Improved implementation of the Alicki–Van Ryn nonclassicality test for a single particle using Si detectors

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Recently a test of nonclassicality for a single qubit was proposed [R. Alicki and N. Van Ryn, J. Phys, A: Math. Theor. **41**, 062001 (2008)]. We present an optimized experimental realization of this test leading to a 46 standard deviation violation of classicality. This factor-of-5 improvement over our previous result was achieved by moving from the infrared to the visible where we can take advantage of higher-efficiency and lower-noise photon detectors.

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I. INTRODUCTION

A simple test of nonclassicality at the single-qubit level was proposed [1,2] to show that some quantum states in a two-dimensional Hilbert space cannot be classical. This test looks very appealing for its simplicity compared to other tests of quantumness (see [3] and references therein) and could represent a useful tool for various applications in the fields of quantum information, fundamental quantum optics, foundations of quantum mechanics, etc.

The criterion for classicality in the Alicki–Van Ryn model is summarized by the following statement: for any pair of observables $\hat{A}$ and $\hat{B}$ that satisfy the condition

$$\hat{B}(x) > \hat{A}(x) > 0 \tag{1}$$

for all states of the system (defined by a hidden variable x), it must always be true that

$$\langle \hat{B}^2 \rangle > \langle \hat{A}^2 \rangle. \tag{2}$$

We note that Eq. (1) defines the class of hidden variable theories (HVTs) considered by this test.

For quantum systems, one can find pairs of observables $\hat{A}, \hat{B}$ such that the minimum eigenvalue of $\hat{B} - \hat{A}$ (minimized over all possible states) is greater than zero, while for certain quantum states

$$\langle \hat{B}^2 \rangle < \langle \hat{A}^2 \rangle. \tag{3}$$

This sharp difference between classicality and nonclassicality in the Alicki–Van Ryn model can be tested experimentally at the single-qubit level [4]. Note that one needs to only find one pair of operators that satisfy Eq. (3) to show violation of the Alicki–Van Ryn classicality.

As this is a test of single-particle states, it does not deal with locality; rather it is a more fundamental test of nonclassicality with respect to the applicability of an underlying HVT. Furthermore, this test addresses only a restricted class of HVTs [4], not every conceivable HVT. Note that the precise identification of this class remains to be determined. Zukowski [5] suggested that the Alicki–Van Ryn classicality criterion is equivalent to the von Neumann theorem. This suggestion is being debated [6]. In addition to the concerns above, the test and its implementation are not loophole-free. Some of the possible loopholes were identified in [4].

One possible pair of operators $\hat{A}$ and $\hat{B}$ are of the form [4]

$$\hat{A} = a\frac{\mathbf{1} + \hat{Z}}{2}, \tag{4}$$

$$\hat{B} = b\frac{\mathbf{1} + r\cos\beta\,\hat{Z} + r\sin\beta\,\hat{X}}{2}, \tag{5}$$

where $\hat{Z}$ and $\hat{X}$ are Pauli matrices. To ensure the positivity of $\hat{A}$ and $\hat{B}$, we require $a>0$, $b>0$, and $0 \leq r \leq 1$ for a, b, and r real.

For it to be true that given $\langle \hat{B} \rangle > \langle \hat{A} \rangle$, at least one state can be found such that $\langle \hat{B}^2 \rangle < \langle \hat{A}^2 \rangle$ (i.e., the condition for the Alicki–Van Ryn nonclassicality), the minimum of the eigenvalue of $\hat{B} - \hat{A}$ should be positive while $\hat{B}^2 - \hat{A}^2$ should be negative for at least one eigenvalue. It can be shown that these conditions correspond to

$$\frac{1 - r^2}{2\sqrt{1 + r^2 - 2r\cos\beta}} < \frac{a}{b} < \frac{1 - r^2}{2(1 - r\cos\beta)}. \tag{6}$$

In this work we present a second experimental realization of the Alicki–Van Ryn test on a single qubit after the one of Ref. [4]. We exploit a simplified and more efficient scheme

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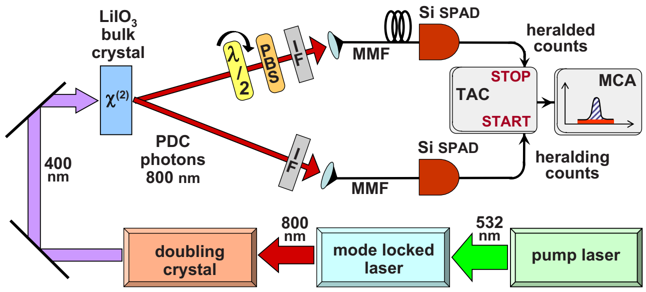

FIG. 1. (Color online) Experimental setup. A PDC heralded single-photon source generates pairs of photons at 800 nm in a $LiIO_3$ bulk crystal pumped by a 400 nm mode-locked laser. The photon registered in the heralding arm announces the presence of its twin in the heralded arm. The coincidence apparatus based on a time to amplitude converter (TAC) and a multichannel analyzer (MCA) is used to discriminate coincidences due to PDC twin photon from the background. The heralded photons are sent to the measurement apparatus designed to evaluate the observables $\langle \hat{A} \rangle$, $\langle \hat{A}^2 \rangle$, $\langle \hat{B} \rangle$, and $\langle \hat{B}^2 \rangle$.

that achieves a larger violation of the classical inequality in Eq. (2) and does so with better uncertainty (a 5× improvement over the previous result).

II. EXPERIMENT

The quantum objects we use to implement this test are linearly polarized single photons ($|\Psi\rangle = \cos\psi|H\rangle + \sin\psi|V\rangle$) produced by a heralded single-photon source based on parametric down conversion (PDC). The main difference of this experiment with respect to the previous one is that in this case both the heralded and the heralding photons are in the visible, while in the previous case the heralded photon was at a telecom wavelength [4]. This allows us to use more efficient and lower-noise detection systems that significantly reduce the experimental uncertainty. We also note that in this experiment measuring the two operators involved manually changing the wave plate angle rather than switching between the two measurements in an automated fashion, an inconsequential difference.

The experimental apparatus is sketched in Fig. 1. The PDC source is a 5-mm-long $LiIO_3$ bulk crystal, pumped by 400 nm light, that produces pairs of correlated photons at 800 nm. The pump light is obtained by doubling the frequency of the output of a mode-locked laser (with a repetition rate of ≈80 MHz) pumped by a 532 nm laser. Two interference filters (IFs) with spectral bandwidth full width at half maximum of 20 nm are placed in both the heralded and heralding arms to reduce background light. Microscope objectives (20×) collect the light into multimode fibers (MMFs) and the photons are counted by Si-single-photon avalanche diodes (Si-SPADs) operating in Geiger mode. A half-wave plate ($\lambda/2$) and polarizing beam splitter (PBS) are used for our polarization projective measurements.

To verify the single-photon nature of our source, which is critical for our test, we quantify the possibility of having more than one photon in the heralded arm after detecting the heralding photon. For this we use the same setup as for the main experiment (Fig. 1), but we substitute into the heralded arm a multimode fiber with an integrated 50:50 beam splitter (not shown) that sends the photons to two Si-SPADs. The purity of a single-photon source can be described by means of the two parameters $\gamma_1 = \theta(1)/\theta(0)$ and $\gamma_2 = \theta(2)/\theta(1)$, where $\theta(0)$, $\theta(1)$, and $\theta(2)$ are the probabilities of the heralded arm producing 0, 1, or 2 counts for each heralding count, respectively.

In general, a heralding detection announces the arrival of a "pulse" containing n photons at the heralded channel. The probability that neither of the Si-SPADs fires for a heralded optical pulse containing n photons is

$$\theta(0|n) = \sum_{m=0}^{n} (1-\tau_A)^m (1-\tau_B)^{n-m} B(m|n;p=0.5) = \left(1 - \frac{\tau_A + \tau_B}{2}\right)^n, \quad (7)$$

where p represents the beamsplitter ratio ($p=0.5$), $B(m|n;p) = n![m!(n-m)!]^{-1} p^m (1-p)^{n-m}$ is the binomial distribution representing the splitting of n photons toward the two Si-SPADs, and τ_A and τ_B are the detection efficiencies of each Si-SPAD (that includes all collection and optical losses in the detection channel). Analogously, the probability of observing 1 or 2 counts due to a heralded optical pulse with n photons is, respectively,

TABLE I. Theoretical and measured photon source parameters compared for an ideal single-photon source and a Poisson source. Standard uncertainties (shown in parentheses) account for both count fluctuations and random misalignment of the polarizers.

Source parameter	Poisson source	Ideal single-photon source	Without background subtracted	Background subtracted
$\theta(0)$	$\exp(-\tau\mu)$	$1-\tau$		
$\theta(1)$	$2[\exp(-\tau\mu/2)-\exp(-\tau\mu)]$	τ		
$\theta(2)$	$1-2\exp(-\tau\mu/2)+\exp(-\tau\mu)$	0		
γ_1	$2[\exp(\tau\mu/2)-1]$	$\tau/(1-\tau)$	0.0578(2)	0.0498(2)
γ_2	$[\exp(\tau\mu/2)-1]/2$	0	0.0013(2)	0.0007(3)
γ_2/γ_1	1/4	0	0.022(3)	0.015(5)

TABLE II. Nonclassicality test experimental results. Standard uncertainties (shown in parentheses) account for count fluctuations and random misalignment of the polarizers. The contribution to the uncertainty budget due to nonideal behavior of our heralded single-photon source is negligible. The expected deviation from classicality for the theoretical prediction, given uncertainties of our experiment, is shown for comparison.

Quantity	Measurement (this work)	Measurement (previous work)	QM theory
$\langle\hat{B}\rangle-\langle\hat{A}\rangle$	0.0701(15)	0.058(11)	0.0685
$\langle\hat{B}^2\rangle-\langle\hat{A}^2\rangle$	−0.0461(10)	−0.0403(66)	−0.0449
Deviation from classicality	46.1σ	9.4σ	45.3σ

$$\theta(1|n)=\sum_{m=0}^{n}\{[1-(1-\tau_A)^m](1-\tau_B)^{n-m}+(1-\tau_A)^m[1-(1-\tau_B)^{n-m}]\}B(m|n;p=0.5)=\left(1-\frac{\tau_A}{2}\right)^n+\left(1-\frac{\tau_B}{2}\right)^n-2\left(1-\frac{\tau_A+\tau_B}{2}\right)^n,$$

$$\theta(2|n)=\sum_{m=0}^{n}[1-(1-\tau_A)^m][1-(1-\tau_B)^{n-m}]B(m|n;p=0.5)=1-\left(1-\frac{\tau_A}{2}\right)^n-\left(1-\frac{\tau_B}{2}\right)^n+\left(1-\frac{\tau_A+\tau_B}{2}\right)^n. \quad (8)$$

From these we get $\theta(0)=\Sigma_n\theta(0|n)\mathcal{P}(n)$, $\theta(1)=\Sigma_n\theta(1|n)\mathcal{P}(n)$, and $\theta(2)=\Sigma_n\theta(2|n)\mathcal{P}(n)$ for $\mathcal{P}(n)$ being the general probability distribution of the number of photons in a heralded optical pulse. Assuming that both Si-SPADs have the same detection efficiencies ($\tau_A=\tau_B=\tau$), we list in Table I the parameters for an ideal single-photon source [$\mathcal{P}(n)=\delta_{n,1}$] and a Poissonian source [$\mathcal{P}(n)=\mu^n e^{-\mu}/n!$] with, on average, μ photons per pulse. Comparing the measured results to the theoretical values for the two types of sources, we see that while our source differs from an ideal single-photon source that emits one photon per pulse, the very small γ_2/γ_1, supports the point that conditional single-photon output does dominate our source's output.

The expectation values $\langle\hat{A}\rangle$, $\langle\hat{B}\rangle$, $\langle\hat{A}^2\rangle$, and $\langle\hat{B}^2\rangle$ can be obtained experimentally by projecting the heralded photons onto the linear polarization states as the operators $\hat{A}$ and $\hat{B}$ can be rewritten as

$$\hat{A}=a\hat{P}_0, \quad (9)$$

and

$$\hat{B}=b\left(\frac{1+r}{2}\hat{P}_{\beta/2}+\frac{1-r}{2}\hat{P}_{(\beta+\pi)/2}\right), \quad (10)$$

where $\hat{P}_\theta$ is the projection operator on the state $|s(\theta)\rangle=\cos\theta|H\rangle+\sin\theta|V\rangle$.

If we choose the parameter values $a=0.74$, $b=1.2987$, $r=3/5$, $\beta=2/9\pi$ [we note that, with this choice, the condition in Eq. (6) is satisfied], and $\psi=-11/36\pi$, the results for $\langle\hat{B}^2\rangle-\langle\hat{A}^2\rangle$ and $\langle\hat{B}\rangle-\langle\hat{A}\rangle$ predicted by quantum theory are those presented in Table II, while the minimum eigenvalue of $\hat{B}-\hat{A}$ is $d_-=0.0189$ (satisfying the $\hat{B}-\hat{A}>0$ requirement for the Alicki–Van Ryn test), where

$$d_-\equiv\frac{1}{2}\{b-a-\sqrt{a^2+b^2r^2-2abr\cos\beta}\}. \quad (11)$$

The experimental results are presented in Table II. From the value of $\langle\hat{B}^2\rangle-\langle\hat{A}^2\rangle$ we see a very large violation, 46.1 standard deviations, of the classical limit of $\langle\hat{B}^2\rangle-\langle\hat{A}^2\rangle>0$. Note that our measurement result agrees with the prediction of quantum theory to within 1 standard deviation, with the theoretically predicted result expected to deviate from classicality by 45.3 standard deviations, given our experiment conditions (see Table II).

III. CONCLUSION

In conclusion, we have presented a very simple and efficient experimental implementation of Alicki–Van Ryn's proposed nonclassicality test that results in a large (46.1 standard deviations) and low-noise violation of the Alicki–Van Ryn classicality condition. This 5× improvement over our previous result was achieved by moving from the infrared to the visible, where we can take advantage of higher-efficiency and lower-noise photon detectors.

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