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Giant Lamb shift in a Kerr nonlinear blackbody

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The Lamb shift in a Kerr nonlinear blackbody (KNB) is studied in the framework of nonrelativistic quantum electrodynamics theory. It is found that the Lamb shift in a KNB depends on the absolute temperature and the Kerr nonlinear coefficient. Below a transition temperature T_c, the Lamb shift is larger than that in a nonabsorbing linear medium. Under some conditions, the Lamb shift in a KNB can become much larger than that in a nonabsorbing linear medium or in free space. Above T_c, the Lamb shift equals that in a nonabsorbing linear medium. The application of our theory is also discussed.

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I. INTRODUCTION

The Lamb shift, which was a real sense stimulus of modern quantum electrodynamics, was first measured by Lamb and Retherford [1] in 1947. In the same year shortly after the experimental result was announced, Bethe developed a simple nonrelativistic theoretical calculation which was in good qualitative agreement with the experimental data [2]. Thereafter, the Lamb shift had gained intense study both theoretically and experimentally [3–8]. Traditionally, due to the coupling of the bound electron to the vacuum modes of the electromagnetic field, the $2S_{1/2}$ level of the hydrogen atom shifts to above the $2P_{1/2}$ level approximately by an amount 1058 MHz. By contrast, according to the Dirac theory, the energies of the two levels should coincide with each other.

In recent years, many efforts have been devoted to the investigation of the Lamb shift [9–23]. In terms of the electromagnetic environment that the atom is located in, the papers mentioned above can be mainly divided into two classes: in free space or in the modified vacuum. In free space, the authors devoted themselves to improve the accuracy of the Lamb shift and several corrections have been made. For instance, Seke [19] studied the Lamb shift including retardation. Pachucki and Jentschura [21] investigated the two-loop Bethe-logarithm correction. Czarnecki *et al.* [22] calculated the one- and two-loop Lamb shift for arbitrary excited hydrogenic states, etc. These corrections are quite significant due to the current experimental accuracy. By contrast, in modified vacuum such as in dielectric mediums [9,10], photonic crystals (PCs) [11–14], optical microwave guides [16], and other kinds of dressed environments [17,18], the results show that the Lamb shift can be modified. For example, Wang *et al.* [14] predicted that the dominant contribution to the Lamb shift comes from emission of real photon in PCs and that the Lamb shift can be enhanced by 1 or 2 orders of magnitude, named *giant* Lamb shift. Sun *et al.* [24] investigated the Lamb shift of a hydrogen atom due to the surface plasmon polariton modes and showed that the modification of the Lamb shift can be adjusted by many parameters both spatially and spectrally. Fundamentally, the modifications of the Lamb shift are caused by the unusual electromagnetic environment that the atom radiates into.

Another interesting electromagnetic environment that the atom can radiate into is a KNB, which was recently developed by Cheng [25]. A KNB is filled with a Kerr nonlinear crystal in the interior of a cavity, as described in Fig. 1. In a KNB below a transition temperature T_c, the interaction between photons and phonons can lead to an attractive effective interaction among the photons themselves. The attractive effective interaction leads to bound photon pairs with opposite wave vectors and helicities. A photon blackbody field in Kerr nonlinear crystal is a squeezed thermal radiation state in which there is a new kind of quasiparticle, the nonpolariton, which is a condensate of virtual nonpolar phonons, with a bare photon acting as the nucleus of condensation. In Ref. [26], it is found that a KNB undergoes a first-order phase transition at a transition temperature T_c. In our recent work [27], it is found that the atomic spontaneous emission can be inhibited in a KNB.

Along with the transition from bare photons to nonpolaritons, many quantities of the photon system have been changed, such as the energy of the modified vacuum, the photonic velocity, and the photonic density of state (DOS), etc. Correspondingly, it is natural to give rise to the question of what will happen to the Lamb shift in a KNB. The aim of the present paper is to answer this question. We find that the Lamb shift in a KNB is modified by two factors: the linear

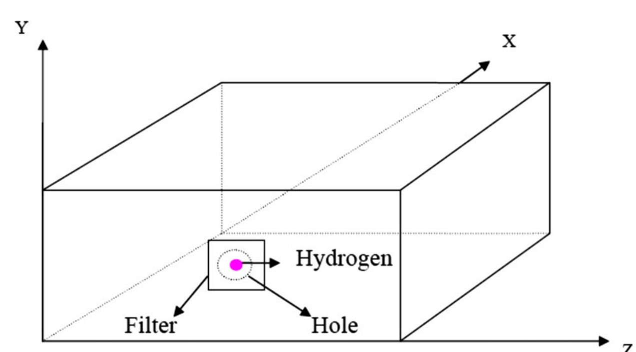

FIG. 1. (Color online) A Kerr nonlinear blackbody: a rectangular Kerr nonlinear crystal enclosed by perfectly conducting walls and kept at a constant temperature; there is a small hole in a wall. A hydrogen atom is put in the vicinity of the hole. The hole is covered by a filter.

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contribution factor and the nonlinear contribution factor. Under some conditions, the Lamb shift in a KNB can become much larger than that in free space or in a nonabsorbing linear medium.

The remainder of this paper is organized as follows. In Sec. II, we make a brief review on the KNB. The expression of the Lamb shift of a hydrogen atom is derived in Sec. III. In Sec. IV, we discussed the potential application of our theory. Finally, we make a brief conclusion in Sec. V.

II. KERR NONLINEAR BLACKBODY

In this section, we will give a brief review of the KNB. The model of a KNB has been described in Sec. I. The crystal under consideration is a covalent one. From the earlier works of Cheng [25], we can find that in a KNB, the interaction between photons and phonons can lead to an attractive effective interaction among the photons themselves. The attractive effective interaction leads to bound photon pairs with opposite wave vectors and helicities. The pair Hamiltonian of the photon system is given by

$$H_{\text{em}} = \frac{1}{2}\sum_{\mathbf{k}\sigma} \hbar\omega_{\mathbf{k}}(a^{\dagger}_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma} + a^{\dagger}_{-\mathbf{k},-\sigma}a_{-\mathbf{k},-\sigma}) + \sum_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'}a^{\dagger}_{\mathbf{k}'\sigma'}a^{\dagger}_{-\mathbf{k}',-\sigma'}a_{-\mathbf{k},-\sigma}a_{\mathbf{k}\sigma}, \tag{1}$$

$$V_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} = \begin{cases} -V_0\hbar\omega_{\mathbf{k}}\hbar\omega_{\mathbf{k}'}, & \text{if } \omega_{\mathbf{k}} \text{ and } \omega_{\mathbf{k}'} < \omega_R, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

where $a^{\dagger}_{\mathbf{k}\sigma}$ and $a_{\mathbf{k}\sigma}$ are, respectively, the creation and annihilation operators of circularly polarized photons with wave vector $\mathbf{k}$ and helicity $\sigma = \pm 1$. The constant V_0 is given by $V_0 = \frac{1}{2}N[\mathcal{P}(\mathbf{0})/2\mathcal{V}\varepsilon_0\varepsilon\omega_R]^2$, where N is the number of primitive cells, ε is the permittivity of the crystal, $\mathcal{V}$ is the volume of the KNB, ω_R is the Raman zero-wave-vector frequency, and $\mathcal{P}(\mathbf{0})$ is the zero-wave-vector Raman coefficient that is characteristic of a crystal. We assume that the crystal has a dispersion-free refractive index $\mathrm{n} \equiv \sqrt{\varepsilon}$, so the photonic frequency is given by $\omega_{\mathbf{k}} = c|\mathbf{k}|/\mathrm{n}$. Unpaired bare photons in the photon system are transformed into a new kind of quasiparticle, the nonpolariton. A nonpolariton is a condensate of virtual nonpolar phonons in momentum space, with a bare photon acting as the nucleus of condensation. The transition from the operators of bare photons to those of nonpolaritons can be effected by a unitary transformation,

$$U = \exp\left[\frac{1}{2}\sum_{\mathbf{k}\sigma} \varphi_{\mathbf{k}\sigma}(a^{\dagger}_{\mathbf{k}\sigma}a^{\dagger}_{-\mathbf{k},-\sigma} - a_{\mathbf{k}\sigma}a_{-\mathbf{k},-\sigma})\right]. \tag{3}$$

The Bogoliubov transformation is

$$c_{\mathbf{k}\sigma} = Ua_{\mathbf{k}\sigma}U^{\dagger} = a_{\mathbf{k}\sigma}\cosh\varphi_{\mathbf{k}\sigma} - a^{\dagger}_{-\mathbf{k},-\sigma}\sinh\varphi_{\mathbf{k}\sigma}, \tag{4a}$$

$$c^{\dagger}_{\mathbf{k}\sigma} = Ua^{\dagger}_{\mathbf{k}\sigma}U^{\dagger} = a^{\dagger}_{\mathbf{k}\sigma}\cosh\varphi_{\mathbf{k}\sigma} - a_{-\mathbf{k},-\sigma}\sinh\varphi_{\mathbf{k}\sigma}, \tag{4b}$$

and the inverse transformation is

$$a_{\mathbf{k}\sigma} = c_{\mathbf{k}\sigma}\cosh\varphi_{\mathbf{k}\sigma} + c^{\dagger}_{-\mathbf{k},-\sigma}\sinh\varphi_{\mathbf{k}\sigma}, \tag{5a}$$

$$a^{\dagger}_{\mathbf{k}\sigma} = c^{\dagger}_{\mathbf{k}\sigma}\cosh\varphi_{\mathbf{k}\sigma} + c_{-\mathbf{k},-\sigma}\sinh\varphi_{\mathbf{k}\sigma}, \tag{5b}$$

where the parameter $\varphi_{\mathbf{k}\sigma}$ is assumed to be real and spherically symmetric: $\varphi_{-\mathbf{k},-\sigma} = \varphi_{\mathbf{k}\sigma}$. $c^{\dagger}_{\mathbf{k}\sigma}$ and $c_{\mathbf{k}\sigma}$ are, respectively, the creation and annihilation operators of nonpolaritons in the photon system, and they obey Bose equal-time commutation relations,

$$[c_{\mathbf{k}\sigma}, c^{\dagger}_{\mathbf{k}'\sigma'}] = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\sigma,\sigma'}, \tag{6a}$$

$$[c_{\mathbf{k}\sigma}, c_{\mathbf{k}'\sigma'}] = 0. \tag{6b}$$

Under the mean-field approximation [28], the pair Hamiltonian of the photon system can be diagonalized into a neat form,

$$H'_{\text{em}} = E_p + \sum_{\mathbf{k}\sigma}\hbar\tilde{\omega}_{\mathbf{k}}(T)c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}, \tag{7}$$

where E_p is the energy of the system of bound photon pairs, and its expression is given by Ref. [29]. $\tilde{\omega}_{\mathbf{k}}(T)$ is the frequency of nonpolaritons and is given by $\tilde{\omega}_{\mathbf{k}}(T) = v(T)|\mathbf{k}|$. $v(T)$ is the velocity of nonpolaritons determined by the equation

$$v(T) = 2V_0\frac{c}{\mathrm{n}}\sum_{\omega_{\mathbf{k}}<\omega_R}\hbar\omega_{\mathbf{k}}\coth\frac{\hbar v(T)|\mathbf{k}|}{2k_BT}. \tag{8}$$

The parameter $\varphi_{\mathbf{k}\sigma}$ in Eq. (3) is determined by the relations

$$\cosh 2\varphi_{\mathbf{k}\sigma} = \frac{\hbar\omega_{\mathbf{k}}}{\sqrt{\hbar^2\omega_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2(T)}}, \tag{9a}$$

$$\sinh 2\varphi_{\mathbf{k}\sigma} = \frac{\Delta_{\mathbf{k}}(T)}{\sqrt{\hbar^2\omega_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2(T)}}, \tag{9b}$$

and

$$v(T) = (c/\mathrm{n})\sqrt{1-\Delta^2(T)}, \tag{10}$$

where $\Delta_{\mathbf{k}}(T) = \hbar\omega_{\mathbf{k}}\Delta(T)$ with $\Delta(T)$ being the order parameter for pairing of photons. From Eq. (10) we can easily find that the order parameter runs from 0 to 1, which is a monotonically decreasing function of temperature T and vanishes at the transition temperature T_c. Thus we can first set $\Delta(T_c) = 0$ in Eq. (10), then substitute $v(T_c) = c/\mathrm{n}$ into Eq. (8) and make the substitution $x = \hbar\omega/2k_BT_c$, finally we convert the summation into the integration over bare frequency ω. As a result, the transition temperature T_c is determined by the equation,

$$1 = \frac{4\gamma}{x_c^4}\int_0^{x_c} x^3\coth x\,dx, \tag{11}$$

where $x_c = \hbar\omega_R/2k_BT_c$ is the upper limit of the integral. The dimensionless constant γ is given by

$$\gamma = \frac{\hbar}{2c^3\Omega n}\left[\frac{\mathcal{P}(\mathbf{0})\omega_R}{4\pi\varepsilon_0}\right]^2, \tag{12}$$

where Ω is the cell volume. Equation (12) shows that γ contains the nonlinear coefficient $\mathcal{P}(0)$ and is characteristic of a nonlinear medium. γ is meaningful only if $0<\gamma<1$. It signifies the coupling strength between a nonpolariton and its virtual nonpolar phonons. The normalized state vector of photon pairs in the photon system may be constructed as $|G\rangle = U|0\rangle$, where $|0\rangle$ is the vacuum state, such that $c_{\mathbf{k}\sigma}|G\rangle = 0$. It is convenient to define the number operators $N_{\mathbf{k}\sigma} = c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}$ for nonpolaritons. The number operators have the eigenvalues $n_{\mathbf{k}\sigma}=0,1,2,\ldots$. The eigenstates of number operators $N_{\mathbf{k}\sigma}$ are given by

$$|\{n_{\mathbf{k}\sigma}\}\rangle = \prod_{\mathbf{k}\sigma}\left[\frac{1}{\sqrt{n_{\mathbf{k}\sigma}!}}(c^{\dagger}_{\mathbf{k}\sigma})^{n_{\mathbf{k}\sigma}}\right]|G\rangle. \tag{13}$$

III. LAMB SHIFT

We consider a hydrogen atom which is embedded in a KNB, as shown in Fig. 1. The atom is put in the vicinity of the hole. The hole is covered by a filter to prevent the atom from running out of the hole. It is necessary to point out that in the present model, we are only interested in the novel effects that are caused by the KNB radiation. The size of the hole is much larger than that of the Hydrogen atom, thus the boundary effects can be neglected. This may be important especially in view of the work of Nakajima *et al.* in Ref. [17]. Here, we are going to derive the expression of the Lamb shift of the hydrogen atom by using a nonrelativistic QED theory. The relativistic correction is neglected because it is much smaller than the nonrelativistic contribution [30]. The Hamiltonian of the field and atom is

$$H = H_0 + H'_{\mathrm{em}} + H_I, \tag{14}$$

where H_0 is the unperturbed atomic Hamiltonian which obeys the eigenvalue equation

$$H_0|n\rangle = E_n|n\rangle, \tag{15}$$

where $|n\rangle$ $(n=1,2,3\ldots)$, with a corresponding eigenvalue E_n, is the eigenstate of the hydrogen atom. H_I is the interaction Hamiltonian and is given by

$$H_I = -\frac{e}{m}\mathbf{p}\cdot\mathbf{A}, \tag{16}$$

where $\mathbf{p}$ is the momentum of the atom and $\mathbf{A}$ is the vector potential of the electromagnetic field. Due to the interaction between bare photons and nonpolar phonons, the vector potential of the electromagnetic field should be changed. In order to show the characteristics of the present system, we can express the vector potential in terms of the annihilation and creation operators of nonpolaritons through Eq. (5). We exploit the property that $c^{\dagger}_{-\mathbf{k},-\sigma}$ is equivalent to $c^{\dagger}_{\mathbf{k}\sigma}$ [25]. Then we can write the vector potential as

$$\mathbf{A} = \sum_{\mathbf{k},\sigma}\left(\frac{\hbar}{2\mathcal{V}\varepsilon_0\varepsilon\omega_{\mathbf{k}}}\right)^{1/2}\exp(\varphi_{\mathbf{k},\sigma})(c_{\mathbf{k}\sigma}+c^{\dagger}_{\mathbf{k}\sigma})\mathbf{e}_{\mathbf{k}\sigma}, \tag{17}$$

where $\mathbf{e}_{\mathbf{k}\pm 1}$ are two orthonormal circular polarization vectors perpendicular to $\mathbf{k}$. In Eq. (17), we have made a dipole approximation. It is worth noting that in Eq. (16), we have neglected the quadratic term of the vector potential because it contributes the same shift to each level and therefore cannot be observed spectroscopically. However, there is a contribution to the total mass. By using the second-order perturbation theory, the energy shift for a reference state $|n_0\rangle$ is given by

$$\Delta E_{n_0} = \sum_n\sum_{\mathbf{k},\sigma}\frac{|\langle n,1_{\mathbf{k},\sigma}|H_I|n_0,G\rangle|^2}{E_{n_0}-E_n-\hbar\tilde{\omega}_{\mathbf{k}}(T)}, \tag{18}$$

where the summation over n includes both discrete and continuous spectra of the hydrogen atom. As we all know, the Lamb shift is the energy difference between two relative levels and is a measurable quantity. The energy shift, which acts as an electromagnetic mass effect, must exist for a bound as well as for a free electron. This effect has already been included in the observed mass of the electron and is named the mass renormalization term. Consequently, it should be subtracted from Eq. (18) [2]. We can get the mass renormalization term by setting $E_n = E_{n_0}$ on the right-hand side of Eq. (18) and so the mass renormalization term is given by

$$\Delta E_{\mathrm{free}} = \sum_n\sum_{\mathbf{k},\sigma}\frac{|\langle n,1_{\mathbf{k},\sigma}|H_I|n_0,G\rangle|^2}{-\hbar\tilde{\omega}_{\mathbf{k}}(T)}. \tag{19}$$

After some direct calculations and performing the mass renormalization procedure, the expression of the Lamb shift of the hydrogen atom in a KNB takes the form

$$S = S_R S_B, \tag{20}$$

where

$$S_R = \frac{1}{\mathrm{n}}\left[\frac{c}{v(T)}\right]^2\sqrt{\frac{1+\Delta(T)}{1-\Delta(T)}} \tag{21}$$

is the relative shift and

$$S_B = \frac{e^2}{6\pi^2\varepsilon_0\hbar m^2c^3}\sum_n|\langle n|\mathbf{p}|n_0\rangle|^2(E_n-E_{n_0})\ln\frac{K}{|E_n-E_{n_0}|} \tag{22}$$

is the usual Lamb shift for a hydrogen atom in free space, and it is calculated by many authors [2,7,21]. The nonrelativistic limit requires a cut-off energy $K=mc^2$ of the nonpolaritons.

IV. RESULTS AND DISCUSSIONS

From Eq. (20), we can see that the Lamb shift in a KNB is modified by a factor S_R compared to that in free space. In order to see the physical origin of S_R, we can write it in another form. First, we can multiply a unity factor $\sqrt{\frac{1+\Delta(T)}{1+\Delta(T)}}$ to the right-hand side of Eq. (21). Then the relative Lamb shift becomes

$$S_R = \frac{1}{\mathrm{n}}\left[\frac{c}{v(T)}\right]^2\frac{1+\Delta(T)}{\sqrt{1-\Delta^2(T)}}. \tag{23}$$

Using the equation $v(T_c)=c/\mathrm{n}$ and Eq. (10), the relative Lamb shift finally becomes

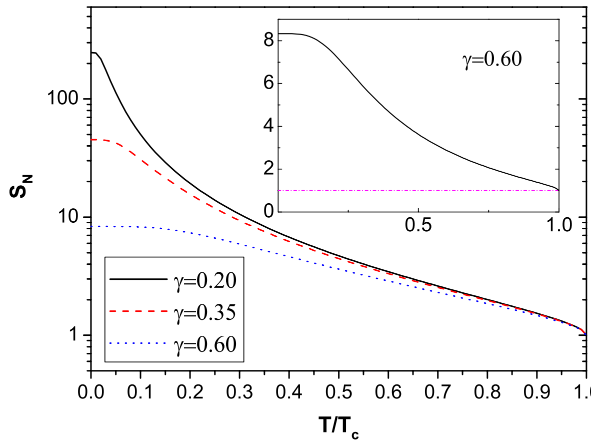

FIG. 2. (Color online) For three values of γ, variation in the nonlinear contribution factor of the Lamb shift S_N with relative temperature T/T_c; the ordinate axis is scaled by logarithm. S_N and T/T_c are all dimensionless. We take the diamond crystal, for example, the value of the zero-wave-vector frequency $\omega_R=2.51\times10^{14}\ \mathrm{s}^{-1}$ [31]. Inset: the ordinate axis is scaled linearly and the dashed-dotted line represents $S_N=1$.

$$S_R = \mathrm{n}[1+\Delta(T)]\left[\frac{v(T_c)}{v(T)}\right]^3. \tag{24}$$

For convenience, we can divide the relative Lamb shift S_R into two parts: the linear contribution part, $S_L=\mathrm{n}$, and the nonlinear contribution part,

$$S_N = [1+\Delta(T)]\left[\frac{v(T_c)}{v(T)}\right]^3. \tag{25}$$

As a result, the relative Lamb shift can be written as

$$S_R = S_L S_N. \tag{26}$$

The Lamb shift in an absorbing linear medium is discussed in Ref. [10]. In the present paper, we are only interested in the nonlinear effect on the Lamb shift. Obviously, the nonlinear contribution factor S_N is a function of absolute temperature T. As a result, the Lamb shift in a KNB is temperature dependent. In addition, the order parameter is also dependent on the dimensionless parameter γ. Therefore, the Lamb shift in a KNB can also be modulated by the Kerr nonlinear coefficient γ. The variation in the nonlinear contribution factor S_N with the relative temperature T/T_c and dimensionless parameter γ is shown in Fig. 2. In drawing Fig. 2, we take the steps as follows. From Eq. (11) we can determine T_c for given γ, provided we know ω_R. Then, from Eq. (8), we can calculate the value of $v(T)$ which depends on the value of γ. From $v(T)$, we can then determine S_N as a function of T/T_c and γ. There are four features in Fig. 2: (1) the nonlinear contribution factor is always larger than unity at $T<T_c$; (2) for fixed γ, the nonlinear contribution factor is a monotonically decreasing function of the relative temperature; (3) for fixed T, the nonlinear contribution factor is also a monotonically decreasing function of the Kerr nonlinear coefficient γ; and (4) for different values of γ, the nonlinear contribution factor goes to unity as the temperature reaches the transition temperature T_c. From Ref. [25], it is found that at $T<T_c$, the photonic system is in a squeezed thermal radiation state. The velocity of the nonpolariton $v(T)$ is always less than $v(T_c)=c/\mathrm{n}$, and the order parameter $\Delta(T)$ is always less than unity. Consequently, the nonlinear contribution factor S_N is always larger than unity at $T<T_c$. At $T\to0$, $\Delta(T)\to\sqrt{1-\gamma^2}$ and $v(T_c)/v(T)\to1/\gamma$, thus

$$S_N \to (1+\sqrt{1-\gamma^2})/\gamma^3. \tag{27}$$

It is easily found that at $T\to0$, S_N is a monotonically decreasing function of γ and can get a very large value on condition that γ is very small. That is to say, the Lamb shift in a KNB can be much larger than that in free space at some conditions. We call this *giant* Lamb shift. At $T\geq T_c$, the KNB behaves as a usual blackbody with a linear crystal in its interior. The nonpolaritons transform into photons and $\Delta(T)=0$, $v(T_c)/v(T)=1$. As a result, $S_N=1$ and the relative Lamb shift equals to that in a nonabsorbing linear medium [see Eq. (5.2) of Ref. [10]].

In order to give a numerical impression of T_c and S_N, we take the diamond crystal. The zero-wave-vector frequency of the Raman-active mode of the diamond crystal is $\omega_R=2.51\times10^{14}\ \mathrm{s}^{-1}$, and the refraction index of the diamond crystal is $n=2.417$ [31]. At $\gamma=0.20,0.35,0.60$, $T_c=3543.1,1962.7,1026.8$ K, respectively. For $\gamma=0.20$ and at room temperature, the blackbody is in a squeezed thermal radiation state. We can choose $T=298.0$ K, then the nonlinear contribution factor $S_N=62.447$. As a result, the Lamb shift $S=150.934S_B\approx159.689$ GHz, which is enhanced by 2 orders of magnitude compared to that in free space.

Up to now, one can raise a question why the Lamb shift can get such a large value in a KNB. We can answer the question as follows. As is well known, the Lamb shift is mainly caused by emitting and reabsorbing virtual photons by atom in free space. The key physical function concerning the atomic QED, not to mention the Lamb shift, is the DOS of the photonic system [13]. In a nonabsorbing linear medium, the DOS is given by

$$\rho(\omega) = \mathcal{V}\omega^2/\pi^2(c/\mathrm{n})^3. \tag{28}$$

However, in a KNB, the photons are replaced by the nonpolaritons. The DOS of the photons is accordingly changed into that of the nonpolaritons. It takes the following form

$$\rho[\tilde{\omega}_{\mathbf{k}}(T)] = \mathcal{V}[\tilde{\omega}_{\mathbf{k}}(T)]^2/\pi^2 v(T)^3. \tag{29}$$

We can see that the DOS in a KNB is a function of the velocity of nonpolaritons $v(T)$, which is a monotonically increasing function of temperature [25]. At low temperature, $v(T)$ can get much smaller value than $v(T_c)=c/\mathrm{n}$. Thus, the DOS in a KNB can be much larger than that in a nonabsorbing linear medium. From the expression of S_N [Eq. (25)], we can also find that S_N is proportional to $\rho[\tilde{\omega}_{\mathbf{k}}(T)]/\rho(\omega)=[v(T_c)/v(T)]^3$ at $\tilde{\omega}_{\mathbf{k}}(T)=\omega$. In addition, the origin of the first factor on the right-hand side of Eq. (25) is the squeezing effect, which is also larger than unity. As a result, the Lamb shift in a KNB can get much larger value than that in a nonabsorbing linear medium.

V. EXPLORING VIEW FOR APPLICATION

From Ref. [1], we can easily find that S_B is proportional to $|\psi_{n_0}(0)|^2$, with $\psi_{n_0}(0)$ being the wave-function value at the center of a hydrogen atom in state n_0. However, for P states, the wave-function value vanishes at the origin. Consequently, the Lamb shift $S_B=0$ for P states. Inasmuch as the Lamb shift in a KNB can get much larger value than that in a nonabsorbing linear medium or in free space, we will expect that the energy level with lower principal quantum number can be higher than those with higher principal quantum numbers. We can take $2S$ and $3P$ states for example. The energy difference between the two states is $\Delta E=5\ \mathrm{Ry}/36$, where Ry is the Rydberg energy. At zero temperature, the Lamb shift S in a KNB for $2S$ state can be larger than ΔE on condition that $\gamma<0.002\,24$. That is to say, the $2S$ level can be higher than $3P$ level at $\gamma<0.002\,24$. What an incredible thing. Actually, for such a large energy shift, nonperturbative effect may have to be taken into account, and we will discuss it in our forthcoming work.

In free space or in a linear dielectric medium, the atomic energy level with $n=2$ always lies below the level with $n=3$, and the number of atoms that occupy the level $n=2$ is larger than the number of atoms that occupy the level $n=3$. However, in a KNB, due to the nonlinearity, the atomic energy level with $n=2$ can be higher than that with $n=3$ at some conditions. As a result, the number of atoms with higher energy level can be larger than that with lower energy level. Thus, the nonlinear pumping can be realized, which can be used to generate lasers. To generate lasers in a KNB, the energy might come from the energy of the effective vacuum. From the aspect of the law of conservation of energy, the statement seems right. However, to obtain the energy from vacuum is a fundamental problem and has long been suspended. It is hard to answer this question theoretically. The only way is to resort to an experiment.

VI. CONCLUDING REMARKS

In summary, we have investigated the Lamb shift in a KNB within the framework of nonrelativistic QED theory. It is found that compared to that in free space, the Lamb shift in a KNB is modified by two factors: the linear contribution factor S_L and the nonlinear contribution factor S_N. The nonlinear contribution factor is a monotonically decreasing function of temperature T and also a monotonically decreasing function of Kerr nonlinear coefficient γ. As a result, the Lamb shift in a KNB can be modulated by T and γ. Below a transition temperature T_c, the value of Lamb shift is always larger than that in a nonabsorbing linear medium or in free space. What is more, the Lamb shift in a KNB can get much larger value than that in free space or in a nonabsorbing linear medium at low temperature as well as small value of γ. Above T_c, the KNB behaves as a normal blackbody with a nonabsorbing linear medium in its interior, and the Lamb shift equals that in a nonabsorbing linear medium. The application of our theory is also discussed. It is hoped that the predicted properties will be verified in physics laboratories for the not too distant future.

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