Comment on "Relevance of Bell's theorem as a signature of nonlocality: Case of classical angular momentum distributions"

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We give some comments on the paper of Matzkin [Phys. Rev. A **77**, 062110 (2008)]. It is argued here that, if the assumption of locality is properly taken into account, Matzkin's model cannot reproduce the quantum-mechanical correlations.

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In a recent article [1] Matzkin claimed to have shown the violation of Bell's inequalities by a classical model that need not assume nonlocality (i.e., the model in Sec. IV of the paper, which will be the sole concern of this Comment since only this model of the paper violates Bell's inequalities). In that model, the hidden variables (HVs) are classical angular momenta of the particles, while spin measurement results are consequences of a random interaction between the measured particle and apparatus, in such a way that the probabilities of the result of measurement depend on the distribution of the hidden variables. To model the singlet state, conservation of classical angular momentum (in this case, anticorrelation) is postulated, not at the individual level but at the ensemble level, since measurement results depend on the distribution of the HVs. In the paper only distributions of a special type is considered—uniform distribution occupying one or both the hemispheres of the HV configuration space. In this model, the quantum-mechanical singlet correlations are reproduced in the following way: obtaining a spin up in direction $\vec{a}$ at one detector tells us that the distribution of the HVs is uniform over the upper hemisphere with axis $\vec{a}$, by conservation of the angular momentum it is known that the distribution on the other particle is uniform over the lower hemisphere with the same axis. This distribution then determines that the result of measuring spin of the other particle in direction $\vec{a}$ gives a spin down. Matzkin then claims (in Sec. V C) that this correlation need not involve nonlocality, since the postulated correlation law is sufficient to ensure that the desired correlation is obtained.

This Comment will focus on whether the model has the significance as claimed, i.e., local yet violates Bell's inequalities by reproducing the quantum correlation statistics. The main problem with this model, according to this author, is that one is unable to discuss the (non)locality of the model without a mechanism that produces the correlation. It is inappropriate to assert that the postulated conservation law does not violate locality, while refraining to give a mechanism or process that would explain it.

However, independent of this specific criticism it will be shown that the model of Matzkin cannot reproduce the quantum-mechanical singlet correlations if it properly takes into account the assumption of locality. Here, a general argument will be given to show, in order for both locality and the conservation of spin to hold in the model in Sec. IV of the paper, that the measurement results must in fact necessarily be determined by the *phase-space position* of *some* spin for both of the particles (despite the claim of Matzkin that in this model the measurement results are only determined by the ensemble and distribution of the spins): if one obtained spin up in direction $\vec{a}$ for detector 1, then from the definition of the model one knows that the distribution of the spin is uniform over the upper hemisphere of the HVs with axis $\vec{a}$, which means that now the angular momentum of particle 1, $\mathbf{J}_1$, must lie somewhere in Σ_{a^+} (the upper hemisphere with axis along direction $\vec{a}$, where we denote $a^{\pm}$ as spin up or spin down in direction $\vec{a}$). By conservation of the angular momentum it is also known that the distribution of the other particle is uniform over the lower hemisphere with the same axis $\vec{a}$, which means that angular momentum for particle 2, $\mathbf{J}_2$, now lies somewhere in Σ_{a^-}. Now by the assumption of locality (at the level of individual spin positions) this fact that $\mathbf{J}_2$ lies somewhere in Σ_{a^-} was neither affected nor created by the measurement of particle 1. This implies that $\mathbf{J}_2$ already lies in Σ_{a^-} even before measurement at particle 1 was carried out. Now instead of measuring the first spin at direction $\vec{a}$, we could have measured in another direction $\vec{b}$, with $\vec{b} \neq \vec{a}$. For simplicity let us assume we get a spin up too, then we know that $\mathbf{J}_2$ now lies in Σ_{b^-}. From these two situations we know that $\mathbf{J}_2$ lies in $\Sigma_{a^-} \cap \Sigma_{b^-}$, even before the measurement at particle 1 is performed [2]. Therefore we can consider continuing such a situation (with a suitable choice of a series of measurements) to continue shrinking the domain of position of the angular momentum, until we finally pinpoint the position of some spin position for particle 2, denoted as $\bar{\mathbf{J}}_2$, to within arbitrary accuracy [3]. Now what is this $\bar{\mathbf{J}}_2$? From the above consideration we saw that $\bar{\mathbf{J}}_2$ always lies in the hemisphere that corresponds to the measurement results in such a way that when it lies in the upper or lower hemisphere centered on some axis, spin measurement in the axis direction will give spin up or spin down, respectively. It is thus obvious that this $\bar{\mathbf{J}}_2$ will completely determine all spin measurement results of particle 2: a spin measurement in direction $\vec{n}$ will give $\mathrm{sgn}(\vec{n} \cdot \bar{\mathbf{J}}_2)$. Thus we have shown that if the assumption of locality is clearly stated from the start, the model in Sec. IV of Matzkin's paper reveals itself to be just a disguise of the second model in his paper (as discussed in Sec. III), and thus cannot violate Bell's inequalities.

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Now we comment on the more conceptual aspects of the paper, particularly the discussion in Sec. V. In point (i) of the Sec. V C, Matzkin claims that conservation may result from symmetry which is a property of spacetime. Although symmetry may give rise to the conservation of quantities, in classical mechanics these conserved quantities exist before and are independent of measurement [4], which is obviously not so in the above model. In fact, the model fails to explain in a clear manner how two non-pre-existing quantities could possibly be conserved or correlated without one affecting another [5]. In point (iii) of this section, the author suggests that the distributions and their changes are epistemic, thus not involving (apparent) nonlocal effects. This view, however, merely pushes the burden of explanation to accounting for the correlation of measurement results, if such correlation is not between quantities that already exist before measurement.

Last, in refraining to explain the mechanism that produces the correlations the model obviously does not play the role of a HV theory, since the purpose of a HV theory is arguably to supply a causal explanation for the quantum correlations. Thus the model also seems to be merely restating the quantum statistics, albeit in the form of classical spins (with nonclassical correlations).

[1] P. Matzkin, Phys Rev. A **77**, 062110 (2008).

[2] It is important to note that both of these measurements need not be performed; they serve as counterfactual consideration. The validity of their usage is guaranteed here by the locality assumption.

[3] Note that by this consideration we can only derive the existence of some angular momentum position $\bar{\mathbf{J}}$, but we are prohibited from finding it experimentally.

[4] Although in classical mechanics not all physical quantities exist before the act of measurement, it is important to realize that it is indeed so for all *conserved* quantities, e.g., momentum, energy, etc.

[5] The author suggests, in order to explain the conservation, that a field may transport the angular momentum between the particles. However, this cannot really explain the correlation of *simultaneous* quantum measurement results if such field propagation moves at a finite speed.