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Spatial and temporal characterization of a Bessel beam produced using a conical mirror

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We experimentally analyze a Bessel beam produced with a conical mirror, paying particular attention to its superluminal and diffraction-free properties. We spatially characterized the beam in the radial and on-axis dimensions and verified that the central peak does not spread over a propagation distance of 73 cm. In addition, we measured the superluminal phase and group velocities of the beam in free space. Both spatial and temporal measurements show good agreement with the theoretical predictions.

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I. INTRODUCTION

Bessel beams [1] are characterized by a transverse field profile defined by the zero-order Bessel function of the first kind. They exhibit several intriguing properties, such as diffraction-free propagation of the central peak over a distance fixed only by the geometry of the source device, and superluminal phase and group velocities in free space.

Diffraction-free characteristics of Bessel beams have found applications in several fields of physics. They have been utilized as pump fields in nonlinear optical processes such as parametric down conversion [2–7], second- [8–11], third- [12], and higher-order harmonic generations [13]. Femtosecond Bessel pulses were used for waveguide writing in glasses [14] and harmonic generation in the femtosecond regime [15]. Furthermore, the accuracy of optical tweezers [16–18], optical trapping [19–23], and the resolution of medical imaging [24,25] have been enhanced by the implementation of diffraction-free beams.

Superluminal properties of Bessel beams have also been a subject of investigation. In contrast to other experiments in which faster-than-light propagation has been observed [26–30], the Bessel beam's apparent superluminal character is not related to absorption or dispersion anomaly in the spectrum of the propagation medium but arises due to interference between the plane-wave components of the beam. As a result, both phase and group velocities of the Bessel beam exceed the speed of light in a wide spectral range. As we explain below, this behavior cannot be used for faster-than-light signaling and does not contradict special relativity.

There have been attempts to demonstrate a superluminal character of microwave Bessel beams [31] but the experimental uncertainties induced by the apparatus were large compared to the magnitude of the superluminal effect [32,33]. Recently, the propagation velocity of an ionization wave front induced by a Bessel pulse was measured [34], but the agreement with the theoretical prediction was reduced due to the dispersion in the propagating medium.

Recently a new technique for generating a Bessel beam by reflection from a conical mirror has been proposed [35,36]. This technique is economical and ensures a high conversion efficiency from the original Gaussian beam into the nondiffractive beam. It also has several advantages with respect to holographic axicons [37], which have a conversion efficiency limited to 40.5% for binary phase holograms and increases to 81.5% for four-level holograms. Holograms can reach the performance of a conical axicon only with a sawtooth profile [37], and therefore better efficiency is achieved at the expenses of a more complicated fabrication procedure and component cost. The same arguments hold for the use of spatial light modulators that provide a reconfigurable solution in terms of multilevel holograms [38]. Refractive axicons are commonly used [39], but thick optical components are not suited for applications that require ultrashort laser pulses due to light dispersion in the medium [40,41]. In contrast, the reflective axicon scheme is well suited for all applications that require both low dispersion and high conversion efficiency from Gaussian to Bessel beam such as waveguide writing [14] and nonlinear processes with femtosecond pulses [15].

Here, we report on complete characterization of spatiotemporal properties of an optical Bessel beam produced with a conical mirror both in continuous wave (cw) and pulsed regime. By measuring the transverse field intensity profile for several transverse planes as well as the on-axis intensity, we determine the spatial properties of the beam in all three dimensions. We find the transverse profile to remain constant over the entire 73 cm diffraction-free propagation distance. We also show a set of measurements of superluminal phase and group velocities in free space that has, to our knowledge, greater accuracy than previously measured and is in agreement with the theoretical expectations.

II. BEAM PROPERTIES

A nondiffractive Bessel beam is realized by a coherent superposition of equal-amplitude equal-phase plane waves whose wave vectors form a constant angle θ, called the axicon angle, with the direction of propagation of the beam

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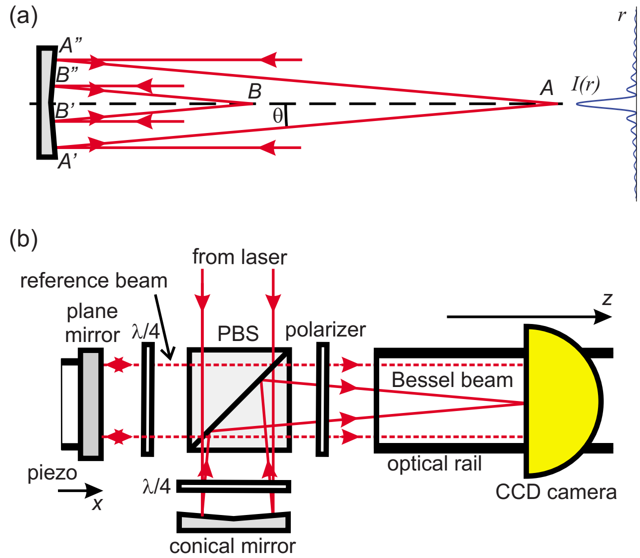

FIG. 1. (Color online) (a) Formation of the Bessel beam from a plane wave by a conical mirror and a plot of its radial intensity profile. (b) Schematic of experimental apparatus used for phase- and group-velocity measurements. PBS means polarizing beam splitter.

which we define as the z axis. As originally proposed by Durnin [1], the analytical solution of the Helmholtz equation for the electric field propagation is given by

$$E(r,z,t) = A \exp[i(\beta z - \omega t)]J_0(\alpha r), \tag{1}$$

where A is a constant, $\beta = k \cos \theta$, $\alpha = k \sin \theta$, $k = \omega / c$ is the wave vector, ω is the angular frequency, r is the radial coordinate, and J_0 is a zeroth-order Bessel function of the first kind. Equation (1) defines a nondiffracting beam since the intensity distribution is independent of z and equal to $J_0^2(\alpha r)$. Furthermore, the field described in Eq. (1) propagates with a superluminal phase velocity given by $v_p = c / \cos \theta > c$. The independence of the phase velocity from the field frequency implies that in free space the group velocity is also superluminal and is equal to the phase velocity.

In practical experiments the diffraction-free region is limited by the finite extent of the beam. As evidenced by Fig. 1(a), the maximum diffraction-free propagation distance depends on the radius R of the optical element used for producing the beam and on the axicon angle θ and is given by $z_{\max} = R / \tan \theta$. At propagation distances greater than $z_{\max}$, the beam diffracts very quickly, spreading the energy over an annular region.

In our experiment, the optical element used to generate the Bessel beam is a conical mirror with a radius of 1.27 cm and an apex angle of $\pi - \theta = 179°$. The mirror parameters correspond to a nondiffractive distance of 73 cm and lead to the phase and group velocities exceeding c by 0.015%.

III. SPATIAL CHARACTERIZATION

The nondiffractive propagation was verified by spatial characterization of the Bessel beam. The experimental setup used for these measurements is shown in Fig. 1(b). The light

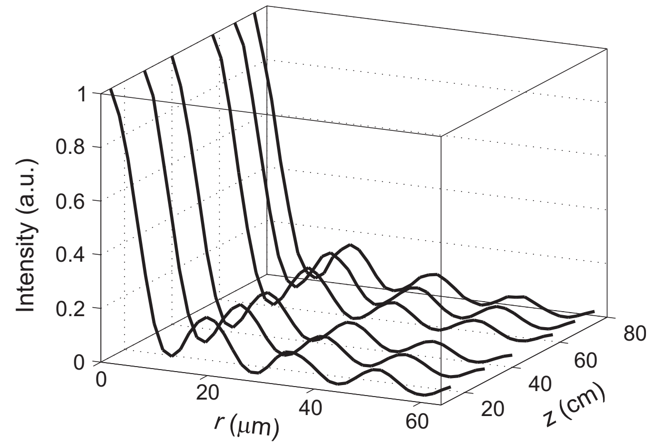

FIG. 2. Normalized radial intensity profile for several CCD camera positions.

source is a cw He:Ne laser operating at 543.5 nm. The horizontally polarized beam is first expanded, so that the resulting collimated light has a waist slightly larger than the diameter of the conical mirror and then is transmitted through a polarizing beam splitter (PBS). After the PBS, the Gaussian beam passes through a $\lambda/4$ wave plate, reflects off the conical mirror, passes through the wave plate a second time, changing to a vertical polarization, and is reflected by the PBS. In this way, 65% of the Gaussian beam power is converted into a nondiffractive beam. The conversion efficiency is limited by the size of the Gaussian beam incident on the conical mirror that has 32% of its power outside the mirror area. Losses at the beam splitter and at the conical mirror are $\sim 3\%$.

The intensity profile at various positions ranging from 18.8 to 74.7 cm along the propagation axis was recorded using a charge-coupled device (CCD) camera. In the acquisition, the closest available distance (18.8 cm) was limited by the presence of the other optical components. A microscope was constructed to magnify the intensity profile by a factor of 3.5, so the camera could resolve fine transverse features of the diffraction pattern. The radial intensity profiles recorded at different distances and normalized to a unitary maximum intensity are shown in Fig. 2: they are almost identical over the entire diffraction-free region with a central peak width that is consistent with the theoretical expectation of $(2 \times 2.405)/\alpha = 24 \pm 1.5$ μm, where 2.405 is the first zero of the Bessel function.

We compared the measured intensity pattern with the pattern calculated numerically by the Rayleigh-Sommerfield diffraction integral [42] [Fig. 3(a)]. For this calculation we assumed the amplitude transmission function of the mirror given by

$$T(r) = e^{-i2kr \tan(\theta/2)}. \tag{2}$$

The intensity of the central peak as a function of the propagation distance is shown in Fig. 3(b) together with a numerical prediction. The central peak intensity behavior is determined by the fact that the light that constructively interferes to create the peak at a longitudinal position z arrives

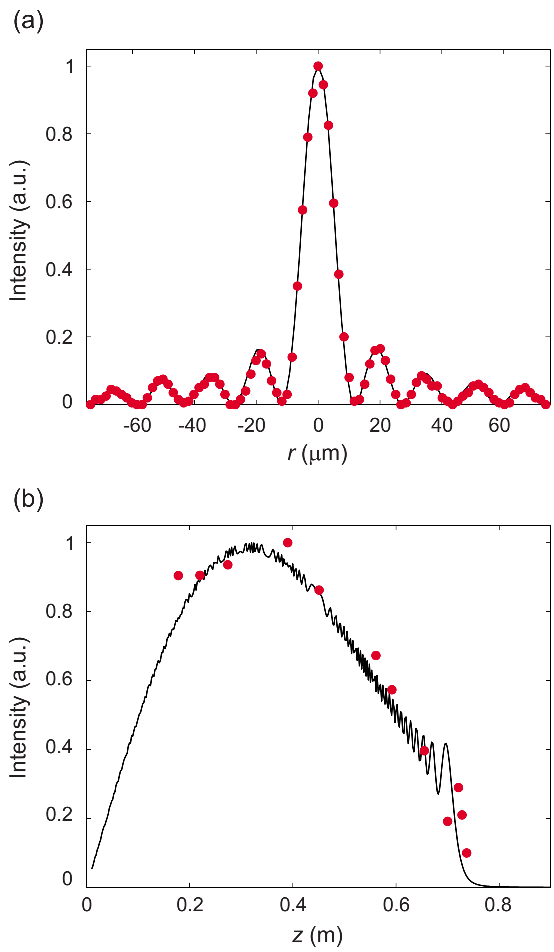

FIG. 3. (Color online) (a) Measured intensity profile (dots) and numerical calculation (line) at 18.8 cm distance from the conical mirror. (b) On-axis intensity: experimental data (dots) and theoretical calculation (line).

from a region of the mirror that has a circumference of $2\pi R = 2\pi z \tan\theta$. The intensity thus grows linearly for small values of z and then decreases when the intensity decrease in the Gaussian profile of the input beam is more rapid than the linear increase in the circumference. The oscillations at the end of the curve are due to the Fresnel diffraction from the outer edge of the mirror. The experimental values reported in Fig. 3(b) display the normalization constants of the profiles in Fig. 2. The combination of both figures provides full three-dimensional spatial information about the Bessel beam. The 73 cm nondiffractive distance estimated with geometrical optics agreed with that obtained from the numerical calculation and with the experimental results.

IV. TEMPORAL CHARACTERIZATION

The phase and group velocities in free space were measured in order to verify that the propagation of the Bessel beam exceeds the speed of light. While each component mode of the Bessel wave propagates at speed c, the wave vector of the Bessel wave front is equal to the z projection of each component's wave vector, i.e., $k = (\omega/c)\cos\theta$, where ω is the optical frequency. Accordingly, the phase and group velocities of the Bessel wave front are $v = \omega/k = c/\cos\theta$, exceeding the speed of light in vacuum.

As in the case of dispersive media, superluminal velocity of the light wave does not imply that faster-than-light signaling is possible [33,32]. Indeed, the electromagnetic fields at points A and B on the Bessel beam axis [Fig. 1(a)] form due to interference of conical waves emanating from different points on the generating optical element (A', A'' and B', B'' respectively). There is thus no causal connection between the signal arriving at points A and B, so the superluminal behavior does not violate causality and is not in contradiction with special relativity. We also note that the superluminal character of the Bessel beam is limited to the same propagation range of $R/\tan\theta$ as is its nondiffracting profile.

We implemented an interferometric design using cw light for the phase-velocity measurement and femtosecond pulses for the group-velocity measurement. The design, shown in Fig. 1(b), is a mixed Michelson interferometer, with a plane mirror in one arm and a conical mirror in the other. In the output channel of the interferometer we placed a CCD camera to record the on-axis intensity as a function of the camera's longitudinal position z. We placed a $\lambda/4$ wave plate into each arm to permit independent manipulation of the intensity of the two beams. This arrangement allowed us to compare the group and phase velocities of the Bessel beam relative to those of the Gaussian beam (that are equal to c) and to observe the relatively small superluminal effect expected.

A. Phase-velocity measurement

The Gaussian and Bessel waves have equal frequency but slightly different phase velocities along the z axis, respectively, $v_{p,G}$ and $v_{p,B}$. They will produce interference fringes of period $\Delta z = 2\pi v_{p,B} v_{p,G}/(\omega|\Delta v_p|)$, where $\Delta v_p = v_{p,G} - v_{p,B}$. If $|\Delta v_p| \ll v_{p,G} \approx c$, then

$$\Delta z = \frac{\lambda c}{|\Delta v_p|}. \tag{3}$$

However, the observation of interference fringes can yield only the absolute value of Δv_p, giving no information about its sign. In order to determine the sign of Δv_p, and hence whether the Bessel beam is propagating superluminally, we varied the angle φ between the direction of propagation of the Gaussian and Bessel beam. By doing so, we set the phase-velocity component of the Gaussian beam in the direction of propagation of the Bessel beam to $v_{p,G} = \omega/k_z = \omega/k\cos\varphi = c/\cos\varphi$. If the phase velocity of the Bessel beam exceeds that of the Gaussian beam then with increasing φ between $0 \le \varphi < \theta$, the interference fringe spacing Δz should also increase. Experimentally, we aligned the Gaussian beam along the optical rail and manipulated φ by slightly tilting the conical mirror. We determined the value of φ by displacing the CCD along the optical rail and recording the relative transverse positions of the centers of the two beams at each location. Figure 4 displays the measured interference period as a function of φ together with a theoretical fit in which the axicon angle θ was the variable parameter. In par-

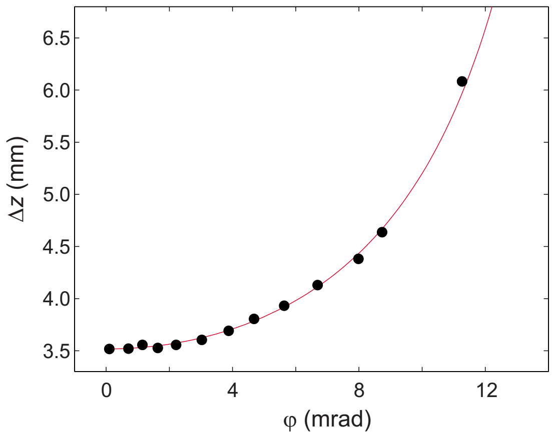

FIG. 4. (Color online) Cw measurements: period of the interference fringes Δz as a function of the angle φ between the direction of propagation of the Gaussian and Bessel beams (dots) together with the theoretical fit (line).

ticular, note that with increasing misalignment, Δz is increasing, indicating that the phase velocity of the Bessel beam is higher than that of the Gaussian beam. The data give a phase velocity of $v_p=(1.000\,155\pm0.000\,003)c$ which is consistent with the expected value $c/\cos(\theta)=1.000\,152c$, corresponding to an axicon angle of $1.009°\pm0.01°$.

B. Group-velocity measurement

The group-velocity measurement was performed using 200 fs pulses at $\lambda=776.9$ nm from a Ti:sapphire mode-locked laser pumped by a 532 nm solid-state laser. The optics were replaced to accommodate the new wavelength. The Gaussian and Bessel beams were aligned with each other to within 0.5 mrad.

The intensity in the output channel of the interferometer was measured with the CCD camera. When there is no temporal overlap between the Gaussian and the Bessel pulses, the CCD measured the sum of the intensities of the two pulses. On the other hand, when the pulse arrival was simultaneous, the intensity was affected by interference. Suppose that for some configuration of the interferometer the CCD is located in a position of maximum visibility. If the camera is now translated along the propagation axis away from the PBS by a distance of Δz, the interferometer arm with the planar mirror must be moved by a distance Δx in order to restore the same visibility. Therefore, the Bessel pulse travels an extra distance of Δz in the time the Gaussian pulse travels a distance of $\Delta z+2\Delta x$, giving

$$\Delta z = \frac{2v_{g,B}}{c-v_{g,B}}\Delta x. \tag{4}$$

Assuming pulses with a Gaussian temporal envelope of duration τ, the measured visibility will have a Gaussian dependence on the relative pulse delay with a width equal to $\sigma_x=2\tau c$.

In the experiment, we acquired several pairs $(\Delta x,\Delta z)$ for which the interference visibility is optimized. The interference was observed by scanning the plane mirror over a distance of 1–1.5 wavelengths with a piezoelectric transducer. For a given position z of the camera, the interference visibility over a range of plane mirror positions was evaluated (Fig. 5)

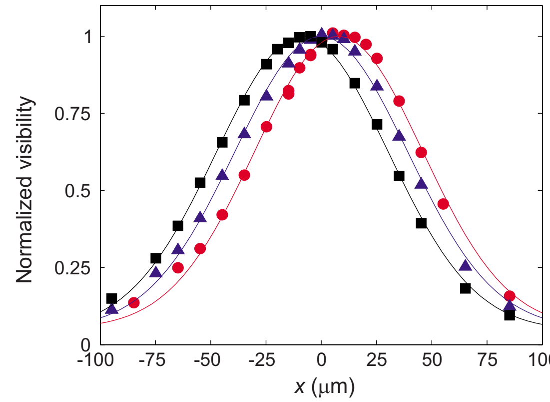

FIG. 5. (Color online) Pulsed measurements: normalized fringe visibility for $z=219$ (■), 119 (▲), and 19 (●) mm as a function of the plane mirror translation Δx. The experimental data are fitted with a Gaussian curve.

The linear relationship predicted by Eq. (4) is evidenced by Fig. 6 and is characterized by a linear regression slope $\partial x/\partial z=(-7.5\pm0.3)\times10^{-5}$. The negativity of the slope indicates that the relative path length of the Gaussian beam had to be reduced as the overall travel distance increased (i.e., larger z values), which corresponds to a superluminal group velocity $v_g=[1-2(\partial x/\partial z)]c=(1.00\,0150\pm0.000\,006)c$. This velocity corresponds to the apex angle of $0.992°\pm0.02°$.

V. CONCLUSION

We have spatially and temporally characterized an optical Bessel beam produced using a conical mirror propagating in free space. We ascertained the beam to have a constant peak size over the propagation distance determined by the properties of the conical mirror. The

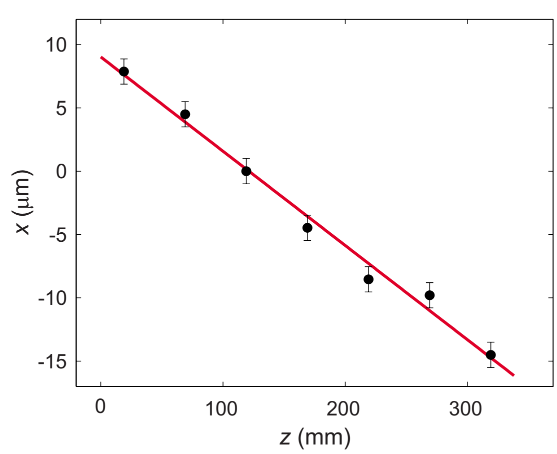

FIG. 6. (Color online) Displacement x of the Michelson interferometer plane mirror, at which the visibility is maximized, as a function of the CCD camera position z (dots) with a linear fit (line).

phase and group velocities of the beam were determined with an interferometric setup to be superluminal, with values of $v_p = (1.000\,155 \pm 0.000\,003)c$ and $v_g = (1.000\,150 \pm 0.000\,006)c$, respectively, which is consistent with the theoretical prediction.

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