Quasiclassical calculations of blackbody-radiation-induced depopulation rates and effective lifetimes of Rydberg *nS*, *nP*, and *nD* alkali-metal atoms with $n \leq 80$

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Rates of depopulation by blackbody radiation (BBR) and effective lifetimes of alkali-metal *nS*, *nP*, and *nD* Rydberg states have been calculated in a wide range of principal quantum numbers $n \leq 80$ at the ambient temperatures of 77, 300, and 600 K. Quasiclassical formulas were used to calculate the radial matrix elements of the dipole transitions from Rydberg states. Good agreement of our numerical results with the available theoretical and experimental data has been found. We have also obtained simple analytical formulas for estimates of effective lifetimes and BBR-induced depopulation rates, which well agree with the numerical data.

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I. INTRODUCTION

Accurate determination of effective lifetimes is important for many theoretical and experimental studies of alkali-metal Rydberg atoms. First measurements and calculations of radiative lifetimes of Na *nS* and *nD* Rydberg states with $n \leq 13$ were done by Gallagher *et al.* [1]. Later on it has been shown that interaction of Rydberg atoms with blackbody radiation (BBR) strongly affects the measured lifetimes [2]. BBR-induced depopulation rates were estimated by Cooke and Gallagher [3] for sodium Rydberg states with $n=18$ and 19, and accurately calculated by Farley and Wing [4] for alkali-metal *nS*, *nP*, and *nD* Rydberg atoms with $n \leq 30$ using Coulomb approximation [5]. Radiative lifetimes of sodium *nS* and *nD* Rydberg states with $17 \leq n \leq 28$ were measured by Spencer *et al.* [6] in a cooled environment in order to reduce the influence of BBR. The temperature dependence of BBR-induced depopulation rate was measured experimentally for the sodium 19*S* state [7] and compared with numerical calculations. Theodosiou [8] performed the model-potential calculations of the effective lifetimes of alkali-metal Rydberg states with $n \leq 21$ for several ambient temperatures in the range from 0 to 720 K. Later on, model-potential calculations of radiative lifetimes for an extended range of n were done by He *et al.* [9]. For most of alkali-metal Rydberg atoms, radiative lifetimes were calculated up to $n=30$, while for rubidium the calculations were done up to $n=50$. For determination of effective lifetimes at 300 K and comparison with available experimental results, the authors of Ref. [9] used the data of Farley and Wing [4]. Galvez *et al.* [10,11] investigated the cascade of BBR-induced transitions from the initially populated $n=24-29$ states of Na, both theoretically and experimentally.

Recent experimental studies of cold alkali-metal Rydberg atoms in magneto-optical traps involved states with relatively high principal quantum numbers $n>50$ [12]. Effective lifetimes and BBR-induced depopulation rates for these states have not been calculated yet, to the best of our knowledge. Commonly used results of [4,8] were limited by $n=21$ and $n=30$, respectively. We note that experiments with cold Rydberg atoms are usually performed at a room temperature of 300 K, when depopulation of Rydberg states by blackbody radiation is the main source of the reduction in radiative lifetimes as it was shown by Gallagher and Cooke [2]. Recently, room-temperature measurements of lifetimes of rubidium *nS*, *nP*, and *nD* Rydberg atoms with $n=26-45$ have been done [13,14] and discussed [15,16].

The present work is devoted to the calculations of the effective lifetimes of *nS*, *nP*, and *nD* Rydberg states of alkali-metal Rydberg atoms with $n \leq 80$ at the ambient temperatures of 77, 300, and 600 K. A simple theoretical model describing spontaneous and BBR-induced transitions between Rydberg states is discussed in Sec. II. Section III is devoted to the calculations of the temperature-dependent BBR-induced depopulation rates. The analytical formulas for estimates of BBR-induced depopulation rates [3] are modified to improve the agreement with numerical results. In Sec. IV the results of the numerical calculations of effective lifetimes of Rydberg states are presented and compared with available experimental and theoretical data. Simple scaling laws for estimates of effective lifetimes are obtained. Atomic units are used, unless specified otherwise.

II. SPONTANEOUS AND BBR-INDUCED TRANSITIONS BETWEEN RYDBERG STATES

A simple model for calculation of effective lifetimes of Rydberg states was developed by Gallagher and Cooke [2]. The rate of a spontaneous transition between nL and $n'L'$ states is given by the Einstein coefficient:

$$A(nL \rightarrow n'L') = \frac{4\omega_{nn'}^3}{3c^3} \frac{L_{max}}{2L+1} R^2(nL \rightarrow n'L'). \qquad (1)$$

Here L_{max} is the largest of L and L', $R(nL \rightarrow n'L')$ is a radial matrix element of the electric dipole transition, and $\omega_{nn'} = |E_{nL} - E_{n'L'}|$ is a transition frequency, where E_{nL} and $E_{n'L'}$ are energies of nL and $n'L'$ states, respectively. Energies of the Rydberg states are expressed through the effective quantum number $n_{eff} = n - \mu_L$, where μ_L is a quantum defect of an nL Rydberg state: $E_{nL} = -1/(2n_{eff}^2)$.

The rate of BBR-induced transitions $W(nL \rightarrow n'L')$ is expressed through the effective number of BBR photons per

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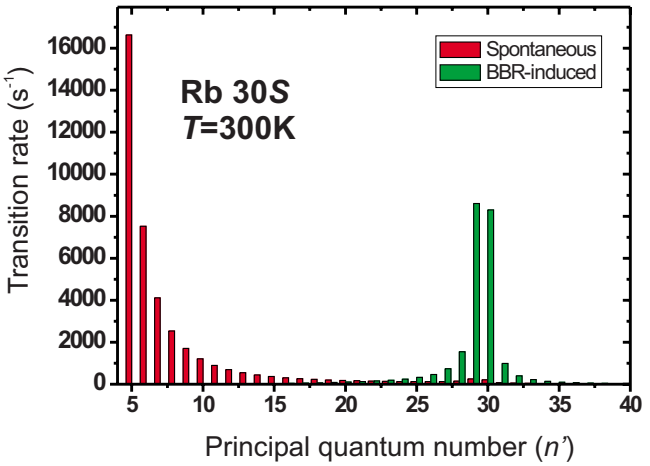

FIG. 1. (Color online) The rates of spontaneous and BBR-induced transitions from the rubidium 30*S* state to $n'P$ states.

mode, $\bar{n}_\omega$, given by the Planck distribution at temperature T,

$$\bar{n}_\omega = \frac{1}{\exp(\omega_{nn'}/kT) - 1}, \tag{2}$$

where k is the Boltzmann constant, and through the Einstein coefficient,

$$W(nL \to n'L') = A(nL \to n'L')\bar{n}_\omega. \tag{3}$$

Radiative lifetime τ_0 of a Rydberg state is determined by the total rate of spontaneous transitions from nL state to all lower-lying states:

$$\frac{1}{\tau_0} = \Gamma_0 = \sum_{E_{nL} > E_{n'L'}} A(nL \to n'L'). \tag{4}$$

The total rate of BBR-induced depopulation can be written in a similar form, taking into account transitions to both lower and higher states:

$$\Gamma_{BBR} = \sum_{n'} A(nL \to n'L') \frac{1}{\exp(\omega_{nn'}/kT) - 1}. \tag{5}$$

Finally, effective lifetime of the nL Rydberg state is determined by the sum of the rates Γ_0 and Γ_{BBR} of spontaneous and BBR-induced $nL \to n'L'$ transitions, respectively:

$$\frac{1}{\tau_{eff}} = \Gamma_0 + \Gamma_{BBR} = \frac{1}{\tau_0} + \frac{1}{\tau_{BBR}}. \tag{6}$$

A calculation of effective lifetimes is thus reduced to a calculation of the radial matrix elements $R(nL \to n'L')$. Exact analytical solution exists only for a hydrogen atom [17]. For alkali-metal atoms, various numerical methods were developed. The Hartree-Fock and multiconfiguration-interaction methods require long calculation time. The Coulomb approximation method was applied by Farley and Wing [4] and Spencer *et al.* [6] for numerical calculations and provided good agreement with experimental results. Theodosiou [8] and He *et al.* [9] used a method of model potential with different atomic potential functions.

A quasiclassical approximation is most suitable for states with large principal quantum numbers $n > 20$. Therefore, in this paper we used a quasiclassical method developed by Dyachkov and Pankratov [18] to calculate radial matrix elements. This approach is helpful for calculations where large number of dipole transitions must be considered. The authors of Ref. [18] showed that by the optimal choice of the mean

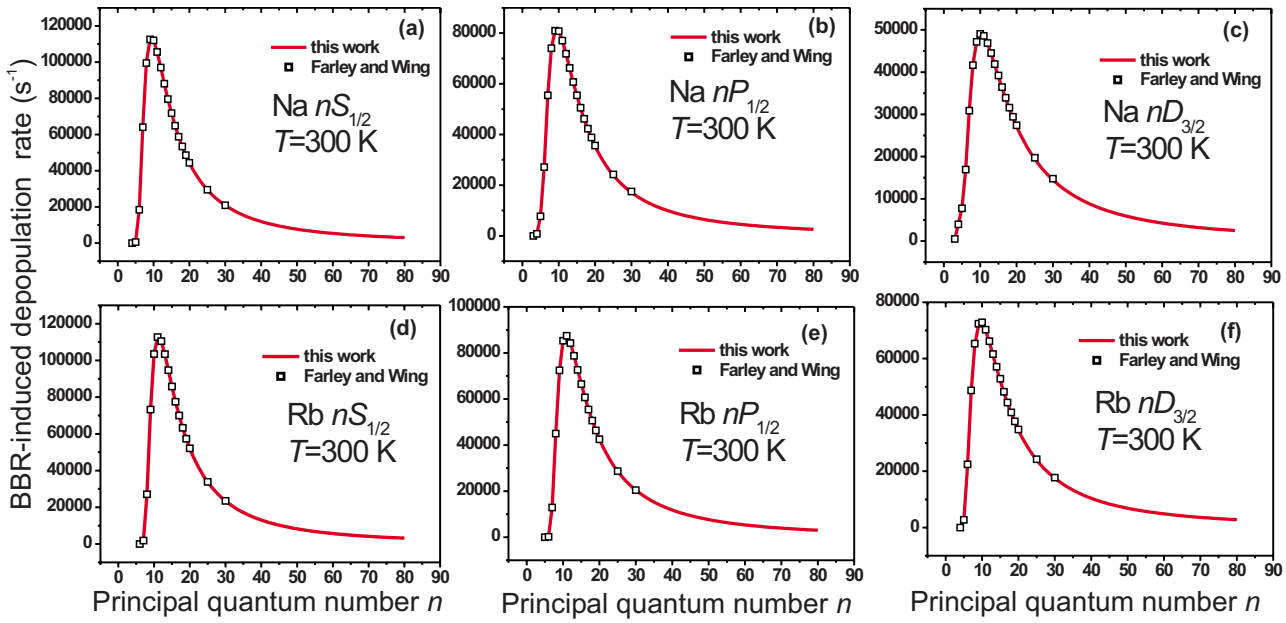

FIG. 2. (Color online) (a) Comparison of the numerically calculated BBR-induced depopulation rates of (a) Na $nS_{1/2}$, (b) Na $nP_{1/2}$, (c) Na $nD_{3/2}$, (d) Rb $nS_{1/2}$, (e) Rb $nP_{1/2}$, and (f) Rb $nD_{3/2}$ Rydberg states (this work) with numerical results of Farley and Wing [4].

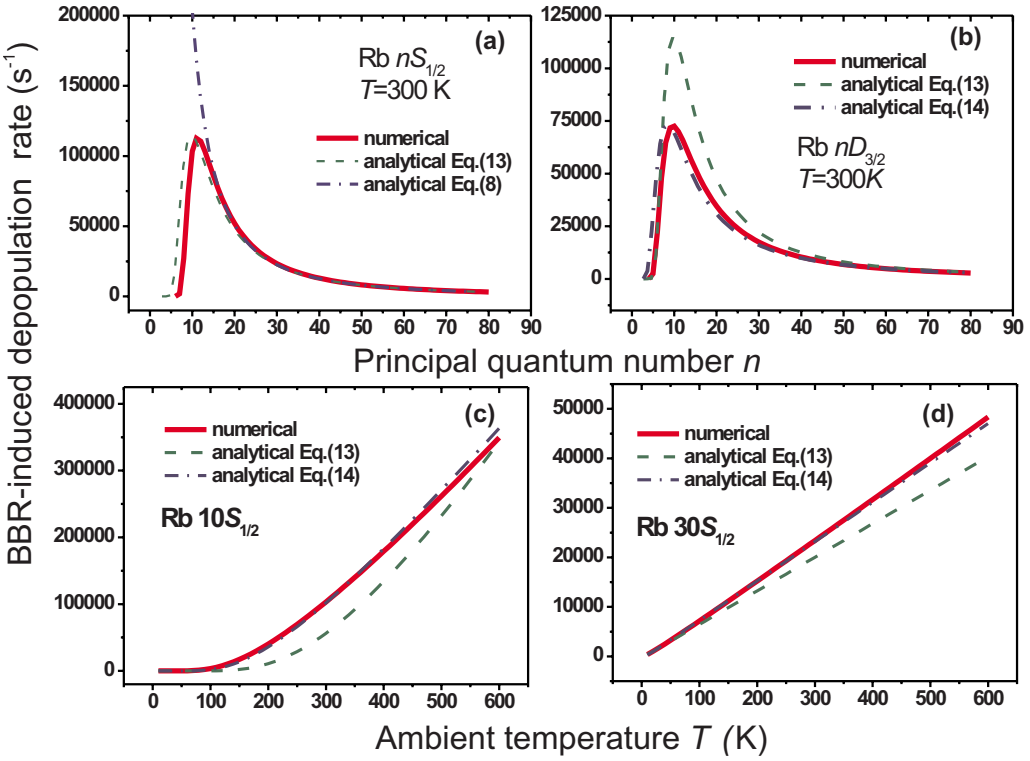

FIG. 3. (Color online) Comparison of our numerically calculated depopulation rates of Rb nS states at $T=300$ K with (a) Eqs. (8) and (13) and (b) Eqs. (13) and (14), and comparison of our numerically calculated temperature dependences of BBR-induced depopulation rates of (c) Rb 10S and (d) Rb 30S states with Eqs. (13) and (14).

energy of the atomic states, it is possible to extend semiclassical approximation and make it applicable even for states with low n. Quantum defects of nS, nP, and nD Rydberg states of Li, Na, K, Rb, and Cs atoms were taken from Refs. [19–24] and used as the input parameters for the calculations. Quantum defects of nF states of Na, K, and Rb Rydberg atoms, required to calculate the effective lifetimes of nD states, were taken from Ref. [21]. For Li and Cs the quantum defects of nF states were taken from [19,23].

For atoms in the ground and low excited states with large frequencies of transitions at $T=300$ K one has $n_\omega \ll 1$, and the rates of BBR-induced transitions are small. Hence, for atoms in such states the interaction with blackbody radiation can be neglected. The situation is different for Rydberg states: at transition frequencies on the order of 10^4 cm^{-1} one has $\bar{n}_\omega \sim 10$, and the rate of BBR-induced transitions can be ten times larger than the rate of the spontaneous decay to neighboring Rydberg states. Hence, depopulation by BBR must be necessarily taken into account when calculating the lifetimes of Rydberg states.

TABLE I. Scaling coefficients A, B, C, and D in Eq. (14).

	$S_{1/2}$				$P_{1/2}$				$D_{3/2}$			
					$P_{3/2}$				$D_{5/2}$			
	A	B	C	D	A	B	C	D	A	B	C	D
Li	0.051	0.097	1.991	3.852	0.040	0.078	1.712	3.610	0.058	0.148	1.934	3.783
Na	0.138	0.259	2.587	4.446	0.074	0.117	2.032	3.977	0.058	0.109	1.816	3.724
					0.074	0.117	2.033	3.978	0.058	0.109	1.816	3.724
K	0.123	0.232	2.522	4.379	0.118	0.257	2.600	4.421	0.044	0.104	2.003	3.831
					0.105	0.236	2.568	4.382	0.044	0.103	2.002	3.830
Rb	0.134	0.251	2.567	4.426	0.053	0.128	2.183	3.989	0.033	0.084	1.912	3.716
					0.046	0.109	2.085	3.901	0.032	0.082	1.898	3.703
Cs	0.123	0.231	2.517	4.375	0.041	0.072	1.693	3.607	0.038	0.076	1.790	3.656
					0.038	0.056	1.552	3.505	0.036	0.073	1.770	3.636

TABLE II. Scaling coefficients τ_s and δ in Eq. (15).

	$S_{1/2}$		$P_{1/2}$		$P_{3/2}$		$D_{3/2}$		$D_{5/2}$	
	τ_s	δ	τ_s	δ	τ_s	δ	τ_s	δ	τ_s	δ
Li	0.8431	2.9936	2.8807	2.9861			0.4781	2.9963		
Na	1.3698	3.0018	12.052	2.9969	11.862	2.9972	0.9555	2.9971	0.9560	2.9971
K	3.6163	2.9966	3.6415	3.0009	3.6163	2.9966	2.3168	2.9829	2.2972	2.9831
Rb	1.368	3.0008	2.4360	2.9989	2.5341	3.0019	1.0761	2.9898	1.0687	2.9897
Cs	1.2926	3.0005	2.9921	2.9892	3.2849	2.9875	0.6580	2.9944	0.6681	2.9941

TABLE III. Comparison of the calculated radiative lifetimes τ_0 (in nanoseconds) of alkali-metal Rydberg states with available theoretical data.

		$S_{1/2}$			$P_{1/2}$			$D_{5/2}$		
	n	This work	[8]	[9]	This work	[8]	[9]	This work	[8]	[9]
Li	3	28.51	30.04		181.3	211.9		13.71	14.64	
	4	52.68	56.29		411.4	391.2		24.32	33.49	
	5	99.05	102.5		704.1	610.3		62.39	63.89	
	10	757.1	783.4		3128	3328		457.4	487.3	
	15	2606	2695		9787	10280		1621	1621	
	20	6263	6457		22610	23520		3811	3818	
Na	3				17.005	16.140	14.900	19.887	19.470	17.300
	4	39.074	37.710	32.900	115.07	107.19	101.00	56.719	52.500	50.600
	5	78.120	77.640	70.700	344.32	369.86	362.00	109.74	108.87	109.00
	10	895.24	888.23	838.00	7849.5	7264.7	8560.0	954.77	956.97	968.00
	15	3506.0	3456.3		32176	27845		3198.3	3218.2	
	20	8920.8	8804.2	8350.0	81359	73653	97500	7539.5	7691.0	7910.0
	30	32397		30300	291860		287530	25391		26700
K	4				27.90	27.51	22.40	281.4	291.2	269.0
	5	46.30	46.50	35.80	134.3	127.1	117.0	627.2	769.6	502.0
	6	86.40	87.12		298.1	321.7		814.8	1169	
	10	626.0	623.3	553.0	2202	2346	2330	2601	3492	1980
	15	2704	2644		8798	9415		7589	10260	
	20	7222	6846	6430	22730	23070	24400	17410		13600
	30	27000		24500	82700		89900	57490		44800
Rb	5				29.233	27.040	20.600	190.88	231.78	247.00
	6	51.554	45.210	38.200	115.07	124.03	83.900	237.21	243.72	251.00
	10	463.92	417.84	402.00	1076.8	1201.4	960.00	820.06	822.10	807.00
	15	2339.8	2092.3		4742.4	5269.0		2828.5	2809.4	
	20	6576.8	5991.4	5870.0	12903		11900	6939.3		6700.0
	30	26594		23800	50010		46100	24471		23600
	40	68751		61400	126540		117000	59516		57700
	50	141310		126000	256280		238000	118210		114000
Cs	5							1402	1283	713.0
	6				32.59	33.66	24.70	61.58	58.39	63.60
	7	55.15	48.17	40.00	130.9	159.3	95.20	93.07	88.06	91.30
	10	293.2	274.0	257.0	893.6	1051	741.0	317.6	315.3	322.0
	15	1746	1618		4650	5318		1357	1336	
	20	5263		4840	13290		11700	3617		3650
	30	22640		20900	53370		47800	13750		14000

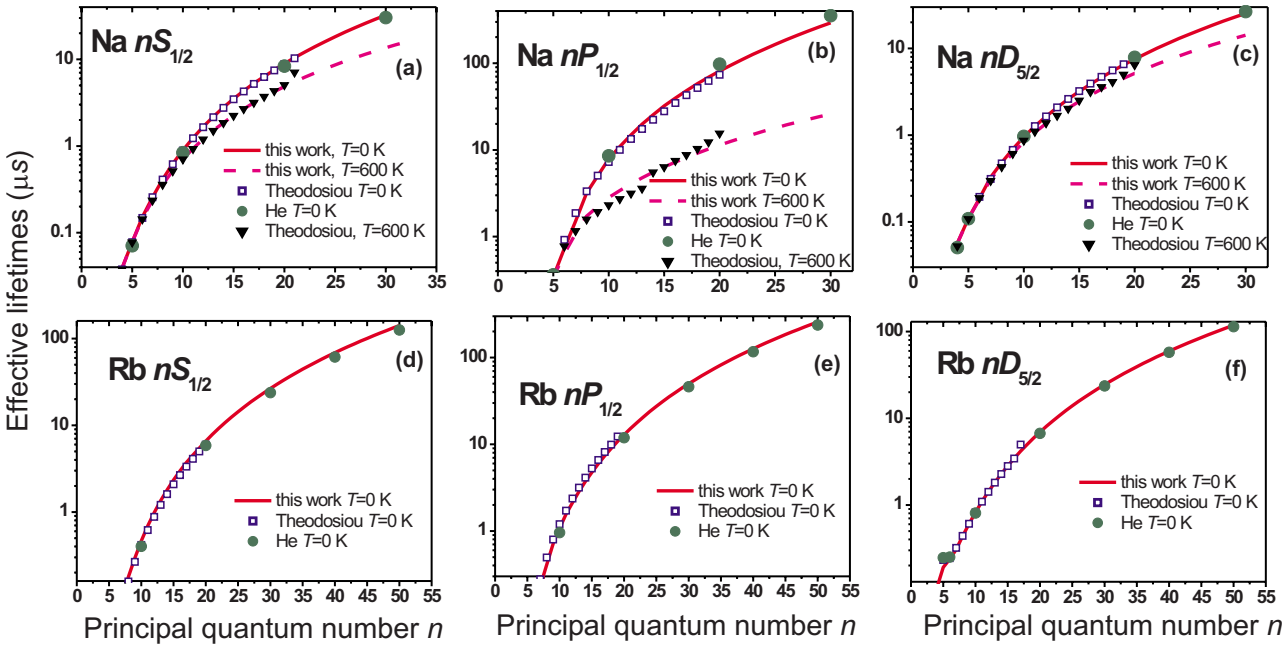

FIG. 4. (Color online) Comparison of the numerically calculated radiative and effective lifetimes of Na (a) nS, (b) nP, and (c) nD and Rb (d) nS, (e) nP, and (f) nD Rydberg states with available numerical data [8,9].

III. BBR-INDUCED DEPOPULATION OF RYDBERG STATES

Following the consideration of interaction of sodium Rydberg atoms with BBR [25], we show numerically calculated rates of spontaneous and BBR-induced transitions from the rubidium 30S state to $n'P$ states in Fig. 1. For a given n spontaneous transitions occur predominantly to the ground and low excited states, while blackbody radiation populates mostly neighboring levels with $n'=n\pm1$. Nevertheless, in order to improve the precision of the numerical calculations of BBR-induced depopulation rates, we took into account transitions to all lower states and to the upper states with $n'<n+40$. Omission of higher discrete states and continuum states reduces the accuracy by less than 0.5% [26,27].

The range of validity of the commonly used theoretical model of interaction of Rydberg atoms with blackbody radiation was discussed by Farley and Wing [4]. They derived a useful formula for determination of the critical values of n

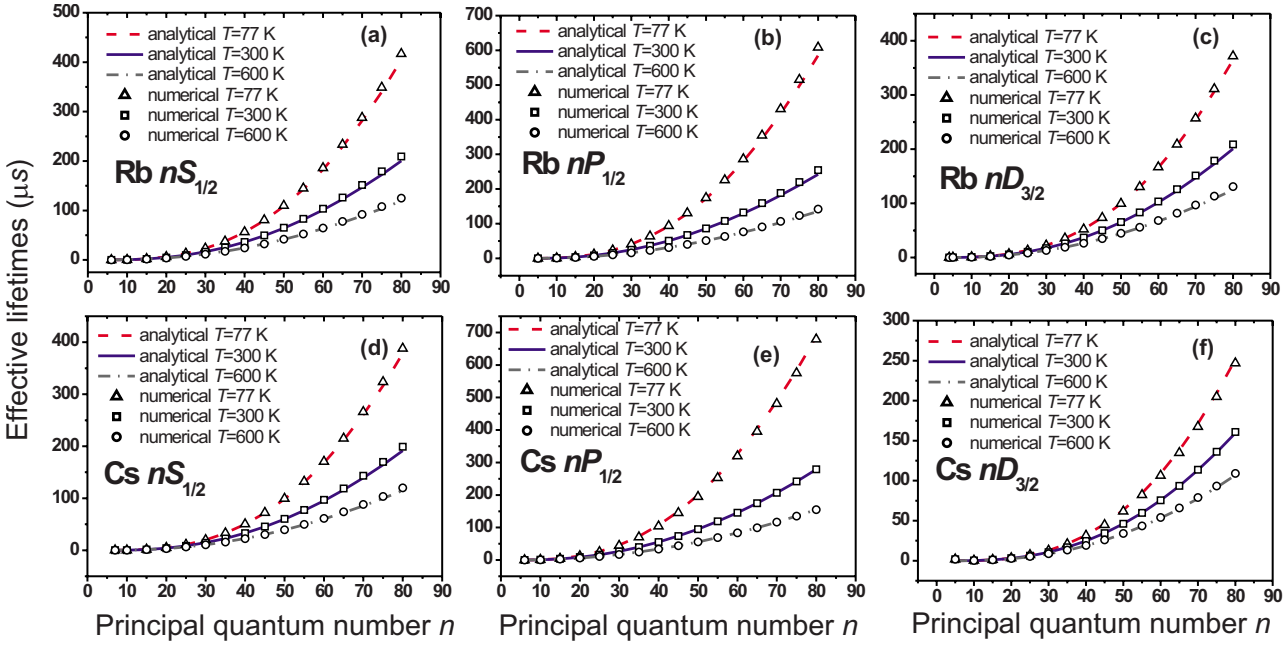

FIG. 5. (Color online) Comparison of the numerically calculated by us effective lifetimes of Rb (a) nS, (b) nP, and (c) nD and Cs (d) nS, (e) nP, and (f) nD Rydberg states at the ambient temperatures T=77, 300, and 600 K with Eq. (16).

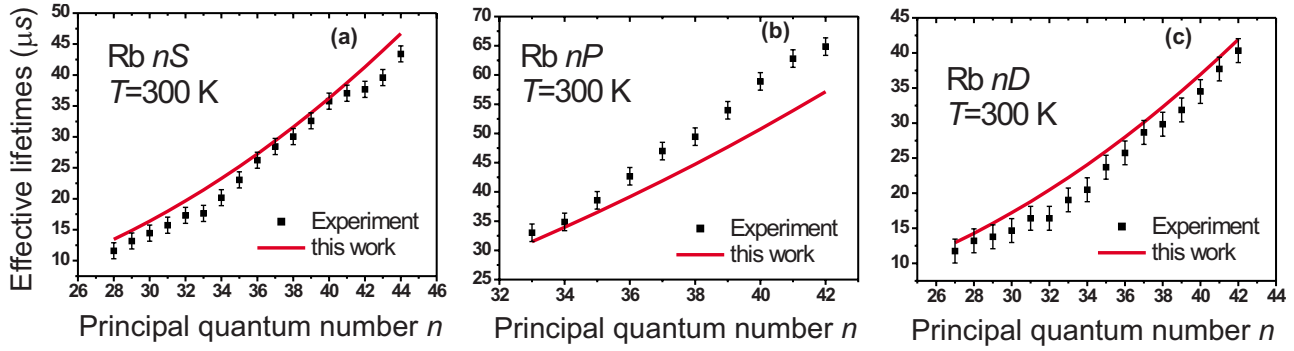

FIG. 6. (Color online) Comparison of the calculated effective lifetimes of Rb (a) nS, (b) nP, and (c) nD Rydberg states with the experiments [13,14] at $T=300$ K.

and T, at which weak-field approximation breaks down and interaction of the Rydberg electron with BBR becomes as intensive as Coulomb interaction of the electron with the ionic core:

$$kT \sim \left(\frac{15}{32\pi^3}\right)^{1/4} \alpha^{5/4} \frac{m_e c^2}{n^2}. \tag{7}$$

Here α is the fine-structure constant, m_e is the electron mass, and c is the speed of light. For $T=300$ K, this formula gives $n=122$, and for 600 K the limit is $n=86$. The condition of the breakdown of the electric dipole approximation is given by a simple formula, $kTn^2 \sim \alpha m_e c^2/3$, which yields $n=219$ for 300 K and $n=155$ for 600 K. The values of n studied in the present work are below the critical limits.

Figure 2 shows a comparison between our numerically calculated BBR-induced depopulation rates of Na and Rb Rydberg states at 300 K with the results of Farley and Wing [4]. It demonstrates that both calculations agree very well.

Cooke and Gallagher [3] used the sum rules to derive a simple approximation for BBR-induced depopulation rate:

$$\Gamma_{BBR} = \frac{4kT}{3c^3 n_{eff}^2}. \tag{8}$$

Later on Farley and Wing [4] showed that this formula overestimates the numerically calculated depopulation rates, especially for low n. Below we show how the accuracy of Eq. (8) can be substantially improved.

Equation (5) can be rewritten as

$$\Gamma_{BBR} = \frac{2}{c^3} \sum_{n'} \omega_{nn'} |f(nL \to n'L')| \left[\frac{\omega_{nn'}}{\exp(\omega_{nn'}/kT) - 1}\right]. \tag{9}$$

Here $f(nL \to n'L')$ is the oscillator strength:

TABLE IV. Effective lifetimes τ_{eff} (μs) of Li nS, nP, and nD Rydberg states.

	$T=0$ K			$T=77$ K			$T=300$ K			$T=600$ K		
n	S	P	D	S	P	D	S	P	D	S	P	D
10	0.757	3.128	0.457	0.753	3.094	0.457	0.711	2.679	0.449	0.650	2.124	0.432
15	2.606	9.787	1.621	2.544	9.290	1.612	2.274	7.033	1.536	1.962	5.119	1.425
20	6.263	22.61	3.811	5.991	20.37	3.757	5.097	13.94	3.484	4.193	9.599	3.142
25	12.32	43.58	7.412	11.55	37.21	7.236	9.396	23.55	6.545	7.427	15.55	5.754
30	21.43	74.45	12.77	19.69	60.41	12.35	15.36	35.86	10.91	11.74	22.95	9.364
35	34.20	117.5	20.25	30.82	90.80	19.38	23.13	51.01	16.74	17.16	31.82	14.06
40	51.24	174.8	30.19	45.30	129.0	28.61	32.80	69.03	24.18	23.72	42.16	19.90
45	73.16	248.3	42.96	63.47	175.4	40.32	44.46	89.89	33.37	31.44	53.95	26.94
50	100.6	339.9	58.89	85.68	230.2	54.74	58.18	113.6	44.40	40.33	67.17	35.21
55	133.6	451.8	78.35	111.8	294.0	72.14	73.83	140.2	57.38	50.32	81.83	44.76
60	173.7	586.1	101.7	142.8	366.8	92.75	91.78	169.6	72.39	61.57	97.93	55.60
65	221.2	744.4	129.2	178.7	449.0	116.8	111.9	201.9	89.50	74.03	115.5	67.77
70	276.6	929.3	161.4	219.7	540.7	144.5	134.3	237.0	108.8	87.68	134.4	81.28
75	340.5	1142	198.5	266.0	642.1	176.1	158.9	275.0	130.3	102.5	154.8	96.14
80	413.7	1386	240.9	317.8	753.4	211.8	185.8	315.8	154.2	118.6	176.6	112.4

TABLE V. Effective lifetimes τ_{eff} (μs) of Na nS, nP, and nD Rydberg states.

	$T=0$ K			$T=77$ K			$T=300$ K			$T=600$ K		
n	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$
10	0.8952	7.8495 7.7377	0.9542 0.9548	0.8905	7.5702 7.4653	0.9524 0.9530	0.8135	4.8014 4.7562	0.9115 0.9120	0.7068	2.8247 2.8078	0.8333 0.8337
15	3.5060	32.176 31.696	3.1970 3.1983	3.3671	25.594 25.279	3.1517 3.1529	2.8010	11.585 11.514	2.8433 2.8442	2.2491	6.3604 6.3366	2.4529 2.4535
20	8.9208	81.359 80.146	7.5357 7.5395	8.2551	52.902 52.363	7.2957 7.2992	6.3928	20.989 20.894	6.2556 6.2580	4.8533	11.364 11.332	5.1582 5.1597
25	18.209	164.29 161.85	14.717 14.725	16.272	89.895 89.119	13.967 13.975	11.853	33.333 33.212	11.444 11.448	8.6116	17.880 17.839	9.0771 9.0797
30	32.397	291.86 287.53	25.376 25.391	28.016	137.57 136.54	23.609 23.621	19.343	48.712 48.563	18.565 18.572	13.564	25.912 25.862	14.246 14.249
35	52.520	474.31 467.29	40.241 40.265	44.024	196.30 195.00	36.708 36.727	28.980	67.105 66.928	27.789 27.799	19.736	35.444 35.385	20.716 20.721
40	79.618	714.32 703.77	60.488 60.525	64.778	265.27 263.69	54.075 54.104	40.851	88.375 88.167	39.435 39.449	27.147	46.433 46.364	28.620 28.626
45	114.71	1025.1 1010.0	86.073 86.125	90.697	345.46 343.59	75.491 75.530	55.017	112.61 112.38	53.250 53.268	35.804	58.901 58.822	37.787 37.795
50	158.86	1413.3 1392.5	118.03 118.10	122.18	436.70 434.53	101.59 101.65	71.530	139.78 139.51	69.458 69.480	45.720	72.836 72.746	48.310 48.320
55	213.09	1890.3 1862.5	157.02 157.12	159.56	539.32 536.83	132.70 132.77	90.430	169.90 169.60	88.105 88.132	56.900	88.241 88.140	60.193 60.204
60	278.44	2462.6 2426.5	203.80 203.93	203.14	653.15 650.33	169.14 169.23	111.74	202.95 202.62	109.25 109.28	69.346	105.11 105.00	73.447 73.459
65	355.95	3141.3 3095.3	259.05 259.21	253.22	778.36 775.21	211.21 211.31	135.49	238.94 238.57	132.92 132.96	83.063	123.44 123.32	88.070 88.084
70	446.64	3933.2 3875.6	323.48 323.68	310.03	914.82 911.34	259.18 259.31	161.70	277.84 277.44	159.16 159.20	98.053	143.24 143.10	104.07 104.09
75	551.57	4846.4 4775.5	397.79 398.04	373.82	1062.5 1058.7	313.30 313.45	190.38	319.66 319.22	187.98 188.03	114.32	164.48 164.34	121.45 121.46
80	671.75	5893.0 5806.9	482.71 483.01	444.78	1221.7 1217.5	373.82 373.99	221.55	364.40 363.93	219.42 219.47	131.86	187.20 187.04	140.20 140.22

$$f(nL \to n'L') = \tfrac{2}{3}\omega_{n'n}R^2(nL \to n'L'). \tag{10}$$

A principal contribution to the BBR depopulation rate is caused by the transitions to neighboring levels with $n' = n \pm 1$ (see Fig. 1). One may note that an expression in the square brackets in Eq. (9) is a slowly changing function of $\omega_{nn'}$ for $n > 15$ and it can be considered independently of the other terms in the sum. For such states, n' and n can be replaced by n_{eff}, and $\omega_{nn'}$ by n_{eff}^{-3}. The remaining sum over the oscillator strengths in Eq. (9) satisfies a sum rule [17]:

$$\sum_{n'} \omega_{nn'} f(nL \to n'L') = \frac{2}{3n_{eff}^2}. \tag{11}$$

From Eqs. (9)–(11) we obtain

$$\Gamma_{BBR} = \frac{4}{3n_{eff}^5 c^3} \frac{1}{\exp(1/n_{eff}^3 kT) - 1}. \tag{12}$$

For large n, Eq. (12) can be expanded and coincides with Eq. (8). It is convenient to rewrite it in the units of s^{-1}, taking the temperature in kelvin:

$$\Gamma_{BBR} = \frac{1}{n_{eff}^5} \frac{2.14 \times 10^{10}}{\exp(315780/n_{eff}^3 T) - 1} \quad (s^{-1}). \tag{13}$$

A comparison of Eqs. (13) and (8) with the numerical results is shown in Fig. 3(a). It is seen that Eq. (13) correctly describes the shape of the numerically calculated dependence, and for higher n it yields the results identical to Eq. (8). However, for nP and nD states [Fig. 3(b)], Eq. (13) overestimates the numerically calculated values at intermediate $n \sim 20$. We have found that a better approximation of the nu-

TABLE VI. Effective lifetimes τ_{eff} (μs) of K nS, nP, and nD Rydberg states.

	$T=0$ K			$T=77$ K			$T=300$ K			$T=600$ K		
n	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$
10	0.6260	2.2016 2.1431	2.6317 2.6011	0.6247	2.1792 2.1218	2.5882 2.5585	0.5854	1.7812 1.7435	2.2115 2.1900	0.5217	1.3424 1.3217	1.7692 1.7556
15	2.7045	8.7975 8.5572	7.6547 7.5885	2.6213	7.9997 7.8021	7.2067 7.1482	2.2384	5.4239 5.3361	5.5668 5.5324	1.8397	3.6878 3.6489	4.1666 4.1478
20	7.2216	22.728 22.098	17.556 17.414	6.7623	18.912 18.480	15.767 15.654	5.3824	11.375 11.225	11.177 11.121	4.1743	7.2867 7.2271	7.9154 7.8881
25	14.902	46.691 45.391	33.732 33.469	13.527	35.877 35.118	28.941 28.748	10.175	19.731 19.508	19.096 19.014	7.5607	12.145 12.062	12.959 12.922
30	27.005	82.702 80.397	57.935 57.489	23.779	59.263 58.094	47.550 47.251	16.965	30.449 30.146	29.468 29.357	12.150	18.233 18.126	19.322 19.276
35	44.344	134.72 130.95	91.709 91.008	37.931	90.324 88.647	72.139 71.708	25.813	43.752 43.363	42.321 42.178	17.931	25.626 25.492	26.990 26.935
40	67.858	205.00 199.25	136.72 135.68	56.451	129.20 126.93	103.24 102.65	36.813	59.567 59.085	57.695 57.519	24.923	34.289 34.126	35.964 35.899
45	98.502	296.32 287.99	194.40 192.93	79.773	176.25 173.32	141.17 140.40	50.040	77.903 77.324	75.580 75.369	33.142	44.224 44.031	46.228 46.154
50	137.21	411.18 399.59	266.59 264.58	108.27	231.65 227.98	186.42 185.45	65.548	98.753 98.071	96.025 95.779	42.598	55.425 55.200	57.795 57.713
55	184.93	552.87 537.28	354.26 351.59	142.29	295.81 291.35	239.00 237.79	83.379	122.15 121.37	118.96 118.68	53.296	67.905 67.646	70.637 70.545
60	242.60	723.70 703.26	459.65 456.19	182.15	368.77 363.44	299.46 298.02	103.56	148.08 147.18	144.48 144.16	65.241	81.654 81.360	84.779 84.679
65	311.16	926.40 900.22	584.42 580.02	228.11	450.67 444.42	368.06 366.35	126.14	176.54 175.53	172.57 172.22	78.438	96.674 96.343	100.22 100.11
70	391.56	1163.8 1130.9	730.09 724.61	280.45	541.65 534.43	444.94 442.94	151.11	207.54 206.41	203.25 202.86	92.889	112.97 112.60	116.96 116.84
75	484.73	1438.8 1398.1	898.07 891.34	339.39	641.87 633.63	530.20 527.89	178.51	241.08 239.83	236.47 236.05	108.60	130.53 130.12	134.98 134.86
80	591.62	1754.0 1704.3	1090.1 1081.9	405.15	751.35 742.05	624.03 621.40	208.35	277.16 275.78	272.27 271.81	125.56	149.37 148.92	154.30 154.17

merical results for large n can be obtained by introducing the fitting parameters A, B, C, and D in Eq. (13):

$$\Gamma_{BBR} = \frac{A}{n_{eff}^{D}} \frac{2.14 \times 10^{10}}{\exp(315780\, B/n_{eff}^{C} T) - 1} \quad (\mathrm{s}^{-1}). \qquad (14)$$

The values of A, B, C, and D, obtained from the best fit of the numerical results at $T=300$ K to Eq. (14), are summarized in Table I. For Li Rydberg states the fine structure is not considered, as it does not affect the results. Figure 3 shows that Eq. (14) provides better agreement with the numerical calculations than Eq. (13), both for dependences on the principal quantum number [Fig. 3(b)] and on the ambient temperature [Figs. 3(c) and 3(d)].

Basically, all scaling laws are better for interpolation than for extrapolation of the precise numerical data. It is difficult to obtain an accuracy better than 50% for $n \sim 80$ using the only available results of Farley and Wing [4] as the input data for scaling with Eq. (8). From Fig. 3(a) one may see that Eq. (13) gives the completely different and more correct results for $n<15$, compared to the commonly used Eq. (8). Furthermore, we have shown that for more accurate analytical calculations of the depopulation rates, scaling coefficients should be introduced, as we did in Eq. (14). Although the data of Ref. [4] with Eq. (14) can be used to obtain the scaling with 10% accuracy at $n=50$ and 18% accuracy at $n=80$, compared to our numerical calculations, the best fit of our numerical results in an extended range of $10<n<80$ allows us to obtain more accurate values of the coefficients in Eq. (14), providing accuracies better than 3.5% for $n \sim 50$ and better than 8% for $n \sim 80$. This improvement can be important for comparison with precise experimental data.

TABLE VII. Effective lifetimes τ_{eff} (μs) of Rb nS, nP, and nD Rydberg states.

	$T=0$ K			$T=77$ K			$T=300$ K			$T=600$ K		
n	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$
10	0.4639	1.0768 1.1724	0.8264 0.8201	0.4637	1.0741 1.1691	0.8230 0.8167	0.4427	0.9862 1.0680	0.7797 0.7744	0.3992	0.8388 0.8998	0.7118 0.7076
15	2.3398	4.7424 5.1076	2.8489 2.8285	2.2774	4.5408 4.8797	2.7839 2.7648	1.9487	3.6041 3.8270	2.4807 2.4667	1.6008	2.7376 2.8719	2.1319 2.1225
20	6.5768	12.903 13.769	6.9892 6.9393	6.1684	11.693 12.422	6.6755 6.6316	4.8990	8.3288 8.7200	5.6284 5.5999	3.7905	5.8832 6.0902	4.5986 4.5816
25	14.332	27.420 29.150	13.995 13.895	12.973	23.534 24.851	13.058 12.975	9.6601	15.352 15.948	10.474 10.425	7.1242	10.296 10.583	8.2014 8.1755
30	26.594	50.010 53.076	24.647 24.471	23.283	40.764 42.877	22.471 22.331	16.392	24.730 25.561	17.218 17.144	11.635	15.964 16.337	13.004 12.968
35	44.386	82.340 87.262	39.679 39.394	37.653	63.935 67.021	35.366 35.152	25.223	36.495 37.573	25.988 25.885	17.352	22.880 23.339	19.032 18.985
40	68.751	126.54 133.99	59.946 59.516	56.588	93.766 98.028	52.256 51.946	36.254	50.745 52.087	36.931 36.795	24.298	31.073 31.619	26.328 26.271
45	100.71	184.36 195.13	86.146 85.526	80.529	130.63 136.27	73.485 73.058	49.554	67.478 69.095	50.098 49.926	32.484	40.531 41.166	34.891 34.823
50	141.31	256.28 272.35	126.53 118.21	109.87	174.26 181.96	105.23 98.874	65.176	86.547 88.576	68.939 65.352	41.921	51.199 51.967	46.860 44.657
55	191.51	346.17 366.02	168.53 158.32	144.94	225.97 234.78	137.30 129.73	83.151	108.23 110.40	87.226 83.124	52.609	63.170 63.978	58.251 55.779
60	252.44	457.66 480.99	218.98 206.67	186.12	286.63 296.24	174.82 165.96	103.53	132.63 134.87	107.92 103.29	64.561	76.478 77.298	70.941 68.200
65	325.03	587.99 617.85	278.60 263.98	233.62	354.55 365.95	218.06 207.83	126.32	159.34 161.85	131.05 125.89	77.774	90.979 91.877	84.929 81.919
70	410.41	740.58 778.04	348.19 331.02	287.78	430.64 443.93	267.29 255.61	151.55	188.53 191.30	156.63 150.93	92.257	106.73 107.70	100.22 96.940
75	509.57	917.87 964.17	428.45 408.48	348.80	515.28 530.57	322.71 309.51	179.25	220.24 223.27	184.69 178.44	108.01	123.75 124.80	116.81 113.26
80	623.54	1121.5 1186.4	500.91 497.28	416.89	608.55 628.32	371.69 369.83	209.42	254.46 258.17	208.93 208.45	125.03	142.02 143.27	131.03 130.89

IV. EFFECTIVE LIFETIMES OF RYDBERG STATES

We have numerically calculated the radiative and effective lifetimes of nS, nP, and nD alkali-metal Rydberg states using Eqs. (1)–(6). A semiempirical formula is commonly used for approximation of numerical results on radiative lifetimes τ_0 [28]:

$$\tau_0 = \tau_s n_{eff}^{\delta} \quad \text{(ns)}. \tag{15}$$

The coefficients τ_s and δ have been obtained from the best fit of our numerical results and are summarized in Table II. The radiative lifetimes τ_0 of alkali-metal Rydberg states calculated by us are compared with the available theoretical data [8,9] in Table III and in Fig. 4 (for Na and Rb). Good agreement between the three data sets is observed.

A combination of Eqs. (14) and (15) with Eq. (6) can be used for estimates of the effective lifetimes of alkali-metal Rydberg states at a given temperature:

$$\tau_{eff} = \left(\frac{1}{\tau_s n_{eff}^{\delta}} + \frac{A}{n_{eff}^{D}} \frac{21.4}{\exp(315780\, B/n_{eff}^{C} T) - 1} \right)^{-1} \quad \text{(ns)}. \tag{16}$$

In Figs. 4(a)–4(c) the effective lifetimes τ_{eff} calculated for Na at $T=600$ K are compared with the results of Theodosiou [8]. Satisfactory agreement with our numerical calculations is observed for low $n<20$. At $n\sim 20$ the results of Theodosiou start to deviate from both a smooth dependence and our calculations. The deviation from the smooth dependence was probably caused by the difficulties of numerical integration of rapidly oscillating wave functions of Rydberg states with large n.

TABLE VIII. Effective lifetimes τ_{eff} (μs) of Cs nS, nP, and nD Rydberg states.

	$T=0$ K			$T=77$ K			$T=300$ K			$T=600$ K		
n	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$	$S_{1/2}$	$P_{1/2}$ $P_{3/2}$	$D_{3/2}$ $D_{5/2}$
10	0.2932	0.8936 1.0165	0.3094 0.3176	0.2932	0.8930 1.0155	0.3092 0.3173	0.2869	0.8611 0.9720	0.3031 0.3109	0.2667	0.7658 0.8525	0.2909 0.2981
15	1.7464	4.6498 5.1490	1.3322 1.3573	1.7138	4.5365 5.0024	1.3186 1.3431	1.5033	3.7020 4.0017	1.2383 1.2603	1.2634	2.8115 2.9832	1.1300 1.1492
20	5.2626	13.287 14.612	3.5556 3.6167	4.9874	12.308 13.407	3.4701 3.5281	4.0594	8.8255 9.3628	3.1316 3.1804	3.2040	6.2018 6.4560	2.7392 2.7791
25	11.912	28.694 31.329	7.4580 7.5803	10.913	25.129 27.059	7.1708 7.2839	8.3287	16.400 17.160	6.2374 6.3267	6.2529	10.962 11.270	5.2594 5.3284
30	22.636	53.372 58.592	13.532 13.746	20.086	44.182 47.568	12.816 13.009	14.492	26.684 27.790	10.771 10.914	10.454	17.158 17.552	8.7940 8.8992
35	38.399	90.019 97.999	22.260 22.607	33.056	70.494 75.065	20.771 21.074	22.691	39.801 41.048	16.901 17.114	15.842	24.804 25.186	13.412 13.560
40	60.186	139.52 151.80	34.107 34.618	50.330	103.83 110.10	31.364 31.799	33.032	55.491 56.961	24.757 25.047	22.442	33.779 34.173	19.157 19.351
45	88.953	204.59 222.68	49.562 50.297	72.339	145.03 153.33	44.925 45.533	45.590	73.906 75.601	34.462 34.848	30.266	44.136 44.531	26.071 26.319
50	125.64	287.56 312.90	69.099 70.116	99.463	194.60 205.06	61.756 62.575	60.414	95.070 96.944	46.112 46.606	39.321	55.876 56.248	34.182 34.489
55	171.30	390.31 424.65	93.227 94.593	132.11	252.69 265.50	82.169 83.240	77.569	118.95 120.98	59.801 60.417	49.624	68.982 69.315	43.520 43.891
60	226.86	514.90 560.22	122.40 124.19	170.58	319.55 334.90	106.42 107.78	97.082	145.53 147.70	75.589 76.340	61.174	83.450 83.730	54.096 54.535
65	293.29	663.62 722.23	157.11 159.40	215.16	395.46 413.53	134.77 136.47	118.98	174.84 177.13	93.540 94.435	73.976	99.281 99.495	65.926 66.436
70	371.58	838.67 912.81	197.83 200.70	266.13	480.61 501.50	167.46 169.54	143.28	206.88 209.24	113.71 114.76	88.034	116.48 116.61	79.023 79.606
75	462.69	1042.0 1134.1	245.06 248.60	323.71	575.10 598.87	204.75 207.25	170.01	241.62 244.03	136.14 137.36	103.35	135.03 135.06	93.398 94.058
80	567.60	1275.9 1388.3	299.26 303.57	388.13	679.12 705.74	246.83 249.80	199.18	279.08 281.50	160.87 162.26	119.93	154.94 154.85	109.06 109.80

A comparison of numerically calculated effective lifetimes of Rb and Cs nS, nP, and nD Rydberg states at the ambient temperatures of 77, 300, and 600 K with Eq. (16) is shown in Fig. 5. Good agreement is observed for all states and n. We have found that in the range $15<n<80$ Eq. (16) can be used for estimates of the effective lifetimes with the accuracy better than 5%. This indicates that Eq. (16) has a good precision and can be used for prompt analytical estimates of the effective lifetimes of all alkali-metal Rydberg states.

We have also compared the calculated effective lifetimes of Rb Rydberg states with those from the experiments of de Oliveira and co-workers [13,14]. The data for nS and nD states were taken from the recent work [14], while for nP states we used the results of measurements published in the earlier work [13]. In the latter paper the authors used an uncommon definition of effective lifetimes, and their results must be divided by a factor of 2, as mentioned in Ref. [14]. Satisfactory agreement between the experiment and our calculations is observed (see Fig. 6). The theoretical curves for nS and nD states go slightly higher than the experimental points, while for the nP states they go below experimental points. A reason for this discrepancy is unclear. On the one hand, it can be caused by inaccuracy of the quasiclassical model applied for calculations of radial matrix elements of transitions to lower states with $n\sim 3-5$, which make the principal contribution to radiative lifetimes (see Fig. 1). On the other hand, we have checked that our quasiclassical model gives better agreement with the experiment and model-potential calculations [8,9], than the commonly used Coulomb approximation method [5]. For nP states, the disagreement between theory and the experiment is more significant, and we may conclude that more accurate experimental measurements would be of great interest.

Finally, we present the results of our numerical calculations of the radiative and effective lifetimes of nS, nP, and

nD alkali-metal Rydberg states with $n=10-80$ at the ambient temperatures of $T=77$, 300, and 600 K in Tables IV–VIII.

V. CONCLUSION

We reported the results of our numerical calculations of BBR-induced depopulation rates and effective lifetimes of *nS*, *nP*, and *nD* alkali-metal Rydberg states, which were extended to higher principal quantum numbers $n \leq 80$ in comparison with previous publications [4,8,9]. Good agreement of the calculated BBR depopulation rates with the results of Farley and Wing [4] for $n \leq 30$ proves the validity of the quasiclassical methods [18] used in the present work. Our results of numerical calculations of spontaneous radiative lifetimes are also consistent with the previous theoretical works [8,9]. We have also obtained satisfactory agreement with the experimental measurements [13,14] of the effective lifetimes of Rb *nS*, *nP*, and *nD* states with $n=26-45$. Nevertheless, the remaining discrepancies between experiment and theory indicate that new experimental measurements for alkali-metal Rydberg states in a wider range of *n* would be of great interest.

We have also derived an improved analytical formula [Eq. (13)] for prompt estimates of the BBR-induced depopulation rates, which better agrees with the results of numerical calculations for lower *n* than the commonly used Eq. (8) [3]. The simple scaling laws [Eqs. (14) and (16)] based on Eq. (13) can be used for accurate approximation of the results of numerical calculations of BBR-induced depopulation rates and effective lifetimes.

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