10^{24}$ cm/s^2, and $a=0$ cm/s^2, respectively.

It is interesting to observe that because of the anisotropy of the function $K_{\ell m}(\omega;z_0,a)$, the energy-level shift in Eq. (34) depends on the direction of the atomic polarizability. For example, for a dipole oriented in a direction parallel to the x-z plane, the energy shift contains an extra term, compared with the case of an atom at rest, which is associated with nondiagonal component xz of the function $K_{\ell m}(\omega;z_0,a)$. This contribution, which is proportional to the atomic acceleration a, is not present in the case of an atom at rest. This suggests the possibility to make the effects of acceleration on the Lamb shift easier to observe for anisotropic polarizability.

We conclude our analysis by comparing the results obtained with the case of an atom at rest in front a perfectly reflecting plate at a temperature different from zero [31]. We have considered the ratio between the energy shifts of an accelerated atom in the presence of a reflecting plate with those obtained for an atom at rest immersed in a thermal bath at a temperature T. In Fig. 3 we have plotted as a function of the distance z_0 the quantity $\delta E^{b\cdot}(T\neq 0)/\delta E^{b\cdot}(a\neq 0)$ for an atom with isotropic polarizability for two different values of temperature ($T=100$ K and $T=300$ K) and for acceleration on the order of 10^{24} cm/s^2 (corresponding to the Unruh temperature $T_U\sim 10^2$ K). The curves obtained show that for distances on the order of 10^{-4} cm, the ratio $\delta E^{b\cdot}(T\neq 0)/\delta E^{b\cdot}(a\neq 0)$ is equal to 1. This suggests that there exist some distances z_0 for specific values of atomic acceleration such that the effect of the atomic acceleration on the energy-level shifts becomes equivalent to a thermal effect. Moreover, Figs. 3 and 4 show that when the temperature or the atomic acceleration increase, the range of distances in which $\delta E^{b\cdot}(T\neq 0)/\delta E^{b\cdot}(a\neq 0)\sim 1$ decreases. In particular, as shown in Fig. 4, for acceleration on the order of 10^{25} cm/s^2, the effect of acceleration on the energy-level shifts becomes equivalent to the thermal one in the limit of small distances ($z_0\sim 0.1$ μm).

IV. CONCLUSIONS

In this paper we have investigated the Lamb shift of a uniformly accelerated hydrogen atom interacting with the electromagnetic field in the presence of a perfectly conduct-

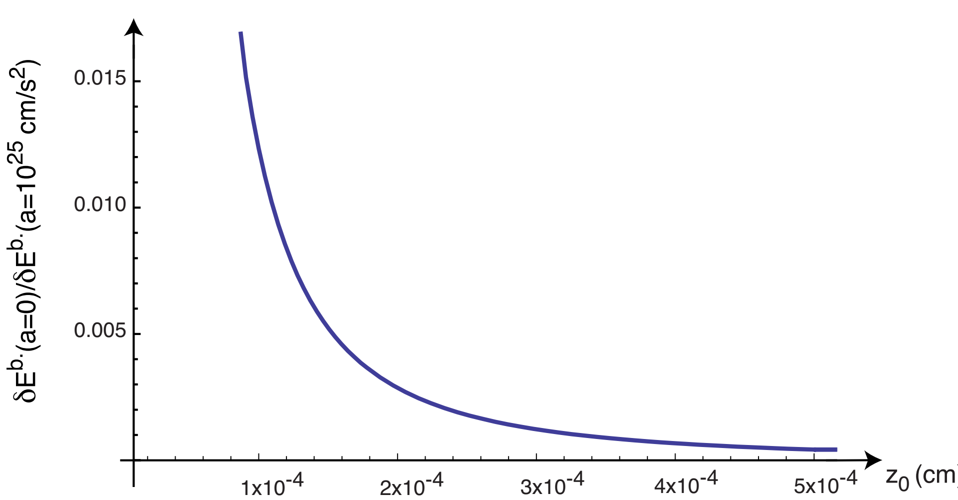

FIG. 2. (Color online) Ratio between the energy-level shift of an atom at rest in front a reflecting plate and the energy-level shift of the atom accelerating along x direction parallel to the plate ($a=10^{25}$ cm/s^2). The effects of acceleration become important when the atom-plate distance z_0 increases.

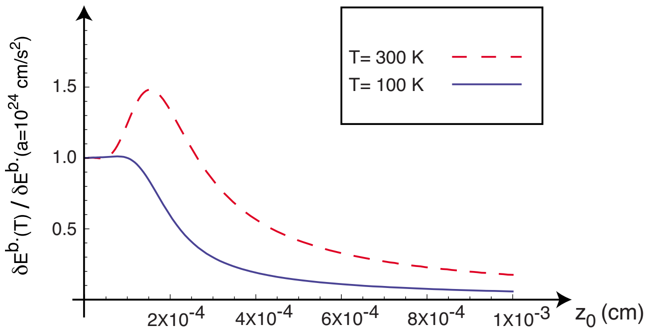

FIG. 3. (Color online) Ratio between the energy-level shift of an atom at rest in front the reflecting plate at temperature T and the energy-level shift of the accelerated atom ($a=10^{24}$ cm/s^2). Red and blue curves (dashed and solid lines, respectively) represent this ratio for two different temperatures, $T=300$ K and $T=100$ K, respectively.

ing plate. The atom accelerates in a parallel direction to the plate. We have evaluated the contribution of vacuum field fluctuations and radiation reaction field to the energy-level shifts of the accelerated atom. We have shown that, due to the presence of the boundary, both these contributions are affected by the atomic acceleration. We have also discussed that the corrections induced by the presence of the boundary are essentially nonthermal. It is argued that for specific values of the atom-plate distance and/or acceleration, nonthermal terms may cancel that already present in the unbounded space. Finally, we have shown that because of the anisotropy of the electric correlation function, the energy-level shifts depend in a significantly way from the orientation of atomic dipole. This suggests that effects of acceleration on the atomic level shifts can become appreciable for atoms with anisotropic polarizability.

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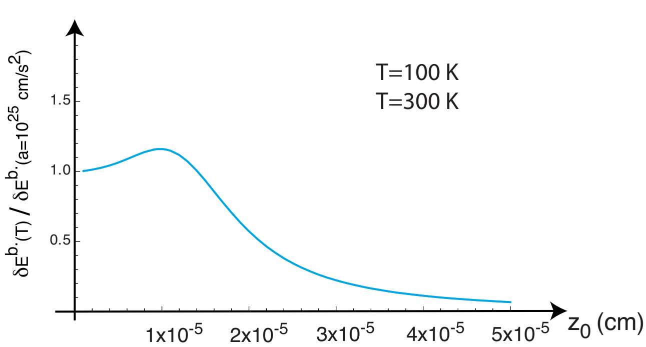

FIG. 4. (Color online) Ratio between the energy shift of an atom at rest in front the plate at temperature T and the energy-level shift of the accelerated atom ($a=10^{25}$ cm/s^2). Note that the curves obtained for two different temperatures ($T=100$ K and $T=300$ K) coincide.

APPENDIX: CORRELATION FUNCTION AND LINEAR SUSCEPTIBILITY AS INTEGRALS OVER FREQUENCIES

In this appendix we outline the calculation of the symmetrical and antisymmetrical correlation functions as integrals over frequencies. We first focus on the symmetrical correlation function

$$C^F_{\ell m}(x,x') = \tfrac{1}{2}\langle 0|\{E_\ell(x(\tau)),E_m(x(\tau'))\}|0\rangle, \tag{A1}$$

From Eq. (25) we obtain

$$\begin{aligned}[C^F_{\ell m}(x,x')]^{(b.)} = &-\frac{a^4}{16\pi^2}\left[\delta_{\ell m}\left\{\sinh^2\left[\frac{a}{2}(\tau-\tau'-i\epsilon)\right]+a^2z^2\right\}\right.\\ &\left.-2n_\ell n_m\sinh^2\left[\frac{a}{2}(\tau-\tau'-i\epsilon)\right]\right]\\ &\times\frac{1}{\left\{\sinh^2\left[\frac{a}{2}(\tau-\tau'-i\epsilon)\right]+a^2z^2\right\}^3}-\frac{a^4}{16\pi^2}\\ &\times[2a^2z^2(\delta_{\ell m}-k_\ell k_m)+2az(n_\ell k_m+k_\ell n_m)]\\ &\times\frac{\sinh^2\left[\frac{a}{2}(\tau-\tau'-i\epsilon)\right]}{\left\{\sinh^2\left[\frac{a}{2}(\tau-\tau'-i\epsilon)\right]+a^2z^2\right\}^3}.\end{aligned} \tag{A2}$$

Observing that $\delta_{\ell m}-2n_\ell n_m=\sigma_{\ell m}$ and defining $\zeta_{\ell m}=az_0(\delta_{\ell m}-k_\ell k_m)+(n_\ell k_m+k_\ell n_m)$, expression (A2) can be written as

$$\begin{aligned}&[C^F_{\ell m}(x,x')]^{(b.)}\\ &=-\frac{a^4}{16\pi^2}\left(\sigma_{\ell n}\frac{\delta_{nm}\sinh^2\left(\frac{a(u-i\epsilon)}{2}\right)+\sigma_{nm}a^2z_0^2}{\left[\sinh^2\left(\frac{a(u-i\epsilon)}{2}\right)-a^2z_0^2\right]^3}\right)\\ &\quad-\frac{a^4}{16\pi^2}\zeta_{\ell m}\frac{\sinh^2\left(\frac{a(u-i\epsilon)}{2}\right)}{\left[\sinh^2\left(\frac{a(u-i\epsilon)}{2}\right)-a^2z_0^2\right]^3},\end{aligned} \tag{A3}$$

where $u=\tau-\tau'$. In order to express this function as integral over frequencies we first observe that

$$\begin{aligned}&\int_0^\infty d\omega\,\sin\left[\frac{2\omega}{a}\sinh^{-1}\left(\frac{\pi\omega}{a}\right)\right]\coth\left(\frac{\pi\omega}{a}\right)(e^{i\omega u}+e^{-i\omega u})\\ &=\frac{a^2z_0(1+a^2z_0^2)^{1/2}}{a^2z_0^2-\sinh^2(au/2)},\end{aligned} \tag{A4}$$

$$\int_0^\infty d\omega\omega \cos\left[\frac{2\omega}{a}\sinh^{-1}\left(\frac{\pi\omega}{a}\right)\right]\coth\left(\frac{\pi\omega}{a}\right)(e^{i\omega u}+e^{-i\omega u}) = -\frac{a}{2}\left\{\frac{2a^2z_0^2(1+a^2z_0^2)}{[a^2z_0^2-\sinh^2(au/2)]^2} - \frac{1+2a^2z^2}{a^2z_0^2-\sinh^2(au/2)}\right\}, \tag{A5}$$

$$\int_0^\infty d\omega\omega^2 \sin\left[\frac{2\omega}{a}\sinh^{-1}\left(\frac{\pi\omega}{a}\right)\right]\coth\left(\frac{\pi\omega}{a}\right)(e^{i\omega u}+e^{-i\omega u})$$
$$= -\frac{a^4z(1+a^2z_0^2)^{1/2}}{2}\left\{\frac{4a^2z_0^2(1+a^2z_0^2)}{[a^2z_0^2-\sinh^2(au/2)]^3} - \frac{3(1+2a^2z_0^2)}{[a^2z_0^2-\sinh^2(au/2)]^2} + \frac{2}{a^2z_0^2-\sinh^2(au/2)}\right\}. \tag{A6}$$

Using these expressions in Eq. (A3), after some algebraic manipulation, we obtain

$$C^F_{\ell m}(x,x')^{(b.)} = -\frac{1}{8\pi^2}\frac{1}{(2z_0)^3(1+a^2z_0^2)^{1/2}}\int_0^\infty d\omega K_{\ell m}(\omega;z_0,a)\coth\left(\frac{\pi\omega}{a}\right)(e^{i\omega u}+e^{-i\omega u}), \tag{A7}$$

where we defined

$$\begin{aligned}
K_{\ell m}(\omega;z_0,a) = {}& \frac{\sigma_{\ell n}}{(1+a^2z_0^2)^{1/2}}\Bigg\{(\delta_{nm}-n_nn_m)\frac{(2\omega z_0)^2}{(1+a^2z_0^2)}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\\
&+(\delta_{nm}-3n_nn_m)\left[\frac{2\omega z_0}{(1+a^2z_0^2)^{3/2}}\cos\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)-\frac{1}{(1+a^2z_0^2)^2}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\right]\\
&+a^2z_0^2\left[(\delta_{nm}+3\sigma_{nm})\frac{2\omega z_0}{(1+a^2z_0^2)^{3/2}}\cos\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)+(\delta_{nm}-2\sigma_{nm})\frac{2}{(1+a^2z_0^2)^2}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\right]\\
&-\sigma_{nm}a^4z_0^4\frac{4}{(1+a^2z_0^2)^2}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\Bigg\}+az_0[(\delta_{lm}-k_lk_m)az_0+n_lk_m+k_ln_m]\\
&\times\left\{\frac{(2\omega z_0)^2}{(1+a^2z_0^2)}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)-\frac{2\omega z_0}{(1+a^2z_0^2)^{3/2}}\cos\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)+\frac{1}{(1+a^2z_0^2)^2}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\right\}\\
&+a^3z_0^3[(\delta_{\ell m}-k_\ell k_m)az_0+n_\ell k_m+k_\ell n_m]\left\{\frac{4\omega z_0}{(1+a^2z_0^2)^{3/2}}\cos\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)+\frac{4}{(1+a^2z_0^2)^2}\sin\left(2\frac{\omega}{a}\sinh^{-1}(az_0)\right)\right\}.
\end{aligned} \tag{A8}$$

Let us now consider the linear susceptibility

$$\chi^F_{\ell m}(x,x') = \tfrac{1}{2}\langle 0|[E_\ell(x(\tau)),E_m(x(\tau'))]|0\rangle. \tag{A9}$$

From Eq. (25) we obtain

$$\begin{aligned}
[\chi^F_{\ell m}(x,x')]^{(b.)} = {}& -i\frac{a^4}{16\pi^2}\{[2\sigma_{\ell n}(\delta_{nm}-n_nn_m)+2az_0\eta_{\ell m}]\Delta_1(u;z_0,a)\\
&-[2\sigma_{\ell n}(\delta_{nm}-3n_nn_m)-2az_0\eta_{\ell m}][\Delta_2(u;z_0,a)-\Delta_3(u;z_0,a)]\},
\end{aligned} \tag{A10}$$

where we defined

$$\begin{aligned}
\Delta_1(u;z_0,a) = {}& \frac{i\pi}{4az_0}\Bigg\{\frac{4}{(1+a^2z_0^2)^{3/2}}[\delta''(au-2\sinh^{-1}(az_0))-\delta''(au+2\sinh^{-1}(az_0))]+\frac{6az_0}{(1+a^2z_0^2)^2}[\delta'(au-2\sinh^{-1}(az_0))\\
&-\delta'(au+2\sinh^{-1}(az_0))]-\frac{1-2a^2z_0^2}{(1+a^2z_0^2)^{5/2}}[\delta(au-2\sinh^{-1}(az_0))-\delta(au+2\sinh^{-1}(az_0))]\Bigg\},
\end{aligned} \tag{A11}$$

$$\begin{aligned}
\Delta_2(u;z_0,a) = {}& \frac{i\pi}{8a^2z_0^2}\Bigg\{-\frac{4}{(1+a^2z_0^2)}[\delta'(au-2\sinh^{-1}(az_0))+\delta'(au+2\sinh^{-1}(az_0))]\\
&-\frac{2az_0}{(1+a^2z_0^2)^{3/2}}[\delta(au-2\sinh^{-1}(az_0))-\delta(au+2\sinh^{-1}(az_0))]\Bigg\},
\end{aligned} \tag{A12}$$

$$\Delta_3(u;z_0,a) = \frac{i\pi}{8a^3z_0^3}\left\{\frac{2}{(1+a^2z_0^2)^{1/2}}[\delta(au - 2\sinh^{-1}(az_0)) - \delta(au + 2\sinh^{-1}(az_0))]\right\}, \quad \text{(A13)}$$

where $\delta'(x)$ and $\delta''(x)$ are the first and the second derivatives of the Dirac's function, $\delta(x)$, respectively.

Observing that

$$\delta(x) = \frac{1}{2\pi}\int_0^\infty d\omega(e^{i\omega u} + e^{-i\omega u}), \quad \text{(A14)}$$

$$\delta'(x) = \frac{i}{2\pi}\int_0^\infty d\omega\omega(e^{i\omega u} - e^{-i\omega u}), \quad \text{(A15)}$$

$$\delta''(x) = -\frac{1}{2\pi}\int_0^\infty d\omega\omega^2(e^{i\omega u} + e^{-i\omega u}), \quad \text{(A16)}$$

we, finally, obtain

$$[\chi_{\ell m}^F(x,x')]^{(b.)} = \frac{1}{4\pi^2}\frac{1}{(2z_0)^3(1+a^2z_0^2)^{1/2}}\int_0^\infty d\omega K_{\ell m}(\omega;z_0,a) \times(e^{i\omega u} - e^{-i\omega u}). \quad \text{(A17)}$$

[1] P. W. Milonni, *The Quantum Vacuum* (Academic, San Diego, 1994).
[2] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
[3] H. B. G. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).
[4] G. Compagno, R. Passante, and F. Persico, *Atom-Field Interactions and Dressed Atoms* (Cambridge University Press, Cambridge, 1995).
[5] S. K. Lamoreaux, Rep. Prog. Phys. **68**, 201 (2005); M. Bordag, U. Mohideen, and V. M. Mostepanenko, Phys. Rep. **353**, 1 (2001); S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997); B. W. Harris, F. Chen, and U. Mohideen, Phys. Rev. A **62**, 052109 (2000); G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, Phys. Rev. Lett. **88**, 041804 (2002).
[6] V. Druzhinina and M. DeKieviet, Phys. Rev. Lett. **91**, 193202 (2003); D. M. Harber, J. M. Obrecht, J. M. McGuirk, and E. A. Cornell, Phys. Rev. A **72**, 033610 (2005); J. M. Obrecht, R. J. Wild, M. Antezza, L. P. Pitaevskii, S. Stringari, and E. A. Cornell, Phys. Rev. Lett. **98**, 063201 (2007); C. I. Sukenik, M. G. Boshier, D. Cho, V. Sandoghdar, and E. A. Hinds, *ibid.* **70**, 560 (1993).
[7] H. B. Chan, V. A. Aksyuk, R. N. Kleiman, D. J. Bishop, and F. Capasso, Science **291**, 1941 (2001).
[8] H. B. Chan, V. A. Aksyuk, R. N. Kleiman, D. J. Bishop, and F. Capasso, Phys. Rev. Lett. **87**, 211801 (2001).
[9] D. Meschede, Phys. Rep. **211**, 201 (1992).
[10] D. Meschede, W. Jhe, and E. A. Hinds, Phys. Rev. A **41**, 1587 (1990).
[11] S. Y. Buhmann and D.-G. Welsch, Prog. Quantum Electron. **31**, 51 (2007).
[12] S. Spagnolo, D. A. R. Dalvit, and P. W. Milonni, Phys. Rev. A **75**, 052117 (2007).
[13] S. Spagnolo, R. Passante, and L. Rizzuto, Phys. Rev. A **73**, 062117 (2006).
[14] D. Iannuzzi, M. Lisanti, and F. Capasso, Proc. Natl. Acad. Sci. U.S.A. **101**, 4019 (2004); A. Lambrecht, J. Opt. B: Quantum Semiclassical Opt. **7**, S3 (2005).
[15] S. Shresta, B. L. Hu, and N. G. Phillips, Phys. Rev. A **68**, 062101 (2003).
[16] L. Rizzuto, R. Passante, and F. Persico, Phys. Rev. A **70**, 012107 (2004).
[17] P. C. W. Davies, J. Phys. A **8**, 609 (1975).
[18] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[19] S. Takagi, Prog. Theor. Phys. Suppl. **88**, 1 (1986).
[20] T. H. Boyer, Phys. Rev. D **21**, 2137 (1980).
[21] A. Belyanin, V. V. Kocharovsky, F. Capasso, E. Fry, M. S. Zubairy, and M. O. Scully, Phys. Rev. A **74**, 023807 (2006).
[22] M. T. Jaekel and S. Reynaud, Quantum Semiclassic. Opt. **7**, 499 (1995).
[23] B. L. Hu and A. Roura, Phys. Rev. Lett. **93**, 129301 (2004).
[24] J. Audretsch and R. Muller, Phys. Rev. A **52**, 629 (1995).
[25] L. Rizzuto, Phys. Rev. A **76**, 062114 (2007).
[26] R. Passante, Phys. Rev. A **57**, 1590 (1998).
[27] J. Dalibard, J. Dupont-Roc, and C. Cohen-Tannoudji, J. Phys. (France) **43**, 1617 (1982).
[28] J. Dalibard, J. Dupont-Roc, and C. Cohen-Tannoudji, J. Phys. (France) **45**, 637 (1984).
[29] H. Yu and Z. Zhu, Phys. Rev. D **74**, 044032 (2006).
[30] N. D. Birrel and P. C. W. Davies, *Quantum Field in Curved Space* (Cambridge Monographs on Mathematical Physics, Cambridge, 1982).
[31] T. Boyer, Phys. Rev. A **11**, 1650 (1975).