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Cooling fermionic atoms in optical lattices by shaping the confinement

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We propose an experimental procedure to cool fermionic atoms loaded into an optical lattice. The central idea is to spatially divide the system into entropy-rich and -poor regions by shaping the confining potential profile. Atoms in regions of high entropy per particle are subsequently isolated from the system. We discuss how to experimentally carry out this proposal and perform a quantitative study of its efficiency. We find that the entropy per particle, s, can typically be reduced by a factor of 10 such that entropies lower than $s/k_B \sim 0.2$ can be reached. Cooling into highly sought-after quantum phases (such as an antiferromagnet) can thus be achieved. We show that this procedure is robust against variations of the experimental conditions.

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I. INTRODUCTION

Rapid experimental progress in manipulating ultracold atomic gases has provided physicists with increased control over quantum many-particle systems [1]. This was recently evidenced by the observation of a Mott-insulating phase of fermionic atoms in a three-dimensional (3D) optical lattice [2,3]. More complex quantum phases, such as Néel antiferromagnets, strongly correlated Fermi liquids, or spin liquids in frustrated geometries, could also be realized using cold atoms. However, such phases typically emerge in a temperature regime lower than currently achievable. In existing experiments, the atomic cloud is precooled by evaporation in a harmonic trap and, in a second step, transferred into the periodic potential of an optical lattice. Loading the atoms into the lattice is ideally performed adiabatically, i.e., conserving the entropy of the system. Present experiments indicate an entropy per particle of $s \approx \pi^2 T_o / T_F \approx 1.5-2$ in the limit of a noninteracting Fermi gas [2,3], with T_o and T_F as the system and Fermi temperatures and k_B set to 1 [4]. These values of s are well above the onset of interesting correlated phases. Thus, developing novel cooling techniques for lattice quantum gases, as we propose in this Rapid Communication, is a crucial step to demonstrate that cold atoms can indeed adequately simulate strongly correlated condensed-matter systems.

Cooling atomic gases in optical lattices is the focus of an increasing number of studies. For bosons loaded into an optical lattice, it was proposed to create entropy-rich regions that are later isolated from the rest of the system [5,6]. These proposals were inspired by earlier experiments in which an adiabatic deformation of the external trapping potential was used to increase the phase-space density of Bose gases [7,8]. For fermions in the absence of a lattice potential, it was suggested to cool the gas by taking advantage of a Feshbach resonance [9,10]. For fermions loaded to an optical lattice very few proposals have been put forward. Most of them apply to noninteracting Fermi gases [11] or are based on the use of a Bose-Einstein condensate as a heat reservoir [12,13]. However, the possible limitation of entropy reduction due to inelastic collisions between bosons and fermions has not been addressed yet.

In this Rapid Communication, we propose an experimentally realistic procedure to cool two-component fermionic mixtures in optical lattices. The key idea is to spatially divide the trapped fermionic gas into regions of low and high entropies per particle by shaping the trapping potential. The two regions are then adiabatically isolated from each other and the atoms from the entropy-rich regions are disposed of. The remaining atoms have a drastically reduced entropy per particle. In fact, we find that the system temperature can be reduced by typically 1 order of magnitude while retaining half of the particles. In addition, the cooling efficiency remains high over a wide range of interatomic coupling strengths, initial particle numbers, and trap anisotropies. Hence, with this method, it should be possible to reach highly anticipated quantum phases not yet observed. Such phases include the Néel antiferromagnet in a cubic lattice and, perhaps even more excitingly, spin liquids or other exotic spin-disordered phases in frustrated lattice geometries [14]. Interestingly, for systems slightly away from half filling, we can also reach sufficiently low entropy per particle to enter the strongly correlated Fermi-liquid regime. Finally, our proposal, which relies only on adding a limited number of lasers to engineer the trap potential, can be well integrated into existing experimental setups.

II. COOLING SCHEME

Let us begin with a spin-$\frac{1}{2}$ mixture of fermionic atoms precooled in a dipole trap. As a first step, we apply a three-dimensional optical lattice potential [Fig. 1(a)]. To allow the atoms to thermalize, the loading is done in the presence of a finite but weak interatomic coupling. We also keep the lattice sufficiently shallow for the atoms to redistribute efficiently. As a second step, we modulate the entropy distribution by creating a potential depression, a dimple, in the middle of the harmonic trap. This dimple must be sufficiently deep and narrow for fermionic atoms to accumulate in it and form a band insulator [15]. The entropy per particle in this "core region" is very small. In contrast, in the outer region, called "storage region," the potential profile is kept shallow in order to create a low-density liquid over a wide volume. Under

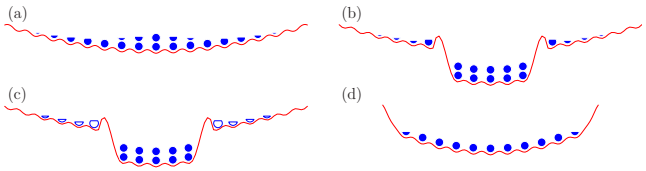

FIG. 1. (Color online) Cooling scheme. (a) The atoms trapped in a parabolic profile are loaded into an optical lattice. (b) A band insulator is created in a dimple at the center of the trap. This core region is isolated from the rest of the system, the storage region, by rising potential barriers. (c) The storage region is removed from the system. (d) The band insulator is relaxed to the desired quantum phase, e.g., a Mott insulator by flattening the dimple and turning off or pushing outwards the barriers.

such conditions and at small interaction strength, the entropy per particle in the storage region is very high. We then separate the core and storage regions by slowly rising potential barriers and obtain the potential profile shown on Fig. 1(b). As a third step, we remove the storage region [Fig. 1(c)]. We are left with a new effective system characterized by a very small entropy per particle [16]. Finally, as a last step, the band insulator is relaxed adiabatically into an experimentally relevant phase. For example, if the aim is to form a Mott-insulating state, the filling can be lowered by slowly flattening the potential in the core region and by turning off or pushing outwards the barriers [Fig. 1(d)].

III. SHAPING THE POTENTIAL PROFILE

The above procedure relies on the ability to add a tailored potential profile on top of a lattice potential with amplitude V_{lattice}. To modulate the entropy distribution, the global potential, shown in Fig. 2, should realize tight trapping in the core region, surrounded by a wide shallow ring in the storage region isolated from the core by high potential barriers. To produce this profile, we envision to use three elements, (i) a shallow harmonic trap (either magnetic or optical), (ii) a dimple which confines atoms in a small region around the trap symmetry axis and helps to create the band insulator, and (iii) a cylindrically symmetric potential barrier to isolate entropically poor and rich regions. The dimple (ii) and potential barrier (iii) are produced by red- and blue-detuned laser beams, respectively, creating attractive or repulsive dipole potentials. The dimple has a Gaussian profile, while the barrier should rather be a narrow annulus. Experimentally this can be realized either by setting a tightly focused laser beam in rapid rotation or by engineering the beam profile using phase plates or other diffractive optics [17]. Consequently, in addition to the lattice potential, the trapping profile is given by

$$V(\mathbf{r}) = V_{\text{harmonic}} + V_{\text{dimple}} + V_{\text{barrier}},$$

with

$$V_{\text{harmonic}}(\mathbf{r}) = V_{\text{h}}(x^2 + y^2 + \gamma^2 z^2)/a^2,$$

$$V_{\text{dimple}}(\mathbf{r}) = -V_{\text{d}} \exp[-2(x^2 + y^2)/w_{\text{d}}^2],$$

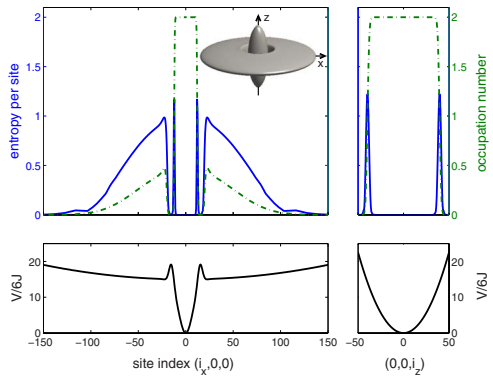

FIG. 2. (Color online) Occupation number (dashed line, upper panels), entropy per particle (solid line, upper panels), and potential profile [18] (solid line, lower panels) in the presence of the dimple and barriers, as a function of the transverse (left) and axial (right) coordinates. The potential is offset such that $V=0$ is at the bottom of the dimple. We chose the following experimentally realistic parameters: $\frac{U}{6J}=0.5$, $\frac{V_{\text{h}}}{6J}=1.8\times10^{-4}$, $\gamma^2=50$, $\frac{V_{\text{b}}}{6J}=6$, $r_{\text{b}}=15a$, $w_{\text{b}}=5a$, $\frac{V_{\text{d}}}{6J}=15$, $w_{\text{d}}=15a$, and 12×10^4 atoms. The average entropy per particle in the total system is $s_T=1.95$ and in the core region $s_C=0.198$. The ratio of particles in the core region versus the total particle number is $\frac{N_C}{N_T}=0.404$. Inset: 3D rendering of the potential profile showing an isopotential surface ($\frac{V}{6J}=16$).

$$V_{\text{barrier}}(\mathbf{r}) = V_{\text{b}} \exp[-2(\sqrt{x^2 + y^2} - r_{\text{b}})^2/w_{\text{b}}^2],$$

where $V_{\{\text{h,d,b}\}}$ are the potential amplitudes, γ is a measure of the anisotropy of the harmonic trap, $w_{\{\text{d,b}\}}$ are the waists of the Gaussian laser beams forming the dimple and barrier, r_{b} is the radius of the cylindrical barrier, and a is the lattice spacing.

IV. EFFICIENCY OF THE PROCEDURE

The efficiency of the proposed cooling scheme can be quantitatively estimated under the assumption that shaping the potential profile is an adiabatic process. Possible deviations from adiabacity will be discussed later on. Under the adiabatic assumption the meaningful quantity is the entropy per particle rather than temperature itself. The cooling efficiency depends on how much entropy per particle is left in the core region, $s_C=S_C/N_C$, at the precise moment when the increasing barrier height causes the two regions to stop exchanging entropy compared to the initial entropy per particle, $s_T=S_T/N_T$. The quantities $S_{C/T}$ and $N_{C/T}$ are the entropy and number of atoms in the core (C) and total system (T). The described situation is shown in Fig. 2. At later times, the core entropy remains unchanged as the two regions are now isolated from one another preventing the backflow of entropy.

To determine the efficiency of the cooling scheme, we describe the two-component mixture of fermions using a Hubbard-type Hamiltonian [19]

$$H = -J\sum_{\langle i,j\rangle\sigma}(c_{i\sigma}^{\dagger}c_{j\sigma} + \text{H.c.}) + U\sum_i \hat{n}_{i\uparrow}\hat{n}_{i\downarrow} - \sum_{i\sigma}\mu_i\hat{n}_{i\sigma}.$$

Here $c_{i\sigma}^{\dagger}$ and $c_{i\sigma}$ are the creation and annihilation operators of the fermions with $\sigma=\{\uparrow,\downarrow\}$, J is the hopping matrix element, U is the on-site repulsion, μ_i is the local chemical potential,

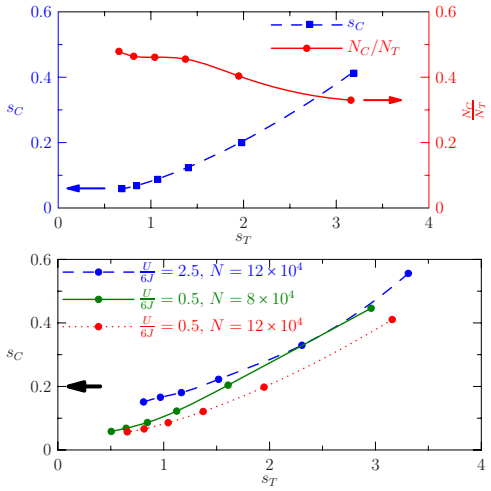

FIG. 3. (Color online) Upper panel: entropy per particle in the core, s_C, and ratio of particles remaining in the core, N_C/N_T, after cooling, as a function of the initial total entropy per particle, s_T, for $\gamma^2=50$, $\frac{U}{6J}=0.5$, and $N_T=12\times10^4$. Lower panel: s_c as a function of s_T for different interaction strengths and total particle numbers at $\gamma^2=50$. The parameters used to shape the trap are the same as in Fig. 2 [22]. The black arrow in the lower panel indicates the pessimistic estimate of the entropy per particle required to reach the antiferromagnet for large interaction strengths [21].

and $\hat{n}_{i\sigma}=c_{i\sigma}^{\dagger}c_{i\sigma}$ is the number operator on site i. All potential profiles are treated in the local-density approximation (LDA), i.e., assuming a spatially varying chemical potential $\mu_i=\mu_0-V_i$. To use LDA, local densities and entropies must be obtained for the homogeneous system. These quantities are calculated using dynamical mean-field theory [20]. In particular, the entropy is calculated as in [21].

In the upper panel of Fig. 3, we show, for a three-dimensional gas ($\gamma^2=50$), that the final entropy per particle in the system core, s_C, can be reduced by a factor of 10 as compared to the initial entropy per particle, s_T. This is done while retaining about half of the particles. For $s_T=1.95$ and $N_T=12\times10^4$ atoms, about 5×10^4 atoms are kept. The lower panel of Fig. 3 shows that the efficiency of our cooling scheme is very stable against variations in the interaction strength and initial particle number. Clearly, the procedure is most efficient at small values of the interaction strength, but the deviations for other interaction strengths are small. Experimentally a compromise has to be found between a small value giving an optimal gain and the time of thermalization for which scattering processes must take place. Finally, we made sure that this cooling scheme is efficient both for quasi-two-dimensional (large γ^2) and three-dimensional (small γ^2) systems. However, in two dimensions, to obtain similar N_C's and maintain the same efficiency, larger radial sizes for both the core and storage regions are required as less particles can be stacked along the z direction.

The reduction in the entropy per particle by one order of magnitude as compared to the current experimental situation opens the door to study a wealth of unexplored phenomena in cold atomic systems. As an example, in Ref. [21], a (pessimistic) lower bound on the entropy per particle needed to stabilize antiferromagnetic long-ranged order was estimated to be $s\simeq0.2$. Using our cooling scheme, entropies per particle lower than this value can actually be reached in the core region. Starting from initial temperatures currently accessible experimentally, $T_o/T_F\approx0.15-0.2$ [2,3], system temperatures of $T_o/T_F\approx0.014-0.02$ are achieved.

Finally, two remarks are in order. First, higher efficiencies could be obtained by removing more atoms or engineering flatter outer regions that can store more entropy. Second, the weak dependence of the efficiency on the initial entropy (Fig. 3) suggests that this procedure can as well be performed several times in a row to reach very low temperatures. However, as all changes have to be performed slowly, the total time required to cool the system will grow with the number of repetitions.

V. REMOVAL OF STORAGE ATOMS

Having shown how the proposed cooling scheme can decrease the entropy in the core region, we now address the fate of the entropy-rich part isolated from the core by the potential barrier. If storage atoms do not disturb the subsequent experimental measurements, they can simply be "pushed outwards" by dynamically increasing the barrier radius and raising its height [23] to avoid "spilling" the storage atoms into the core region. However, in many cases, getting rid of these atoms or transferring them to a different hyperfine state could be advantageous for later detection. Several different removal or transfer schemes may be envisaged depending on the details of the experimental setup. These schemes do not need to be adiabatic as the storage atoms are already isolated from the core.

One possible removal scheme relies on applying a linear potential gradient $-Fx$ (which could be due to gravity or to an intentionally applied magnetic gradient) and weakening the shallow trap along the x direction. Under the influence of the applied force, storage atoms will undergo Bloch oscillations [24] interrupted by Landau-Zener (nonadiabatic) transitions to higher bands. These transitions can lead to outcoupling of "atom bursts" at multiples of the Bloch period $T_B=h/Fa$ [25]. Atoms in the core region are confined by the combined dimple/barrier potentials, and the potential gradient merely shifts the potential minimum by a small amount. To achieve significant outcoupling rates, one should also significantly lower the lattice depth along x. In the weak-binding limit, the Landau-Zener formula indeed predicts a transition rate $\Gamma_{\text{out}}\sim\frac{1}{T_B}e^{-A_{LZ}}$ [24], where $A_{LZ}=ma\Delta^2/4\hbar^2F$ and where Δ is the band gap which should be as small as possible. For instance, for a lattice depth of $0.5E_R(\Delta\sim0.2E_R)$, $a=266$ nm, and $F/m\sim10$ m/s^2, we find $\Gamma_{\text{out}}\sim10$ s^{-1} for ^{40}K atoms ($A_{LZ}\approx3$) and essentially zero for ^{6}Li atoms ($A_{LZ}\approx144$) [26]. Another possibility to decrease the band gap is to excite the storage atoms to a higher Bloch band [24]. In order to leave the core atoms untouched, the excitation beams should have a "hollow" profile (created using the same techniques as the potential barriers) to suppress the transition probability near the center of the cloud.

VI. DEVIATION FROM ADIABATICITY

In an experimental setup, one has to find a compromise between changing the potential profile slowly (which favors adiabacity) or quickly (which subjects the system to external disruptions only for a short time). To approximate the heating induced by nonadiabaticity, we perform time-dependent simulations of the cooling and subsequent relaxation into a Mott-insulating state within an experimentally realistic time. As a measure of the induced heating, we determine the energy absorbed by the core region during the process. We use the adaptive time-dependent density-matrix renormalization-group method [27,28] to simulate the procedure in a one dimensional fermionic system described by the Hubbard model. The simulations follow this sequence: we (i) shape the trap (by increasing the dimple amplitude and the barrier height), (ii) relax the band insulator (by simultaneously pushing outwards the barriers, decreasing the dimple amplitude, and adjusting the density by changing the parabolic trapping potential), and (iii) tune the interaction strength to its final value. We assume a linear variation in each parameter with time. For a total procedure time of the order of 500 $\frac{\hbar}{J}$ (700 ms for ^{40}K atoms in a lattice with $V_{\text{lattice}}=8E_R$), the system remains very close to its ground state [29] and the energy absorbed by the system is smaller by more than 1 order of magnitude than the superexchange coupling $4J^2/U$. Consequently, the heating induced by the nonadiabaticity is small enough not to hinder the efficiency of our cooling scheme. We expect that for a three-dimensional system the time scales for the redistribution of atoms are even more favorable than for the one dimensional case simulated here.

VII. CONCLUSION

We proposed an efficient scheme to cool fermionic atoms confined to optical lattices. This cooling procedure relies on spatially dividing the trapped fermionic gas into regions of low and high entropies per particle using a complex potential profile. We find that, for a two-component fermionic mixture loaded into a cubic lattice potential, this scheme reduces the system temperature by typically 1 order of magnitude while keeping approximately half of the atoms. The procedure remains efficient over a wide range of interatomic coupling strengths, initial particle numbers, and trap anisotropies. This method can be used to cool atoms into highly sought-after quantum phases such as the Néel antiferromagnet, spin liquids, and strongly correlated Fermi liquids.

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[1] I. Bloch *et al.*, Rev. Mod. Phys. **80**, 885 (2008).
[2] R. Jördens *et al.*, Nature (London) **455**, 204 (2008).
[3] U. Schneider *et al.*, Science **322**, 1520 (2008).
[4] For a system of interacting fermions in an optical lattice, this relation is more involved. See F. Werner *et al.*, Phys. Rev. Lett. **95**, 056401 (2005); A. M. Dare *et al.*, Phys. Rev. B **76**, 064402 (2007); A. Koetsier *et al.*, Phys. Rev. A **77**, 023623 (2008).
[5] M. Popp *et al.*, Phys. Rev. A **74**, 013622 (2006).
[6] B. Capogrosso-Sansone *et al.*, Phys. Rev. A **77**, 015602 (2008).
[7] P. W. H. Pinkse *et al.*, Phys. Rev. Lett. **78**, 990 (1997).
[8] D. M. Stamper-Kurn *et al.*, Phys. Rev. Lett. **81**, 2194 (1998).
[9] L. D. Carr *et al.*, Phys. Rev. Lett. **92**, 150404 (2004).
[10] R. Haussmann *et al.*, Phys. Rev. A **78**, 063602 (2008).
[11] P. B. Blakie and A. Bezett, Phys. Rev. A **71**, 033616 (2005); P. B. Blakie *et al.*, *ibid.* **75**, 063609 (2007).
[12] A. Griessner *et al.*, Phys. Rev. Lett. **97**, 220403 (2006).
[13] T.-L. Ho and Q. Zhou, Proc. Natl. Acad. Sci. U.S.A. **106**, 6916 (2009).
[14] A. P. Ramirez *et al.*, Phys. Rev. Lett. **84**, 2957 (2000); S. Nakatsuji *et al.*, Science **309**, 1697 (2005); Y. Okamoto *et al.*, Phys. Rev. Lett. **99**, 137207 (2007).
[15] In the core, any phase with sufficiently low entropy can be used. The low entropy of a band insulator was also put to use in Ref. [13].
[16] The low entropy per particle in the core region could be, for example, deduced from a measurement of the double occupancy [2].
[17] N. Friedman *et al.*, Adv. At., Mol., Opt. Phys. **48** 99 (2002).
[18] For a 3D lattice loaded with ^{40}K atoms, we can use beams of $P=2.7$ W at $\lambda_L=532$ nm focused to a waist of 310 μm. For the optical dipole trap, we can use two beams in the x and y directions of $P=15$ W at $\lambda_{\text{DT}}=1064$ nm focused to the same waist as the lattice. $E_R=h^2/(2m\lambda_L^2)$ is the recoil energy. For $V_{\text{lattice}}=8E_R$, $6J=0.19E_R$. To create the potential profile, all energy scales were chosen to be smaller than the energy required to excite atoms from the first to the second band as $\Delta E_{1\to 2}/6J\sim 19$.
[19] D. Jaksch *et al.*, Phys. Rev. Lett. **81**, 3108 (1998).
[20] A. Georges *et al.*, Rev. Mod. Phys. **68**, 13 (1996).
[21] L. De Leo *et al.*, Phys. Rev. Lett. **101**, 210403 (2008).
[22] Only the height of the barrier is adjusted to adequately suppress the entropy exchange between the core and storage regions.
[23] For the example shown in Fig. 2, the depth of the shallow trap is on the order of $25J$. A barrier with height exceeding $\geqslant 36J$ and radius ~ 150 a implies a realistic tenfold increase in laser power compared to the initial configuration.
[24] E. Peik *et al.*, Phys. Rev. A **55**, 2989 (1997).
[25] B. P. Anderson *et al.*, Science **282**, 1686 (1998).
[26] The large difference stems from the quadratic dependence of A_{LZ} on the recoil energy, which is about 6.7 times larger for ^{6}Li than for ^{40}K.
[27] A. J. Daley *et al.*, J. Stat. Mech.: Theory Exp. (2004) P04005.
[28] S. R. White *et al.*, Phys. Rev. Lett. **93**, 076401 (2004).
[29] The switching on can be performed on a relatively short time scale of the order of $100\hbar/J$, whereas the relaxation process is more critical and has been performed on the order of $400\hbar/J$.