Observation of Landau-Zener tunneling through atomic current in the optical lattices

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The atomic current in the Fourier-synthesized optical lattices under a constant external force is investigated theoretically. Based on a two-band model, the atomic current is derived by solving the Boltzmann equations. We find that the stationary atomic current changes with the probability of Landau-Zener tunneling, depending on the adjustable energy structure of the optical lattices. In contrast to the classical results of an electron in superlattices given by the Esaki-Tsu equations, the relation between the stationary atomic current and the strength of the external force in optical lattices is modified significantly. Both these characteristics can be taken as an effective way to observe the Landau-Zener tunneling in the optical lattices.

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I. INTRODUCTION

Landau-Zener tunneling is an important quantum phenomenon and has attracted lots of interests since it was predicted [1]. Originally, Zener calculated the tunneling rate of an electron from the valence band to the conduction band due to an external electric field. However this process is actually a kind of nonresonant tunneling [2], and the tunneling rate is too small for electrons in usual crystal lattices because of the large energy-band gap (in the regime of several eV's). Although the situation in semiconductor superlattices is improved notably (the miniband gap goes down to the order of meV), it is still demanding for the strength of the external field. In these years, the achievements in the atom cooling technique and the accelerating optical lattices provide a great platform [3] to demonstrate the Landau-Zener tunneling [4–8]. The energy-band gap of the optical lattices reaches down to several peV's, so that the atom could jump easily from one band to the other by the Landau-Zener tunneling with the help of the external force, such as its gravity. Moreover, the energy-band gap is adjustable in experiment by the relative phase of these two bichromatic lasers constructing the optical lattices [9]. Thus the Landau-Zener tunneling could be well controlled.

Then one expects to find a measurable macroscopic quantity to observe the controllable Landau-Zener tunneling. Atomic current is undoubtedly a good choice for its clear physical meaning and possible direct measurement in optical lattices [10]. A cold atom driven by the external force in optical lattices behaves like Bloch oscillation [11,12] in one band and produces oscillatory atomic current. Landau-Zener tunneling makes it possible to elevate the atom up to the excited band. Then the so-called Bloch-Zener oscillation happens [13,14]. Both intraband and interband behaviors of the atom have an influence on the total atomic current. In the absence of damping factors the long time average atomic current is zero. However in real systems there always exist some kinds of damping factors, such as the spontaneous emission [15,16] and collisions [17]. With the help of damping factors, the atomic current would approach a net nonzero value after a long time and therefore transport is possible [18]. The atomic current through the optical lattices without Landau-Zener tunneling has been studied in Ref. [19].

In this paper we get the information of the Landau-Zener tunneling from the stationary atomic current. The objective is to find the relation between the stationary atomic current I and the probability of the landau-Zener tunneling, which could be adjusted by the relative phase of the two bichromatic lasers or the external force F. Based on a two-band model, we give the formula for the atomic current by solving the Boltzmann equations. The stationary atomic current is calculated with the realistic parameters of the Fourier-synthesized optical lattices. Numerical results show that the stationary atomic current increases with the decreasing energy-band gap tuned by the relative phase. For a typical value of relative phase, the I-F curves are found to be modified significantly in comparison with its counterpart, the I-V (V is the external voltage) curve of electrons in the semiconductor superlattices, which was first introduced in the famous work done by Esaki and Tsu [20]. We find the Landau-Zener tunneling may produce more than one Easki-Tsu peak in particular parameters. All these results provide an effective way to observe the Landau-Zener tunneling in optical lattices.

II. MODEL AND SOLUTION

For an atom in the optical lattices, it could jump from one energy band to another by the Landau-Zener tunneling. Since all the gaps between excited energy bands are almost zero and controllable Landau-Zener tunneling mainly takes place between the two lowest energy bands [9,21] in the optical lattices, we model the system as a two-band one to manifest the influence of Landau-Zener tunneling on the atomic current. Assume that the lattice constant is d and the energy dispersions for band j ($j=1,2$) are $\epsilon_j(q)$, respectively, where q is the momentum along the direction of periodic lattice. Due to the periodicity of the potential, we have the Fourier transformation of the energy dispersion as

$$\epsilon_j(q) = \sum_{n=0}^{\infty} R_n^j \cos(nqd) \quad (j=1,2), \tag{1}$$

where R_n^j are the Fourier coefficients of the energy dispersion of jth band. The atomic velocity is obtained correspondingly as

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$$v_j(q) = \frac{1}{\hbar}\frac{\partial \epsilon_j(q)}{\partial q}. \tag{2}$$

The distribution function $f_j(q,t)$ $(j=1,2)$ of the states satisfies the Boltzmann equations

$$\frac{\partial f_{jq}}{\partial t} + \frac{Fd}{\hbar}\frac{\partial f_{jq}}{\partial q} = -g_1^j(f_{jq} - f_{jq}^0) + \Gamma(f_{\bar{j}q} - f_{jq}), \tag{3}$$

where $j=1,2$ and $j \neq \bar{j}$, f_{jq}^0 represents the equilibrium distribution in band j, and g_1^j is the jth band's damping rate to the equilibrium state. The last term on the right-hand side of the above equations is the change in the distribution due to the Landau-Zener tunneling. As we all know, the atom, supposed to be in the low energy band, would behave like Bloch oscillation as driven by the external force in the absence of Landau-Zener tunneling. In every time period of the Bloch oscillation T_{BO}, it will run one time across the Brillouin zone and pass through the point where the band gap lies (usually at $q=0$). When the Landau-Zener tunneling takes effect, it will have a certain probability to tunnel to the upper energy band, which leads to the redistribution of the population located in each band. The Landau-Zener tunneling probability can be estimated to be $e^{-a_c/a}$, where $a_c = d\Delta^2/(4\hbar^2)$, a is acceleration, and Δ is the energy-band gap. The exponential function implies that the Landau-Zener tunneling could be assumed to happen just at point $q=0$. Therefore we have the formula

$$\Gamma = e^{-a_c/a}\delta(q)/T_{BO}. \tag{4}$$

Here we consider the ultracold atoms without interactions. It is known that the interactions between atoms can be tuned by Feshbach resonance (see Ref. [22] and referred experiments therein) and the nonlinear term can be viewed as an effective potential for the weak interaction when the interaction energy is small compared to the lattice depth. Since our formalism is applied for general lattice potential, the interaction is partially included in the effective optical potential through parameters R_n as long as the nonlinear interaction is too weak to produce the loop structure of energy band [23]. Moreover, as we introduce the damping factor in the Boltzmann equations, the interactions between the atoms could be taken as one kind of mechanism contributing to the damping factor g_1 and free of more detailed investigation here.

For the periodicity of the energy dispersion, the distribution function can also be expanded in the form of a Fourier series,

$$f_j(q,t) = \sum_{n=-\infty}^{\infty} f_n^j(t) e^{-inqd}, \tag{5}$$

where $j=1,2$ and i is the imaginary unit. Then the Fourier transforms $f_n^j(t)$ satisfy the equations

$$\frac{df_n^j}{dt} = in\frac{Fd}{\hbar}f_n^j - g_1^j(f_n^j - f_n^{j0}) + e^{-a_c/a}\sum_m (f_m^{\bar{j}} - f_m^j)/T_{BO}, \tag{6}$$

where f^{j0} is the Fourier transformation of equilibrium distribution f_{jq}^0. Finally we can express the transient current as

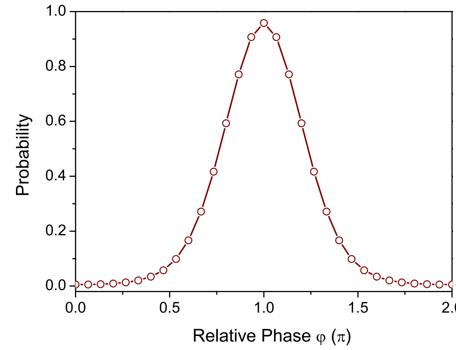

FIG. 1. (Color online) The Landau-Zener tunneling probability in the optical lattices adjusted by the relative phase φ.

$$j(t) = \int v(q)f(q,t)dq, \tag{7}$$

$$= -i\frac{\pi d}{\hbar}\sum_j \sum_{n=1}^{\infty} nR_n^j[f_n^j(t) - f_{-n}^j(t)]. \tag{8}$$

The atomic current would get stationary after a long time because of the damping factor. The stationary atomic current is then defined as

$$I = \lim_{t\to\infty} j(t). \tag{9}$$

III. RESULTS AND DISCUSSION

In the Fourier-synthesized optical lattices, the potential felt by the atom is

$$V(z) = \frac{V_1}{2}\cos(2kz) + \frac{V_2}{2}\cos(4kz + \varphi), \tag{10}$$

where V_1 and V_2 are the potential depths of two lattice harmonics, respectively, and φ is the relative phase. The spatial period of the optical lattice is $d=\lambda/2$, where $\lambda=2\pi/k$.

We use the following parameters: ^{87}Rb atoms, $\lambda=800$ nm, $V_1=4E_r$, and $V_2=1.2E_r$, where E_r is the recoiling energy. In this paper, we use E_r and λ as the units of energy and length, respectively. Then the unit of atomic current is E_r/h, where $h=2\pi\hbar$. The external force is added by tilting the lattices and so the acceleration of the atom is $a=g\cos\alpha$, where g is the gravity acceleration and α is the angle between the direction of the optical lattices and the gravity. The structure of energy band depends on the relative phase; therefore, one can tune the Landau-Zener probability by adjusting the relative phase as shown in Fig. 1. We can see that the relative phase is really an effective way to control the Landau-Zener tunneling probability, varying from almost 0 to almost 1.

When the damping factor is taken into account, the atom in either band would oscillate in the external field and jump from one band to the other with certain probability due to the Landau-Zener tunneling. The Landau-Zener tunneling plays different roles in the process of Bloch-Zener oscillation with different φ. We choose a typical value $\varphi=0.75\pi$ to demon-

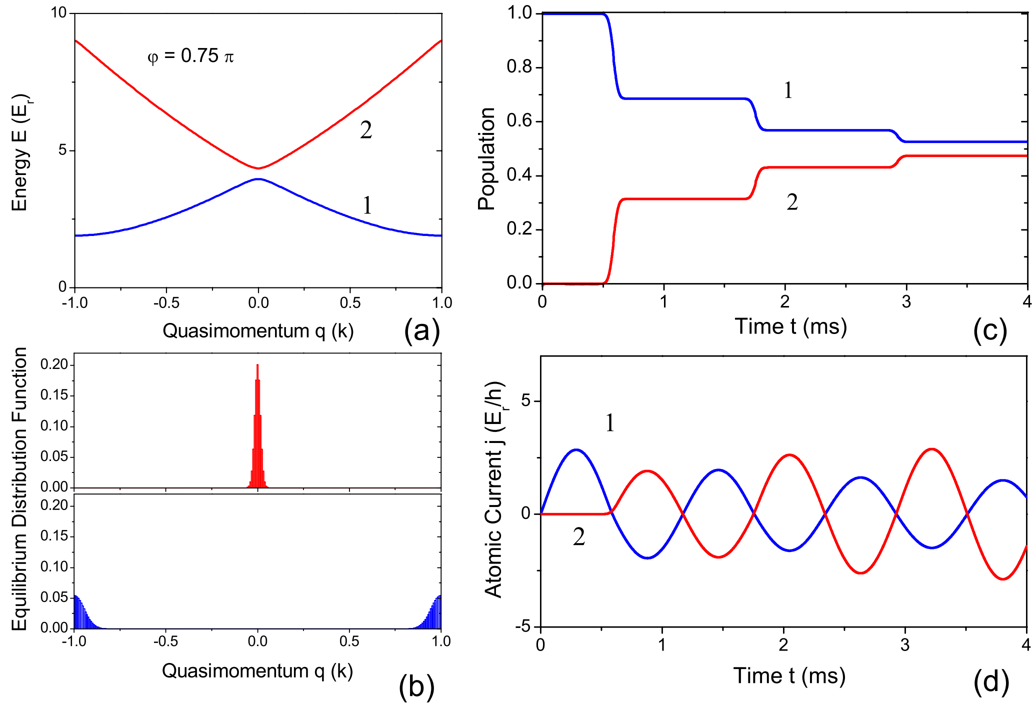

FIG. 2. (Color online) Demonstration of the atomic current in the Bloch-Zener oscillation. (a) Energy dispersion of the atom in the optical lattices with φ=0.75π. The lowest two bands are labeled with 1 and 2, respectively. (b) The equilibrium distribution functions of band 1 (lower) and band 2 (upper). (c) The time evolutions of the populations in bands 1 and 2, respectively. (d) The time evolutions of atomic currents contributed from bands 1 and 2, respectively.

strate the Bloch-Zener oscillation and its influence on the atomic current. Figure 2(a) gives the energy dispersion and Fig. 2(b) is the corresponding equilibrium distribution function for each band, respectively. The atom is supposed to stay in band 1 in equilibrium at the initial time. As the external force is tuned on, the atom population and the atomic current contributed from each band would evolve with time as plotted in Figs. 2(c) and 2(d), respectively. In Fig. 2(c), we see that the population in band 1 decreases while that in band 2 increases until they get almost equal. The stepwise jumping matches the Bloch oscillation period T_{BO}. In Fig. 2(d), the atomic current contributed from each band demonstrates the characteristic of Bloch oscillation. The changing amplitude of the oscillation means the variation in the population in each band, which proves the existence of Landau-Zener tunneling.

When the damping factor is considered, the net atomic current would not be zero any more after a long time. On one side, the external force drives the atom to perform Bloch oscillation; on the other side, the damping factor pulls the atom back to the equilibrium state. Finally, when the system gets stationary, there is a net atomic current. A typical evolution of the atomic current with time as the damping exists is shown in the inset of Fig. 3. As the stationary atomic current is an easily measured quantity in experiment, we could connect it with the relative phase φ and hence with the probability of the Landau-Zener tunneling. In other words, the Landau-Zener tunneling can be observed by measuring the stationary atomic current with the variation in φ. Figure 3 shows the relation between the stationary atomic current and the relative phase φ. For φ=0 the Landau-Zener probability is almost zero, so the atom mainly stays in one band (band 1). With φ increasing, the gap of energy band decreases and the probability for the atom to tunnel into the upper band (band 2) increases correspondingly. In this process the weight of the contribution to the stationary atomic current from band 2 increases gradually. The total stationary atomic current increases simultaneously because the atom in band 2 has a bigger contribution to the stationary atomic current than that of band 1 due to their different energy-band structures. With φ increasing further, the Landau-Zener tunneling

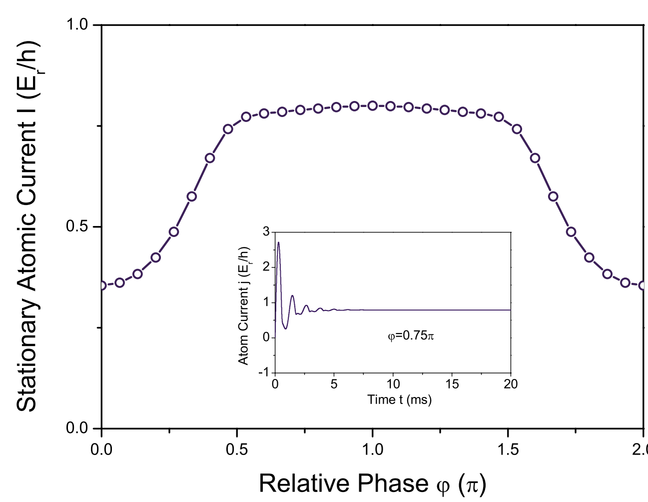

FIG. 3. (Color online) The stationary atomic current with the variation in φ in the optical lattices with the damping factor considered. The damping rate is $0.02E_r$. Inset is the time evolution of the atomic current at a typical value of φ=0.75π.

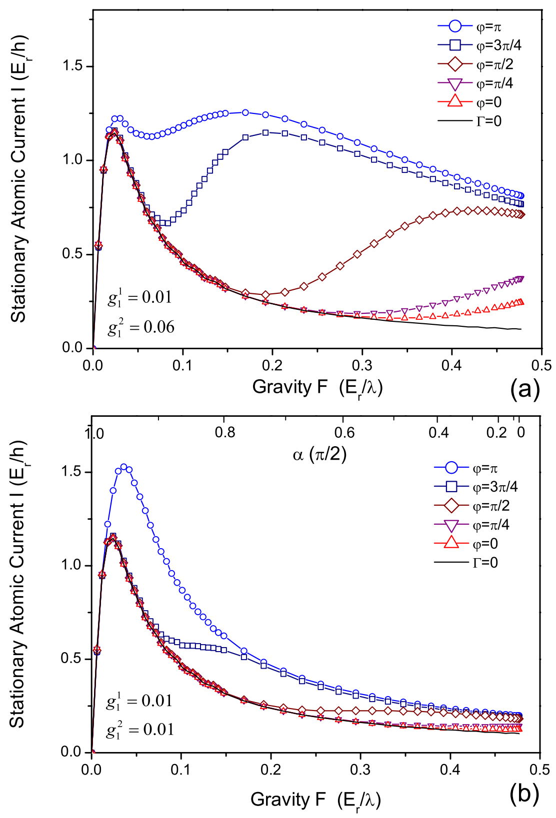

FIG. 4. (Color online) The stationary atomic current with the variation in the gravity for different values of φ and the damping rates (a) $g_1^1=0.01E_r$ and $g_1^2=0.06E_r$; (b) $g_1^1=g_1^2=0.01E_r$. The gravity is adjusted by changing the angle α. Because the value of φ can always be transferred to a corresponding value in the interval $[0,\pi]$ with the optical lattices unchanged, we only consider the values of φ in the interval.

probability becomes large enough to mix two bands before damping factor takes effect and consequently results in almost equivalent atom population between two bands. Once the populations get unchanged between two bands, the stationary atomic current reaches maximum.

Besides the energy-band gap, the Landau-Zener tunneling probability is also dependent on the strength of the external force F. Then how does the I-F curve change in the presence of the Landau-Zener tunneling? If the relative phase φ is so small that the Landau-Zener tunneling could be neglected, the relation of the stationary atomic current to the external force should have the same characteristics as the I-V curve for electrons in single band in semiconductor superlattices [20]. When the Landau-Zener tunneling begins to play a role, however, the stationary atomic current would change due to the joined contribution from band 2. The modification of the external force F is usually realized by adjusting the component force of gravity $F=mg\cos\alpha$ in experiment. The calculated stationary atomic currents are plotted with the variation in angle α for different values of φ in Fig. 4. In Fig. 4(a), we set the damping rate as $g_1^1=0.01E_r$ and $g_1^2=0.06E_r$. It is shown that, when $\varphi=0$, the I-F curve has almost the same shape as that in the absence of Landau-Zener tunneling, meaning that the behavior of the atom is almost determined by the single band. But the difference at the tail of the curve indicates the existence of Landau-Zener tunneling. The role played by the Landau-Zener tunneling becomes more and more obvious with the increasing value of φ. Finally there appears an additional peak in the case of $\varphi=\pi$. The appearance of the new appeared peak is mainly contributed by band 2. As the location of Esaki-Tsu peak is closely connected with the damping rate, we find that, if the energy-band gap is small enough and the damping rates of these two bands have an appropriate difference, the two-peak phenomenon could be observed in the I-F curve in optical lattices. However this phenomenon is usually impossible in the semiconductor superlattices. If the damping rates of these two bands are not so distinct, the Esaki-Tsu peaks contributed from these two bands may be difficult to distinguish from each other. For example, in Fig. 4(b), we set the damping rates as $g_1^1=g_1^2=0.01E_r$. For $\varphi=\pi$, the population in band 2 is accumulated more enough even at the small value of F and therefore the peaks of both bands overlap with each other. However the Landau-Zener tunneling could still be observed by the shoulder of the peak for the cases of $\varphi=3\pi/4$ and $\pi/2$. That means the curve of dI/dF, the counterpart of differential conductance dI/dV in semiconductor superlattices, is reshaped by the Landau-Zener tunneling. On all accounts, the I-F curve is determined by the interplay between the damping rates and the probability of Landau-Zener tunneling, and the latter is determined by the relative phase φ. Consequently, the I-F curve is an effective way to observe the Landau-Zener tunneling in optical lattices.

IV. CONCLUSION

Based on a two-band model, we get the atomic current in optical lattices by solving the Boltzmann equations. With the realistic parameters in the Fourier-synthesized optical lattices, we calculate the stationary atomic current with the Landau-Zener tunneling taken into account. We find the stationary atomic current goes with the increasing probability of Landau-Zener tunneling, adjusted by the relative phase. Moreover, in contrast to the case of electrons in the semiconductor superlattices governed by the classical Esaki-Tsu equation, the I-F curve in optical lattices is modified significantly by the Landau-Zener tunneling. Two peaks may appear in the I-F curves. All of the results are the reflection of the changes in population in the upper energy band caused by the Landau-Zener tunneling. Therefore the well-controlled Landau-Zener tunneling in the optical lattices can be observed through the stationary atomic current, a measurable quantity in experiment.

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