Experimental evidence for the thermal origin of 1/*f* frequency noise in erbium-doped fiber lasers

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We present experimental evidence in support of the recent theoretical proposal that intrinsic $1/f$ frequency noise in short cavity erbium-doped fiber lasers is of thermal origin. We demonstrate that the power spectral density of frequency noise in distributed-feedback fiber lasers (DFB FL) exhibits predicted T^2 temperature dependence across all frequencies over a temperature range of almost 200 K. This temperature dependence is observed both in direct interferometric measurements of frequency noise in a single mode DFB FL and noninterferometric measurements of polarization-beat-frequency noise in a dual frequency DFB FL. It is also shown that frequency noise of orthogonal polarization modes in the dual frequency DFB FL is substantially correlated providing a strong indication of a common origin.

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I. INTRODUCTION

One of the most important properties of distributed-feedback fiber lasers (DFB FL) and similar short cavity erbium-doped fiber lasers (EDFL) is their innate capacity for extremely narrow linewidth single frequency operation. This, along with their compact size, compatibility with wavelength division multiplexing and amenability to optical pumping, has led to important applications in high performance interferometric sensing as narrow linewidth sources [1,2], and as sensor elements in their own right [3–5]. The recent renewal of interest in coherent optical communications raises the possibility that short cavity EDFL may also offer advantages over traditional semiconductor sources in emerging high-bandwidth telecommunications systems.

The linewidth of a single frequency DFB FL may be as narrow as 3–4 kHz and, as such, very difficult to observe directly. For such devices, it is natural to focus on laser frequency noise, which is readily observed as intensity fluctuations at the output of an unbalanced interferometer. Indeed, the linewidth may be derived from the (suitably weighted) integral of the frequency noise spectrum over all fluctuation frequencies [6]. In many applications, such as interferometric sensing, it is the frequency noise spectrum, rather than linewidth, which is of primary interest.

It has been understood for some time that cavity length perturbations caused by equilibrium temperature fluctuations impose a lower limit on frequency noise in short cavity EDFL at room temperature [7]. The importance of fundamental thermal noise in practice is supported by experimental measurements showing DFB FL frequency noise close to the theoretical limit [1,7,8]. However, these measurements also indicate a dominant $1/f$ characteristic at low frequency that is evidently not consistent with conventional theory [8]. This $1/f$ noise persists over several decades from below 10 Hz up to several kHz. At frequencies beyond 10 kHz the noise spectrum appears consistent with well understood equilibrium thermal noise.

This inherent $1/f$ noise should not be confused with pump absorption dependent self-heating noise which is well known as a source of frequency noise in monolithic Nd:YAG lasers [9] and has also been cited as a source of line broadening in highly doped DFB FL [10]. Such noise does not have a $1/f$ spectrum. Also, in [8] it was found that $1/f$ laser frequency noise was independent of pump power and exhibited no discernible correlation with pump fluctuations. The transition from pump-independent $1/f$ noise to pump-dependent self-heating noise has recently been observed at high (980 nm) pump power in an erbium-doped DFB FL with moderate dopant concentration [11]. Another remarkable feature of $1/f$ frequency noise that rules out a pump related origin is that it exhibits no dependence on pump wavelength; i.e., pumping at 1480 or even 1530 nm yields essentially identical $1/f$ frequency noise as 980 nm pumping.

Recently, it has been proposed that nonequilibrium thermal fluctuations associated with radiative transitions (spontaneous emission) in the active gain medium introduce a fundamental $1/f$ contribution to the thermal noise spectrum that fully accounts for the dominant low-frequency noise [12]. According to this theory the spectrum of temperature fluctuations $S_T(f)$ within the laser cavity consists of a conventional equilibrium contribution $S_e(f)$ plus a nonequilibrium contribution due to the active gain medium $S_{\mathrm{Er}}(f) \propto 1/f$,

$$S_T(f) = S_e(f) + S_{\mathrm{Er}}(f). \quad (1)$$

The frequency noise spectrum $S_\nu(f)$ is derived from thermally induced cavity perturbations and is entirely characterized by $S_T(f)$,

$$S_\nu(f) = \nu_l^2 q^2 L_2^{-1} S_T(f), \quad (2)$$

where ν_l is the laser frequency, q is the thermo-optic coefficient and L_2 is a measure of laser mode length [8]. This theory makes a number of specific predictions about the behavior of $1/f$ frequency noise that have yet to be subjected to experimental test. In particular, the magnitude of the $1/f$ contribution should depend on the erbium ion concentration. Also, if the $1/f$ part of the spectrum is indeed of thermal origin it should exhibit strong temperature dependence. Specifically, one expects [12]

$$S_{\mathrm{Er}}(f) \propto \frac{N^2T^2}{f}, \tag{3}$$

where N is the ion concentration and T is temperature. Although one might reasonably expect the temperature dependence to be readily observable in the laboratory, verifying the N^2 dependence presents numerous technical challenges since it requires fabrication of an adequate sample of separate lasers, nominally identical except for N. A less ambitious task is to verify that the noise is of thermal origin [i.e., that Eq. (2) is true]. Such a result is highly significant in its own right since it is well known that it is extremely difficult to generate $1/f$ noise from conventional thermal fluctuations in a physical system [8,13,14]. Thus the confirmation of $1/f$ thermal noise implies a new, most likely nonequilibrium, process—which can only arise within the laser gain medium itself.

In addition to the strong temperature dependence, another distinguishing feature of thermal noise is that it is associated with cavity length perturbations. In instances where the cavity supports multiple modes the frequency noise of the modes should be correlated, since each mode sees essentially the same cavity. Such correlations are a useful discriminator between cavity noise and dynamical noise (such as conventional spontaneous emission noise [15]), since each mode is generally sustained by a separate ensemble of ions.

The aim of the current paper is to present direct experimental evidence of a thermal origin to fundamental $1/f$ frequency noise in DFB FL. In Sec. II we shall outline the necessary theoretical background. In Sec. III we shall present results of laser frequency noise measurements for a single frequency DFB FL. These demonstrate that the $1/f$ part of the spectrum exhibits the T^2 temperature dependence predicted in Eq. (3). In Sec. IV we show that T^2 temperature dependence is also manifest in $1/f$ polarization-beat-frequency (PBF) noise for a dual polarization mode DFB FL. This is interpreted as indicating T^2 dependence of $1/f$ laser frequency noise for each of the polarization modes. This interpretation is supported by comparison between PBF noise and direct measurements of laser frequency noise at room temperature. Substantial correlation between frequency noise of the two modes is observed, providing evidence of a common (thermal) origin. We conclude with a brief discussion in Sec. V.

II. THEORY AND DEFINITIONS

At any given point in space, the optical output of a single frequency laser may be expressed in the general form,

$$E(t) = A(t)\exp\{i[2\pi\nu_l t + \phi(t)]\} \tag{4}$$

where $A(t)$ and $\phi(t)$ are the slowly varying amplitude and phase, respectively. The instantaneous frequency is defined as the time derivative of the total phase (modulo 2π),

$$\nu(t) = \nu_l + \dot{\phi}(t)/2\pi = \nu_l + \Delta\nu(t), \tag{5}$$

where $\Delta\nu = \dot{\phi}/2\pi$ is the *frequency noise* and is assumed to be a stationary, zero-mean random variable. Frequency noise may be observed by injecting the laser output into an unbalanced interferometer and monitoring intensity fluctuations at the output (see Fig. 1). The statistics are characterized by the temporal correlation function,

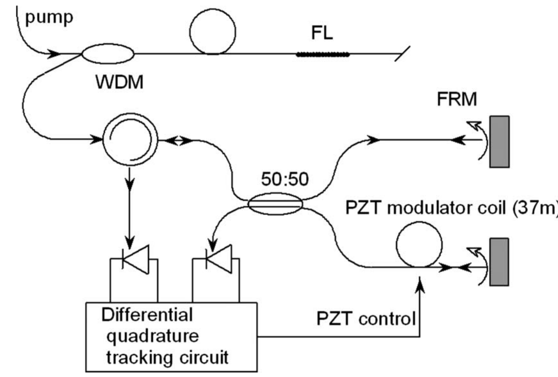

FIG. 1. The experimental setup used to measure frequency noise.

$$C_\nu(\tau) = \langle \Delta\nu(t)\Delta\nu(t+\tau)\rangle, \tag{6}$$

(where the angled brackets indicate the mean value) or its Fourier transform, the frequency noise spectral density function,

$$S_\nu(f) = \int_{-\infty}^{\infty} C_\nu(\tau)e^{i2\pi f\tau}d\tau. \tag{7}$$

$S_\nu(f)$ corresponds physically to the mean squared frequency deviation within a 1 Hz band centered at (fluctuation) frequency f. This interpretation follows readily from Eqs. (6) and (7) since

$$\langle \Delta\nu^2\rangle = C_\nu(0) = \int_{-\infty}^{\infty} S_\nu(f)df = \int_0^{\infty} 2S_\nu(f)df, \tag{8}$$

where we have used the inverse Fourier transform. The last equality follows from the symmetry of S_ν and is important in practice since laboratory spectral analyzers do not distinguish positive and negative frequencies and typically measure $2S_\nu(f)$—the so-called one-sided spectral density function.

An important means of characterizing the overall magnitude of frequency noise is the root-mean-square (rms) frequency deviation, defined as

$$\Delta\nu_{\mathrm{rms}} = \sqrt{\langle \Delta\nu^2\rangle} = \left[\int_0^{\infty} 2S_\nu(f)df\right]^{1/2}. \tag{9}$$

This quantity is a natural measure of the spectral purity of the laser line and is closely related (although not equivalent) to linewidth [6]. Strictly speaking both the linewidth and $\Delta\nu_{\mathrm{rms}}$ are ill defined for lasers with $1/f$ noise, since the integral (9) diverges in the limit $f\to 0$[22]. However, this problem may be avoided in practice by specifying the duration τ over which the frequency noise (or linewidth) measurement is taken, since the linewidth cannot contain contributions from fluctuations at frequencies lower than $f_c = 1/\tau$.

According to [12], the frequency noise in the absence of any external perturbations is dominated by thermal fluctuations and S_ν is related to the spectral density of temperature fluctuations by Eq. (2). As expressed in Eq. (1) the thermal noise is composed of an equilibrium and nonequilibrium contribution. For completeness we state here the explicit theoretical expressions for S_e and S_{Er},

$$S_e(f) = \frac{KT^2}{2\pi\kappa_t}\mathrm{Re}[e^{ik_1^2a^2/2}E_1(ik_1^2a^2/2)], \quad (10)$$

where $E_1(.)$ is the exponential integral function [16], $K \simeq 1.38\times10^{-23}$ J/K is Boltzmann's constant, a is the radius of the fiber mode field and $k_1(f)=\sqrt{\pi f c_v/\kappa_t}$, where $\kappa_t \simeq 1.37$ W/mK is the coefficient of thermal conductivity for silica and $c_v \simeq 1.67\times10^6$ J/m^3 K is the specific-heat capacity (per unit volume) of silica. This rather complicated looking distribution was first given in [8]. It is qualitatively similar to an earlier expression for the thermal fluctuation spectrum given by Wanser [17], but contains fewer parameters and differs by 3 dB in the high-frequency limit.

The nonequilibrium contribution is predicted to take the approximate form [12],

$$S_{\mathrm{Er}}(f) \simeq \frac{(a_l KTN\lambda^2 s_1)^2}{128\pi^2\Delta\lambda(a_l+g_l)^2\tau\kappa_t c_v f}, \quad (11)$$

where a_l and g_l are the absorption and emission coefficients of the gain medium, respectively, N is the active ion concentration, λ is the nominal laser wavelength, $\Delta\lambda$ is the emission bandwidth, τ is the spontaneous emission lifetime, and s_1 is a dimensionless parameter of order unity. Evidently, S_{Er} represents a fundamental noise mechanism closely tied to the laser emission process. The quantum origin of the noise is implicit from the presence of the spontaneous emission lifetime τ in Eq. (11).

Note from Eqs. (10) and (11) that both the equilibrium and nonequilibrium contributions to thermal noise exhibit a T^2 dependence. Thus, not withstanding the weak temperature dependence of various material parameters, the laser frequency noise would be expected to vary with temperature as T^2.

To conclude this section, consider a dual polarization, single longitudinal-mode laser. The only difference between this and the single frequency case is that the laser output is now composed of two orthogonally polarized fields E_j, $j=1,2$ such that

$$E_j(t) = A_j(t)\exp\{i[2\pi\nu_j t + \phi_j(t)]\}. \quad (12)$$

The frequency noise for each of the two modes is defined as for a single mode laser (with ϕ_j replacing ϕ) and may readily be measured by arranging a polarizer at the laser output so as to extinguish one or other of the modes and then proceeding with interferometric phase demodulation as usual. The frequency noise characteristics of each of the modes should be essentially similar to that of a single mode laser. However, it is possible to extract additional information about the origin of the noise by examining the correlation statistics *between* modes. In particular, for noise of thermal origin we expect a strong correlation due to the common cavity.

It is possible to obtain information about frequency noise noninterferometrically by scrambling the polarization at the output of the laser and beating the two modes together at a photodetector. The measured intensity is then given by

$$I = A_1^2 + A_2^2 + 2A_1A_2\cos\{2\pi\nu_{12}t + [\phi_1(t) - \phi_2(t)]\}, \quad (13)$$

where $\nu_{12}=\nu_1-\nu_2$ is the mean polarization-beat frequency (PBF) and is assumed to be in the MHz–GHz range. The polarization-beat-frequency noise is defined as

$$\Delta\nu_b(t) = (2\pi)^{-1}[\dot{\phi}_1(t) - \dot{\phi}_2(t)] = \Delta\nu_1(t) - \Delta\nu_2(t). \quad (14)$$

In other words, the PBF noise is equal to the difference in frequency noise between the dual polarization modes. Assuming that the noise in each mode is of similar origin, the temperature dependence of the PBF noise should therefore follow that of the laser frequency noise. Calculating the spectrum of PBF noise it is readily verified that

$$S_{\nu_b}(f) = S_{\nu_1}(f) + S_{\nu_2}(f) - 2S_{12}(f), \quad (15)$$

where

$$S_{12}(f) = \int_{-\infty}^{\infty}\langle\Delta\nu_1(t)\Delta\nu_2(t+\tau)\rangle e^{i2\pi f\tau}d\tau \quad (16)$$

is the cross-spectral density, from which may be derived the coherence function,

$$\gamma(f) = \left[\frac{|S_{12}(f)|^2}{S_{\nu_1}(f)S_{\nu_2}(f)}\right]^{1/2}. \quad (17)$$

III. TEMPERATURE DEPENDENCE OF DFB FL LASER FREQUENCY NOISE

In this experiment a 5 cm long single frequency erbium-doped DFB FL with wavelength 1549 nm was used. The mode field diameter was approximately 4 μm and the effective mode length parameter L_2 was approximately 1 cm. The peak absorption at 980 nm was approximately 10 dB/m. The erbium ion concentration was not known precisely but was certainly less than 1000 ppm. In our modeling we assumed a nominal erbium ion concentration of 10^{19} cm^{-3} (about 190 ppm). The fiber cladding had a diameter of 125 μm and was coated with an acrylate buffer out to 250 μm. The pump wavelength was 980 nm.

The laser frequency noise was measured from 10 to 10^5 Hz using an acoustically isolated unbalanced Michelson interferometer with a 74 m path imbalance (i.e., a 37 m delay coil in 1 arm) as shown in Fig. 1. A piezoelectric stretcher in one arm, driven by low-frequency feedback electronics at the photodiode output, kept the mean ($\leqslant$10 Hz) phase difference between the arms locked on quadrature. The active homodyne configuration was used because it is relatively simple and direct and eliminates the possible issue of RF noise associated with heterodyne phase modulation methods. Even so, any possibility of the homodyne measurement system injecting noise into the system was carefully examined and ruled out. Faraday rotating mirrors were used in the interferometer to eliminate polarization fading effects.

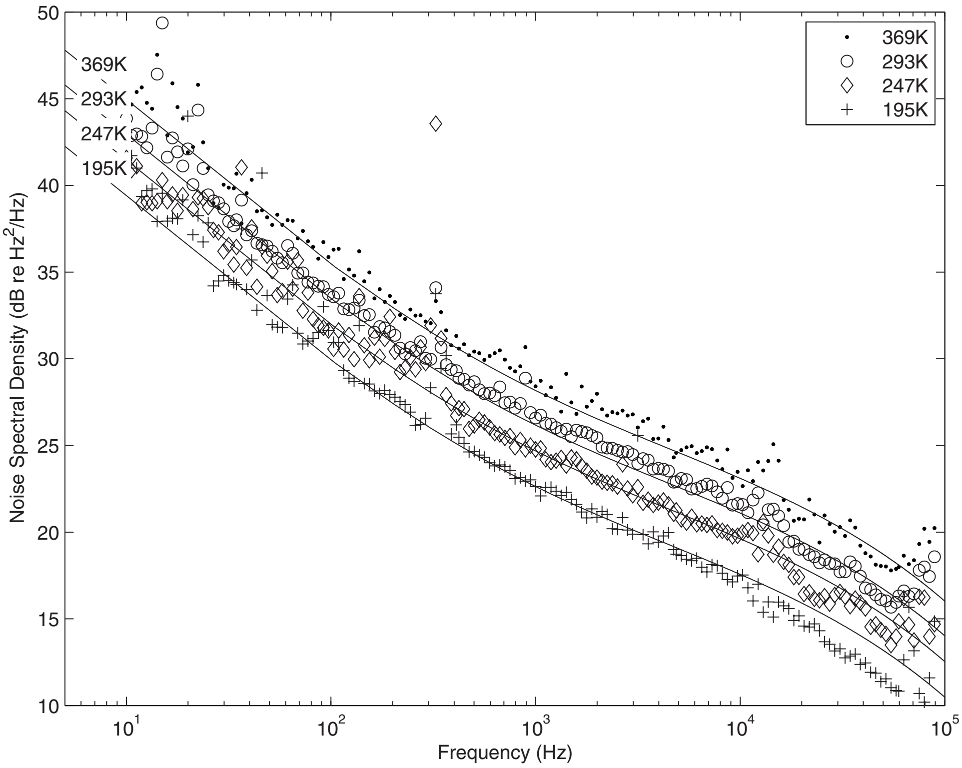

FIG. 2. Experimental one-sided DFB FL frequency noise $[2S_\nu(f)]$ data measured at 369 K ("·"), 293 K ("○"), 247 K ("◇") and 195K ("+"). Corresponding theoretical curves are shown as solid lines.

The laser was placed in an insulated chamber purged with vapor from a liquid nitrogen reservoir. This enabled the temperature of the laser to be cooled from room temperature (293 K) to −110 °C (163 K). The rate of flow of N_2 vapor (and therefore the temperature of the chamber) could be controlled by adjusting the current passing through a heating element submerged in the liquid N_2. A slightly modified arrangement was used to heat the laser up to 96 °C (369 K).

The results of frequency noise measurements made at four different temperatures spanning the range 369—195 K are shown in Fig. 2. These data are in excellent agreement with corresponding theoretical curves derived using Eqs. (10) and (11). Note the dominant $1/f$ behavior at low to intermediate frequencies due to the contribution of S_{Er}. Note also that each curve is equivalent modulo a constant factor T^2. The predicted temperature dependence is clearly evident across a temperature range of over 170 K.

Additional data taken at 171 and 163 K are similar to that at 195 K and has been omitted from Fig. 2 to avoid clutter. The lack of strongly evident temperature dependence below 195 K suggests a "bottoming out" of the frequency noise which may be due to limitations of the measurement procedure or changes to the material (and optical) properties of the fiber at very low temperature. The thermo-optic coefficient is dominated by the value of dn/dT at room temperature, which is fairly constant (albeit with a weak quadratic dependence [18]) over the range 379–200 K but decreases at lower temperatures to around half its room temperature value by 100 K [19]. Perhaps more significantly, the thermal expansion coefficient of silica changes dramatically at low temperature and actually passes through zero (and changes sign) somewhere between 150 and 200 K [20]. Evidently the material properties of silica undergo a significant transition in this domain and it is not reasonable to extrapolate the expected T^2 temperature dependence to temperatures below 200 K. Note that the lack of strong temperature dependence below 195 K was apparent at all frequencies, including above 10 kHz where conventional equilibrium thermal noise is believed to dominate.

Figure 3 shows normalized estimates of the rms frequency deviation (obtained by summing the measured noise data over all frequencies) verses temperature. As can be seen from Fig. 2, the frequency data were adversely affected by mechanical resonances below 60 and at 300 Hz. Data below 60 Hz, and in the range 250–350 Hz was therefore excluded from all estimates. The best estimates of $\Delta\nu_{\mathrm{rms}}$, obtained by summing over the maximum 10^5 Hz measurement bandwidth, are shown as boxes (labeled $\Delta\nu_5$). The additional points labeled $\Delta\nu_4$ (crosses) and $\Delta\nu_3$ (circles) correspond to partial sums of noise data up to 10^4 and 10^3 Hz, respectively. To enable direct comparison of temperature dependence, each of the three different estimates has been normalized to unity at the maximum temperature (369 K).

The reason for including the partial sums is that $\Delta\nu_4$ and $\Delta\nu_3$ are dominated by $1/f$ noise, whereas the total frequency deviation $\Delta\nu_5$ is dominated by the equilibrium thermal noise $S_e(f)$. The outstanding coincidence of these three estimates between 369 and 195 K indicates that the shape of the spectrum is remaining invariant with temperature and, importantly, that the $1/f$ and "thermal" part of the spectrum have, essentially, identical temperature dependence.

By Eq. (9), $\Delta\nu_{\mathrm{rms}}$ is predicted to vary linearly with temperature if the spectrum S_ν is proportional to T^2. The frequency deviation estimates plotted in Fig. 3 exhibit excellent linearity between 369 and 195 K and are in strong agreement with the exact linear curve $y=T/369$ (shown as a solid line).

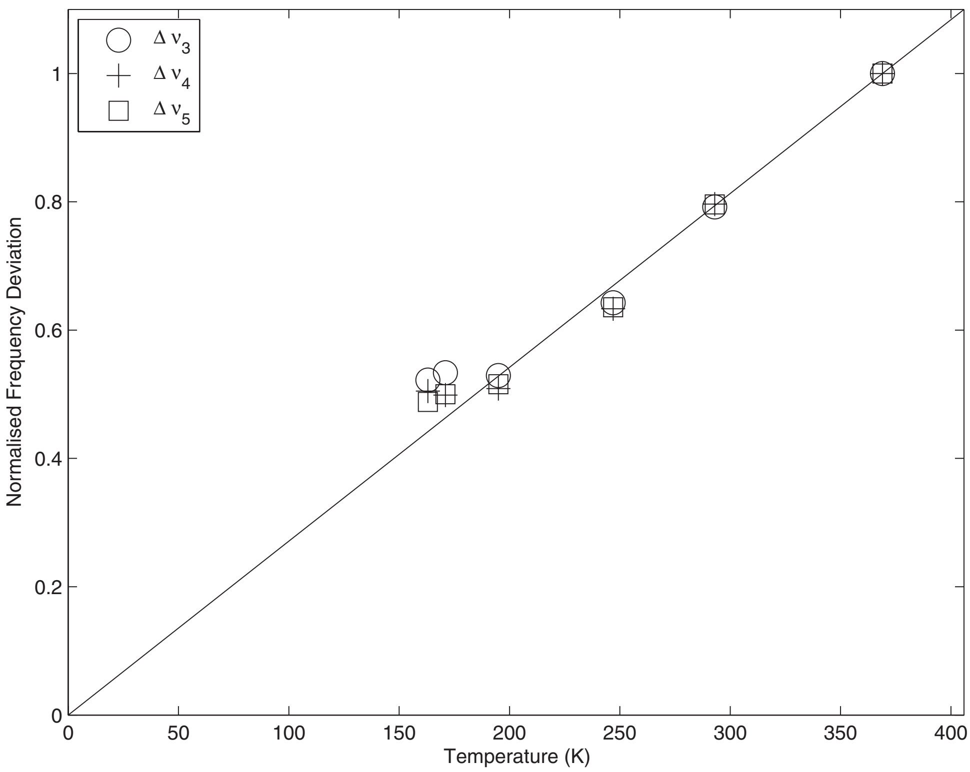

FIG. 3. Estimated rms laser frequency deviation verses temperature (normalized to unity at 369 K): $\Delta\nu_5$ ("□") includes noise contributions up to 10^5 Hz; $\Delta\nu_4$ ("+") includes contributions up to 10^4 Hz; $\Delta\nu_3$ ("○") includes contributions up to 10^3 Hz. Noise contributions below 60 Hz and between 250 and 350 Hz have been excluded.

The previously discussed rolling off of the temperature dependence below 195 K is also evident.

The standard error in $\Delta\nu_5$ over repeated measurements was less than 0.1 dB (corresponding approximately to the diameter of the markers in Fig. 3) for all measurement temperatures.

IV. FREQUENCY NOISE IN DUAL POLARIZATION MODE DFB FL

Further evidence for the thermal origin of $1/f$ noise has been obtained from a separate set of experiments carried out with a dual polarization DFB FL. These measurements were carried out earlier than those described in Sec. III and provided the first direct support for the conjecture of strong temperature dependence in DFB FL frequency noise. The aim of the first experiment was to establish the degree of correlation between the frequency noise of the individual polarization modes of the fiber laser and the PBF noise as implied by Eq. (15). The experimental setup used to characterize the polarization resolved frequency noise and PBF noise is shown in Fig. 4.

As with the previous measurement the fiber laser was pumped at 980 nm. An acoustically isolated fiber-optic Michelson interferometer formed with a 3×3 coupler with negligible polarization dependent loss, incorporating a fiber path imbalance of 222 m was used to interrogate frequency noise. In contrast to the configuration shown in Fig. 1, a sinusoidal modulation was applied to a piezoelectric stretcher placed in one arm of the interferometer in order to permit implementation of phase-generated carrier demodulation (PGC) [21]. This technique permits the phase of the interferometer to be tracked and requires only one output of the interferometer. Faraday rotation mirrors placed on each interferometer arm ensure that the output polarization state for each laser polarization mode is exactly orthogonal to the input polarization state for the corresponding mode. Orthogonality of the individual polarization modes is thus maintained in transit through the interferometer. The output of the interferometer was injected into a polarizing beam splitter (PBS), prior to detection on photodetectors. To confirm that the two laser polarization modes were orthogonal at the input of the PBS, one output of the PBS was injected into a scanning fiber Fabry-Pérot, with a finesse sufficiently high to resolve the two polarization modes. By adjusting the input polarization to the PBS with a polarization controller (PC), it was possible to reduce the relative power of each polarization mode by greater than a factor of 10, confirming that the two modes were nominally orthogonal. This also serves as

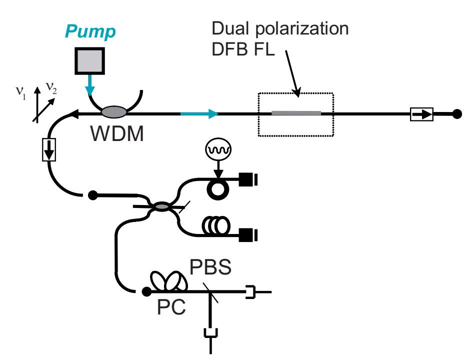

FIG. 4. (Color online) Experimental setup for measuring the polarization resolved frequency noise and temperature dependence of the polarization-beat frequency in a dual-polarization DFB fiber laser

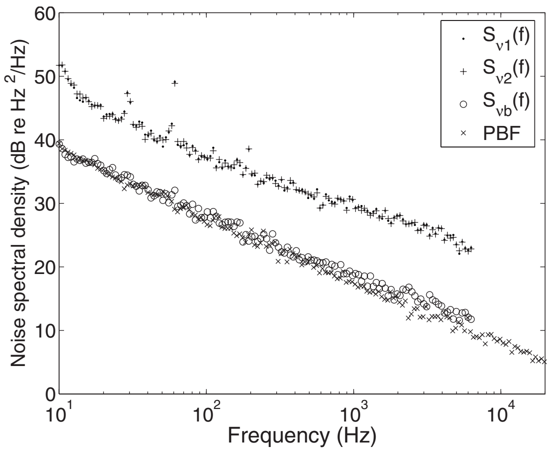

FIG. 5. Polarization resolved (one-sided) frequency noise spectra of a dual-polarization DFB fiber laser, labeled S_{ν_1} and S_{ν_2} for the respective polarization modes. Also shown is the difference between the two spectra (S_{ν_b}) and the polarization-beat-frequency noise measured directly (PBF).

the setup procedure for a polarization resolved frequency noise measurement.

To measure the polarization-beat-frequency noise directly, the interferometer was removed from the setup and the laser output injected directly into the PBS and amplified with an erbium-doped fiber amplifier. The amplified output of the PBS was detected on a high-bandwidth photodetector yielding a beat frequency of $\simeq 28$ MHz, which was mixed with a stable oscillator to yield a difference frequency of approximately 100 kHz and sampled with an analog to digital converter. The instantaneous frequency of this signal may be calculated from the time derivative of the instantaneous phase of its analytic signal, which is calculated using the Hilbert transform.

The one-sided spectral density of the frequency noise spectra, S_{ν_1} and S_{ν_2}, for the two polarization modes are shown in Fig. 5. Also shown is the spectral density of the difference in frequency noise, $S_{\nu_b}(f)$, between the two modes along with the spectral density of the PBF noise measured directly. The ordinary coherence function γ [see Eq. (17)] between the frequency noise for each mode was measured to be between 0.75 and 0.8 over the frequency range 10 Hz to 10 kHz, indicating a high degree of correlation of the frequency noise between the two modes. This is confirmed by the spectral density of the difference of the frequency noise, $S_{\nu_b}(f)$, falling approximately 9 dB (± 1 dB) below the frequency noise of the two modes at 100 Hz. The spectral density of the PBF noise measured directly is found to overlap very closely with the differential frequency noise, $S_{\nu_b}(f)$, indicating that the origin of the noise causing laser frequency noise and PBF noise is most likely closely related. This behavior was also found for two other lasers fabricated using different Er^{3+} doped fibers. A slight deviation in the differential polarization mode frequency noise and the PBF noise measured directly is observed at frequencies greater than 1 kHz. This is believed to be an artifact of the PGC interrogation scheme associated with noise aliasing due to the limited measurement bandwidth causing the calculated differential polarization noise to increase slightly.

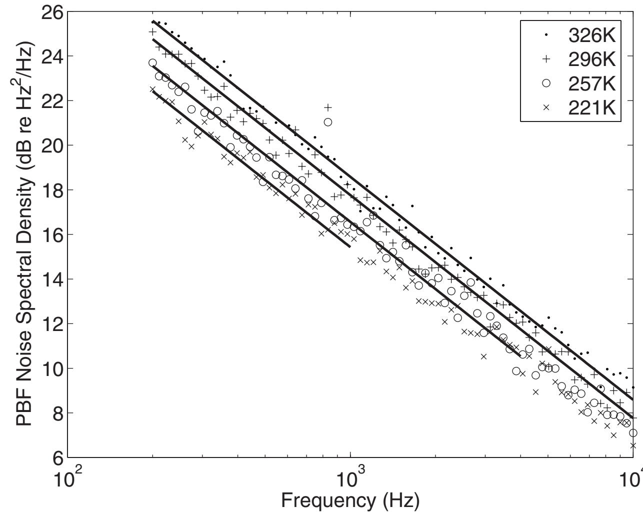

FIG. 6. One-sided power spectra density of the polarization-beat-frequency noise vs temperature.

Both the PBF noise and the frequency noise spectra conform to a $1/f$ curve over the (6 kHz) range that they are directly comparable. Interestingly, the PBF noise appears to be dominated by a $1/f$ characteristic over the full 20 kHz measurement band. This is curious since the plots in Fig. 2 suggest that one might expect a flattening off of the spectrum above 1 kHz out to about 20 kHz (beyond which it rolls off as $1/f^2$). It is possible that the shape of the PBF noise spectrum deviates from that of the frequency noise at high frequencies, however, the current data are insufficient to draw any firm conclusion.

Having established a likely equivalence between the difference frequency noise of the laser and PBF noise, the temperature dependence of the PBF noise was characterized using a similar method to that described in Sec. III, by placing the laser in a chamber purged with vapor from a liquid nitrogen reservoir. The low-frequency limit of the measurement was determined by the time over which the PBF is sufficiently stable to allow an accurate measurement. The temperature of the laser was varied over the range 170–326 K by varying the flow rate of the nitrogen vapor. Figure 6 shows the PBF spectral density vs temperature. Measurements taken below 221 K have been omitted since below this temperature the PBF was not sufficiently stable to permit an accurate measurement and caused the PBF spectra to exhibit anomalously high levels of excess noise. The exact cause of this increased instability was not determined and may be experimental in origin or possibly caused by changes in the properties of the optical fiber at low temperatures. However, the temperature range of the presented data is considered to be sufficiently representative.

A nonlinear least-squares curve fit of the empirical function, $S_{\nu_b}(f) = af^{-1}$ has been applied to each data set in Fig. 6 using the Levenberg-Marquardt algorithm. The range of the fit to the data sets taken at 221 and 257 K was restricted to frequencies less than 1 and 3 kHz, respectively, since above these frequencies the data sets exhibited slightly elevated noise levels believed to be experimental in origin. These fits are also shown in Fig. 6. The dependence of the scaling

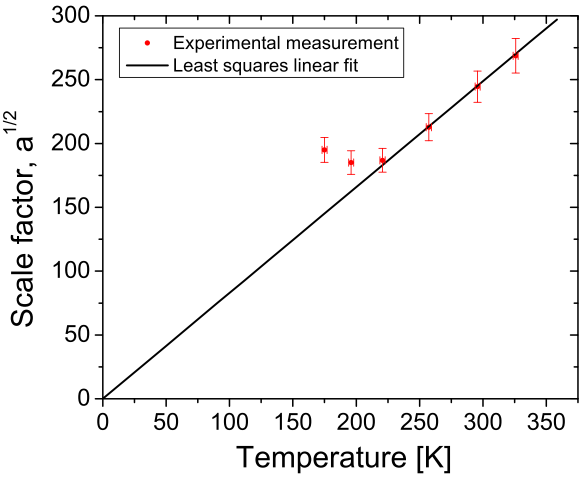

FIG. 7. (Color online) Scale factor $a^{1/2}$ obtained from a nonlinear curve fit of polarization-beat-frequency noise vs temperature.

factor on temperature is shown in Fig. 7. A linear fit of the data points between 221 and 326 K clearly shows the linear dependence of the square root of the PBF noise spectrum on temperature as predicted by Eq. (3) for temperatures above 221 K. Data taken below 221 K imply invariance to temperature, but as described above this is thought likely to be experimental in origin.

V. CONCLUSIONS

We have observed theoretically predicted T^2 temperature dependence in DFB FL $1/f$ laser frequency noise using both a direct interferometric phase demodulation technique and noninterferometric measurements of polarization-beat-frequency noise. T^2 temperature dependence is strongly characteristic of fundamental thermal fluctuations. However, it has previously been shown that wide bandwidth $1/f$ noise is almost certainly not consistent with conventional equilibrium thermal noise in optical fiber lasers [8]. On this basis we propose that these observations provide evidence of thermal fluctuations generated by nonequilibrium process within the laser gain medium. Since the noise is not related to pump absorption and is independent of pump wavelength we are led to conclude that it is associated with relaxation processes from the laser excited state. These results therefore provide strong support for the theory proposed in [12]. Although the experimental evidence does not yet constitute conclusive proof of this theory it is very difficult to conceive of an alternative explanation which explains all of the observed properties of the noise.

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[22] Such divergence is actually unphysical in the current context, where the $1/f$ noise is (apparently) thermal, since it implies infinite energy fluctuations. In a system of finite extent, there must be a fundamental physical cutoff frequency below which the noise rolls off, however, this limit may not be measurable in practice.