Comparative study of the finite-temperature thermodynamics of a unitary Fermi gas

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The finite-temperature thermodynamics of a unitary Fermi gas is studied in detail. The chemical potential, energy density, and entropy are given analytically with the quasilinear approximation. The ground-state energy agrees with previous theoretical and experimental results. Recently, the generalized exclusion statistics is applied to the discussion of the finite-temperature unitary Fermi-gas thermodynamics. A concrete comparison between the two different approaches is performed. Emphasis is made on the behavior of the entropy per particle. In physics, the slope of entropy gives the information on the effective fermion mass m^*/m in the low-temperature strongly degenerate region. Compared with $m^*/m \approx 0.70 < 1$ given in terms of the generalized exclusion statistics, our quasilinear approximation determines that $m^*/m \approx 1.11 > 1$.

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I. INTRODUCTION

In recent years, strongly interacting fermion physics has become the focus of theoretical and experimental attention [1]. This is much attributed to the rapid progress of the atomic Fermi-gas experiments.

By tuning the external magnetic field, one can control the *s*-wave scattering length a or interaction strength between two atomic fermions. The crossover from Bardeen-Cooper-Schrieffer (BCS) regime to Bose-Einstein condensation (BEC) can be realized by the so-called Feshbach resonance [2]. At the resonance point, the scattering length can be singular with the existence of a zero-energy bound state. Although the scattering length is singular, the scattering cross section is saturated as $\sigma \sim 4\pi/k^2$ (with k being the relative momentum between two atomic fermions) due to the unitary property limit. The divergent scattering fermion thermodynamics is referred to as the unitary Fermi-gas thermodynamics in the literature [3]. Dealing with the strongly interacting matter is related with a variety of realistic many-body topics.

Usually, the thermodynamics of dilute fermion system is determined by the two-body scattering length a, particle number density n, and temperature T. In the unitary limit with $a = \pm\infty$, the dynamical scattering limit should drop out in the thermodynamic quantities. At unitarity, the dynamical detail should not affect the thermodynamics; i.e., the unitary fermion system can manifest the universal properties [3].

Due to lack of any small expansion parameter, the unitary Fermi gas provides an intractable problem in statistical physics. The fundamental issue is on the zero-temperature ground-state energy. Based on the dimensional analysis, the ground-state energy should be proportional to that of the ideal Fermi gas with a universal constant $\xi = 1+\beta$, which excites many theoretical and experimental efforts. The world average value of ξ is 0.42–0.46 [4–8]. Recently, we have attempted a quasilinear approximation method to explore the strongly interacting limit fermion thermodynamics [9]. The obtained ground-state energy or the universal constant $\xi = \frac{4}{9}$ is reasonably consistent with some theoretical or experimental investigations.

Generally, the finite-temperature thermodynamics is as intriguing as the zero-temperature ground-state energy. There have been several Monte Carlo finite-temperature calculations of a unitary Fermi gas [10,11]. In the strongly correlated unitary fermions, the nonlinear quantum fluctuations and correlations compete with dynamical high-order effects. In the weakly degenerate Boltzmann regime, the nonlinear correlations make the second-order virial coefficient a_2 vanish. To a great extent, the vanishing leading-order quantum correction reflects the *intermediate* crossover characteristics of a unitary Fermi gas [9].

Can the intermediate characteristics be described in another way? In [12,13], the generalized exclusion statistics was developed to describe the anyon behavior in the low-dimensional strongly correlated quantum system. Physically, the behavior of a unitary Fermi gas is between those of Bose gas and Fermi gas [10]. Similarly, the behavior of anyons is also between those of bosons and fermions. Can one use the anyon statistics to describe the intermediate unitary Fermi gas? Recently, the generalized exclusion statistics has been generalized to describe the unitary Fermi-gas thermodynamics [14,15]. As a hypothesis, the priority is that the thermodynamics at finite temperature can be investigated quantitatively.

From the general viewpoint of statistical mechanics, calculating entropy is not a simple task. In either classical or quantum theory, the entropy describes how the microscopic states are counted properly. From the quantum degenerate viewpoint, the low-temperature behavior of the entropy is a characteristic quantity. For example, according to the Landau theory for the strong-correlation Fermi liquid, the slope of entropy per particle versus temperature is related to the effective fermion mass m^*/m. In physics, the dynamical parameter m^*/m is very important for the phase-separation discussion of the asymmetric fermion system with unequal populations [16–18]. Like the universal constant $\xi = 1+\beta$, the effective fermion mass m^*/m is an another universal constant for the BCS-BEC crossover thermodynamics. Obviously, the physics beyond the mean-field theory should be reasonably well understood.

Unlike the ground-state energy or the universal constant ξ with the world average value $\xi \approx 0.44$, the effective fermion mass is an unknown parameter up to now. For example, the effective fermion mass is estimated to be $m^*/m \approx 1.04$ with a

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quantum Monte Carlo calculation [16]. A quantitative study of the phase diagram at zero temperature along the BCS-BEC crossover using fixed-node diffusion Monte Carlo simulations shows that $m^*/m \approx 1.09$ [17]. A many-body variational wave function with a T-matrix approximation leads to a larger value $m^*/m \approx 1.17$ [18]. What is the exact value of m^*/m?

In a quantitative way, we make a comparative study of the finite-temperature thermodynamic properties of the unitary fermion gas with the two formulations. The behavior of entropy per particle based on the quasilinear approximation and the generalized exclusion statistics is discussed in detail. Indirectly, the effective fermion mass is determined from the entropy. The results are further compared with the Monte Carlo calculations.

The paper is organized in the following way. In Sec. II, the relevant thermodynamic expressions are given by the quasilinear approximation. Correspondingly, the thermodynamics given by the generalized exclusion statistics is presented in Sec. III. The numerical calculations and concrete comparisons between the two methods are given in Sec. IV. In this section, the entropy per particle and corresponding effective fermion mass m^*/m are discussed. In Sec. V, we present the conclusion remarks.

II. THERMODYNAMICAL QUANTITIES GIVEN BY STATISTICAL DYNAMICS WITH QUASILINEAR APPROXIMATION

Strongly correlated matter under extreme conditions often requires the use of effective field theories in the description of the thermodynamic properties, independently of the energy scale under consideration. In the strongly interacting system, the central task is how to deal with the nonperturbative fluctuation and correlation effects. In Ref. [9], a quasilinear approximation is taken to account for the nonlocal correlation effects on the unitary Fermi-gas thermodynamics.

With the quasilinear approximation method, the obtained grand thermodynamic potential $\Omega(T,\mu)$ or pressure $P = -\Omega/V$ can be described by two coupled parametric equations through the intermediate variable effective chemical potential μ^*,

$$P = \frac{2T}{\lambda^3} f_{5/2}(z') + \frac{\pi a_{\rm eff}}{m} n^2 + n\mu_r, \tag{1}$$

$$\mu = \mu^* + \frac{2\pi a_{\rm eff}}{m} n + \mu_r. \tag{2}$$

In the above equations, $\lambda = \sqrt{2\pi/mT}$ is the thermal de Broglie wavelength and m is the bare fermion mass (with natural units of $k_B = \hbar = 1$ throughout the paper).

The effective chemical potential μ^* is introduced by the single-particle self-consistent equation. μ^* makes the thermodynamic expressions appear as the standard Fermi integral formalism

$$f_\upsilon(z') = \frac{1}{\Gamma(\upsilon)} \int_0^\infty \frac{x^{\upsilon-1} dx}{z'^{-1} e^x + 1}, \tag{3}$$

where $\Gamma(\upsilon)$ is the gamma function, and $z' = e^{\mu^*/T}$ is the effective fugacity. For example, the quasiparticle Fermi-Dirac distribution function gives the particle number density according to

$$n = \frac{2}{\lambda^3} f_{3/2}(z'). \tag{4}$$

In coupled equations (1) and (2), the shorthand notations are defined as

$$a_{\rm eff} = -\frac{m}{2\pi m_D^2}, \quad m_D^2 = \left(\frac{\partial n}{\partial \mu^*}\right)_T. \tag{5}$$

The shift term $\propto \mu_r$ characterizes the high-order nonlinear contributions, which strictly ensures the energy-momentum conservation law. In the quasilinear approximation, this significant high-order correction term can be fixed in a thermodynamic way. It is worthy noting that the terms $\propto \mu_r$ can be exactly canceled by each other in the Helmholtz free-energy density

$$\frac{F}{V} = f = -P + n\mu, \tag{6}$$

where V is the system volume. However, the high-order correlation term $\propto \mu_r$ can be obtained in terms of the thermodynamic relations [9]

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T,N} = -\left(\frac{\partial \frac{F}{N}}{\partial \frac{V}{N}}\right)_T = n^2 \left(\frac{\partial \frac{f}{n}}{\partial n}\right)_T, \tag{7}$$

and

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V} = \left(\frac{\partial \frac{F}{V}}{\partial \frac{N}{V}}\right)_T = \left(\frac{\partial f}{\partial n}\right)_T. \tag{8}$$

Comparing those obtained from Eqs. (7) and (8) with Eqs. (1) and (2), the explicit expression of μ_r is

$$\mu_r = \frac{1}{2}\left(\frac{\partial m_D^2}{\partial n}\right)_T \left(\frac{2\pi a_{\rm eff}}{m}\right)^2 n^2. \tag{9}$$

The integrated expressions of the pressure and chemical potential for the unitary Fermi gas are

$$P = \frac{2T}{\lambda^3}\left(f_{5/2}(z') - \frac{f_{3/2}^2(z')}{2f_{1/2}(z')} + \frac{f_{3/2}^3(z') f_{-1/2}(z')}{2f_{1/2}^3(z')}\right), \tag{10}$$

$$\mu = \mu^* - T\frac{f_{3/2}(z')}{f_{1/2}(z')} + \frac{T}{2}\frac{f_{3/2}^2(z') f_{-1/2}(z')}{f_{1/2}^3(z')}. \tag{11}$$

In the quasilinear approximation, the auxiliary implicit variable μ^* is introduced to characterize the nonlinear fluctuation and correlation effects. As indicated by Eqs. (10) and (11), the μ^* or z' makes the realistic grand thermodynamic potential $\Omega(T,\mu)$ appear as the set of highly nonlinear parametric equations, which can be represented by the standard Fermi integral. By eliminating the auxiliary variable μ^*, the equation of state will uniquely be determined.

From the underlying grand thermodynamic potential-partition function, one can derive the analytical expressions for the entropy density $s=S/V$ and internal energy density $\epsilon=E/V$. The following partial derivative formulas will be used:

$$\left(\frac{\partial\mu^*}{\partial T}\right)_{\mu}\left(\frac{\partial T}{\partial\mu}\right)_{\mu^*}\left(\frac{\partial\mu}{\partial\mu^*}\right)_{T}=-1,$$

$$\left(\frac{\partial m_D^2}{\partial T}\right)_{n}=\left(\frac{\partial m_D^2}{\partial T}\right)_{\mu^*}+\left(\frac{\partial m_D^2}{\partial\mu^*}\right)_{T}\left(\frac{\partial\mu^*}{\partial T}\right)_{n}. \tag{12}$$

The entropy is derived according to

$$\frac{s}{n}=\frac{1}{n}\left(\frac{\partial P}{\partial T}\right)_{\mu}=\frac{5}{2}\frac{f_{5/2}(z')}{f_{3/2}(z')}-\ln z'+\frac{3f_{-1/2}(z')f_{3/2}^2(z')}{4f_{1/2}^3(z')}-\frac{f_{3/2}(z')}{4f_{1/2}(z')}. \tag{13}$$

Correspondingly, the explicit energy density expression is calibrated to be

$$\epsilon=\frac{3T}{\lambda^3}\left(f_{5/2}(z')-\frac{f_{3/2}^2(z')}{2f_{1/2}(z')}+\frac{f_{3/2}^3(z')f_{-1/2}(z')}{2f_{1/2}^3(z')}\right). \tag{14}$$

Essentially, the entropy density includes the high-order nonlinear contribution. What we want to emphasize is that the third law of thermodynamics is exactly ensured as expected. The analytical analysis indicates that the energy density at zero temperature gives the dimensionless universal coefficient according to $\xi=\mu/E_F=\frac{4}{9}$ or $E/(\frac{3}{5}NE_F)=\xi$, where the Fermi energy is $E_F=(3\pi^2 n)^{2/3}/2m$ and T_F is the Fermi characteristic temperature in the unit Boltzmann constant. The universal coefficient $\xi=\frac{4}{9}$ has attracted much attention in the literature and is reasonably consistent with some Monte Carlo calculations [4,10].

III. THERMODYNAMICS GIVEN BY THE GENERALIZED EXCLUSION STATISTICS

A. Generalized exclusion statistics

The generalized exclusion statistics is proposed in [12,13]. If the dimension of the Hilbert space is d and the particle number is N, then d and N are connected by $\Delta d=-g\Delta N$, where the shift in the single-particle states' number is Δd. The shift in the particle number for identical particle system is ΔN and g is a statistical parameter, which denotes the ability of one particle to exclude other particles in occupying single-particle state. When $g=0$ the intermediate statistics returns to the Bose-Einstein statistics and when $g=1$ to the Fermi-Dirac statistics.

For anyons, the number of quantum states W of N identical particles occupying a group of G states are determined by the interpolated statistical weights of the Bose-Einstein and Fermi-Dirac statistics. A simple formula with the generalized exclusion statistics is used to describe the microscopic quantum states [13],

$$W=\frac{[G+(N-1)(1-g)]!}{N![G-gN-(1-g)]!}. \tag{15}$$

One can divide the one-particle states into a large number of cells with $G\gg 1$ states in each cell, and calculate the number with N_i particles in the ith cell. The total energy and the total number of particles are fixed and given as

$$E=\sum_i N_i\epsilon_i,\quad N=\sum_i N_i, \tag{16}$$

with ϵ_i defined as the energy of particle of species i. By generalizing Eq. (15), we have

$$W=\prod_i\frac{[G_i+(N_i-1)(1-g)]!}{N_i![G_i-gN_i-(1-g)]!}. \tag{17}$$

We consider a grand canonical ensemble at temperature T. For very large $G_i\gg 1$ and $N_i\gg 1$, using the Stirling formula $\ln N!=N(\ln N-1)$ and introducing the average occupation number defined by $\bar{N}_i\equiv N_i/G_i$, one has

$$\ln W=\sum_i\ln\left\{\frac{[G_i+(N_i-1)(1-g)]!}{N_i![G_i-gN_i-(1-g)]!}\right\}$$
$$\simeq\sum_i\{G_i[1+(1-g)\bar{N}_i]\ln G_i[1+(1-g)\bar{N}_i]-G_i(1-g\bar{N}_i)\ln G_i(1-g\bar{N}_i)-G_i\bar{N}_i\ln G_i\bar{N}_i\}. \tag{18}$$

Through the Lagrange multiplier method, the most probable distribution of $\bar{N}_i$ is determined by

$$\frac{\partial}{\partial\bar{N}_i}\left[\ln W-\sum_i\frac{G_i\bar{N}_i(\epsilon_i-\mu)}{T}\right]=0, \tag{19}$$

with chemical potential μ. It follows that

$$\bar{N}_i e^{(\epsilon_i-\mu)/T}=[1+(1-g)\bar{N}_i]^{1-g}(1-g\bar{N}_i)^g. \tag{20}$$

Setting $\omega_i=1/\bar{N}_i-g$, we have the anyon statistical distribution

$$\bar{N}_i=\frac{1}{\omega_i+g}, \tag{21}$$

where ω obeys the relation

$$\omega^g(1+\omega)^{1-g}=e^{(\epsilon-\mu)/T}. \tag{22}$$

One can define ω_0 of ω at $\epsilon=0$ with Eq. (22),

$$\mu=-T\ln[\omega_0^g(1+\omega_0)^{1-g}]. \tag{23}$$

The relation between μ and T has been established indirectly through ω_0 and g. From Eq. (22), the ω and ω_0 are related with each other through single-particle energy ϵ,

$$\epsilon=T\ln\left[\left(\frac{\omega}{\omega_0}\right)^g\left(\frac{1+\omega}{1+\omega_0}\right)^{1-g}\right], \tag{24}$$

which gives

$$d\epsilon = \frac{T(g+\omega)}{\omega(1+\omega)} d\omega. \tag{25}$$

For $T=0$, the average occupation number can be explicitly indicated as

$$\bar{N} = \begin{cases} 0 & \text{if } \epsilon > \mu \\ \dfrac{1}{g} & \text{if } \epsilon < \mu, \end{cases} \tag{26}$$

which is quite similar to the Fermi-Dirac statistics.

B. Particle number and energy densities

In the anyon statistics, the density of states is also given by

$$D(\epsilon) = \alpha(2m)^{3/2} V \epsilon^{1/2}/4\pi^2, \tag{27}$$

where α is the degree of the spin degeneracy and m is the bare fermion mass.

At $T=0$, the particle number is explicitly given by

$$N = \frac{1}{g}\int_0^{\tilde{E}_F} D(\epsilon)d\epsilon = \frac{\alpha(2m)^{3/2}}{6\pi^2} V E_F^{3/2}, \tag{28}$$

where $\tilde{E}_F$ is related to the Fermi energy E_F through $\tilde{E}_F = g^{2/3}E_F$. With the $\tilde{E}_F$ symbol, the system energy can be represented as

$$E = \frac{1}{g}\int_0^{\tilde{E}_F} \epsilon D(\epsilon)d\epsilon = \frac{3}{5} g^{2/3} N E_F. \tag{29}$$

As we will see, once g is fixed, one can discuss the general finite-temperature thermodynamic properties. Therefore, the essential task in the generalized exclusion statistics is fixing the statistical factor g. This can be determined by the zero-temperature ground-state energy or the universal constant ξ according to $\xi = g^{2/3}$. Various theoretical or experimental attempts have been made in the literature for determining the ground-state energy. With the universal coefficient $\xi = \frac{4}{9}$ [9], the expected statistical factor can be identified to be $g = \frac{8}{27}$.

For the general finite-temperature scenario, the particle number and energy can be rewritten as

$$N = \int_0^\infty \frac{D(\epsilon)d\epsilon}{\omega + g}, \tag{30}$$

$$E = \int_0^\infty \frac{\epsilon D(\epsilon)d\epsilon}{\omega + g}. \tag{31}$$

By replacing Eqs. (24), (25), and (28) into Eqs. (30) and (31), one can have

$$\frac{3}{2}\left(\frac{T}{T_F}\right)^{3/2} a(\omega_0) = 1, \tag{32}$$

$$\frac{E}{NE_F} = \frac{3}{2}\left(\frac{T}{T_F}\right)^{5/2} b(\omega_0), \tag{33}$$

$$a(\omega_0) = \int_{\omega_0}^\infty \frac{d\omega}{\omega(1+\omega)}\left[\ln\left(\frac{\omega}{\omega_0}\right)^g \left(\frac{1+\omega}{1+\omega_0}\right)^{1-g}\right]^{1/2},$$

$$b(\omega_0) = \int_{\omega_0}^\infty \frac{d\omega}{\omega(1+\omega)}\left[\ln\left(\frac{\omega}{\omega_0}\right)^g \left(\frac{1+\omega}{1+\omega_0}\right)^{1-g}\right]^{3/2}.$$

Equation (32) determines ω_0 for a given temperature T. E/NE_F can be obtained by a given ω_0 through Eq. (33).

For giving the explicit entropy density expression with the generalized exclusion statistics in Sec. III C, let us make further discussion on the energy density. By eliminating N with Eqs. (28) and (33), the energy can be alternatively expressed as

$$E = \frac{\alpha(2m)^{3/2}}{4\pi^2} V T^{5/2} b(\omega_0). \tag{34}$$

The partial derivative of the internal energy E to T for fixed μ is given by

$$\left(\frac{\partial E}{\partial T}\right)_\mu = \frac{\alpha V(2m)^{3/2}}{4\pi^2} T^{3/2}\left[\frac{5}{2} b(\omega_0) + T\left(\frac{\partial b(\omega_0)}{\partial T}\right)_\mu\right]. \tag{35}$$

Furthermore, the variable ω_0 of the integral function $b(\omega_0)$ can be converted into μ and T through Eq. (23)

$$b(\omega_0,\mu,T) = \int_{\omega_0}^\infty \frac{d\omega}{\omega(1+\omega)}\left\{\ln[\omega^g(1+\omega)^{1-g}] + \frac{\mu}{T}\right\}^{3/2}. \tag{36}$$

Therefore, one can have

$$\left(\frac{\partial b}{\partial T}\right)_\mu = \frac{3}{2T}\ln[\omega_0^g(1+\omega_0)^{1-g}] a(\omega_0). \tag{37}$$

C. Entropy per particle

Due to the scaling properties, the thermodynamics of a unitary Fermi gas also satisfies the ideal gas virial theorem [3,9,19]

$$P = \frac{2}{3}\frac{E}{V}. \tag{38}$$

According to the thermodynamic relation for the entropy S and pressure P, one can have

$$S = \frac{2}{3}\left(\frac{\partial E}{\partial T}\right)_\mu. \tag{39}$$

By substituting Eqs. (35) and (37) into Eq. (39), the explicit expression for the entropy per particle is derived to be

$$\frac{S}{N} = \frac{5}{2}\left(\frac{T}{T_F}\right)^{3/2} b(\omega_0) + \ln[\omega_0^g(1+\omega_0)^{1-g}], \tag{40}$$

where ω_0 is given by Eq. (32) for a given T.

IV. NUMERICAL RESULTS AND COMPARISONS

Based on the above analytical expressions, we will give the numerical results.

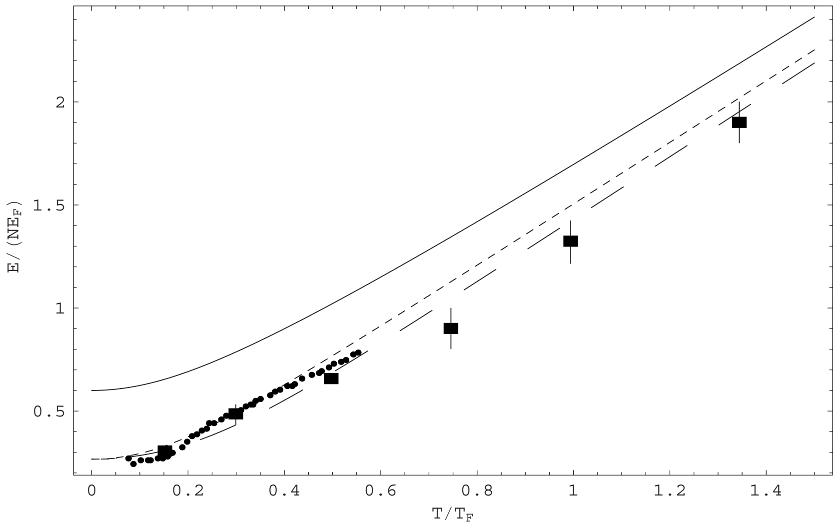

FIG. 1. The internal energy per particle versus the rescaled temperature. The solid curve denotes that for the ideal Fermi gas, and the short-dashed one is that given by the quasilinear approximation. The long-dashed curve represents the result in terms of the generalized exclusion statistics model. The dots and solid squares are the Monte Carlo calculations of [10,11], respectively.

A. Internal energy and chemical potential

From Eqs. (32) and (33), the energy per particle versus the rescaled temperature can be solved. As indicated by Fig. 1, the internal energies for the unitary Fermi gas based on the quasilinear approximation and the generalized exclusion statistics have similar analytical properties; i.e., the internal energy increases with the increase in temperature. The two approaches both show that the energy density of a unitary Fermi gas is lower than that of the ideal Fermi gas. However, the shift in the internal energy given by the quasilinear approximation is quicker than that determined by the generalized exclusion statistics model.

With Eqs. (32) and (23), we also show the chemical potential versus the rescaled temperature in Fig. 2. The chemical potential given by the two formalisms decreases with the increase in temperature. The departure between them becomes bigger with increasing temperature.

The results for the energy per particle shown in Fig. 1 in terms of the two different analytical approaches are reasonably consistent with the Monte Carlo calculations [10,11], while the chemical potential differs explicitly from the Monte Carlo result [11] for $T/T_F > 0.8$, as shown in Fig. 2.

B. Entropy

With Eqs. (32) and (40), the entropy per particle versus the rescaled temperature curve is presented in Fig. 3. The quasilinear approximation predicts that the curve is higher than that of the ideal Fermi gas, while the generalized exclusion statistics model gives lower values compared with that of the ideal Fermi gas. With the increase in temperature, the entropy per particle given by the generalized exclusion statistics becomes closer to and almost overlaps with that of the ideal Fermi gas. In terms of the quasilinear approximation, the ratio of entropy to that of the ideal Fermi gas approaches a constant in the Boltzmann regime.

Especially in the low-temperature strongly degenerate regime, the slopes of the entropy per particle versus the scaled temperature curves given by these two approaches are different. The low-temperature behavior is determined by the effective fermion mass according to the Landau theory of strongly correlated Fermi liquid. In turn, from the entropy curve, one can derive the effective fermion mass indirectly. The careful study shows that the quasilinear approximation indicates that $m^*/m \approx 1.11 > 1$, while the latter predicts $m^*/m \approx 0.70 < 1$. Compared with the latter, the quasilinear

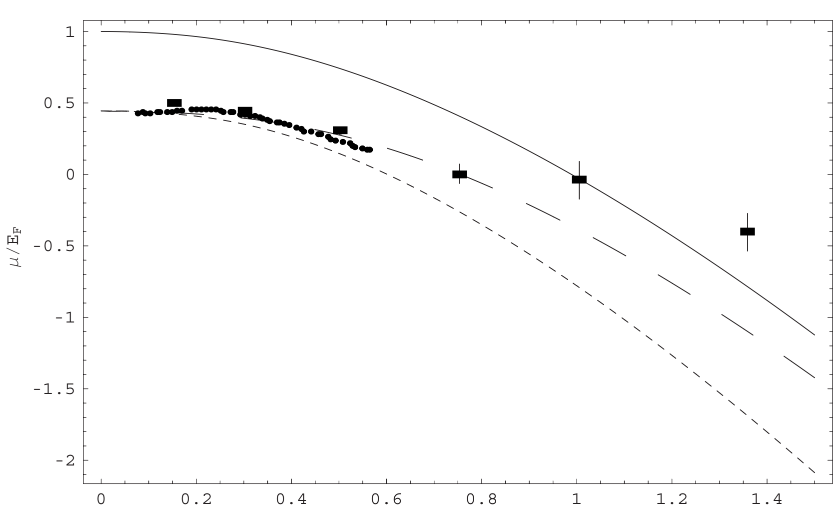

FIG. 2. Physical chemical potential versus the rescaled temperature. The line styles are similar to those in Fig. 1.

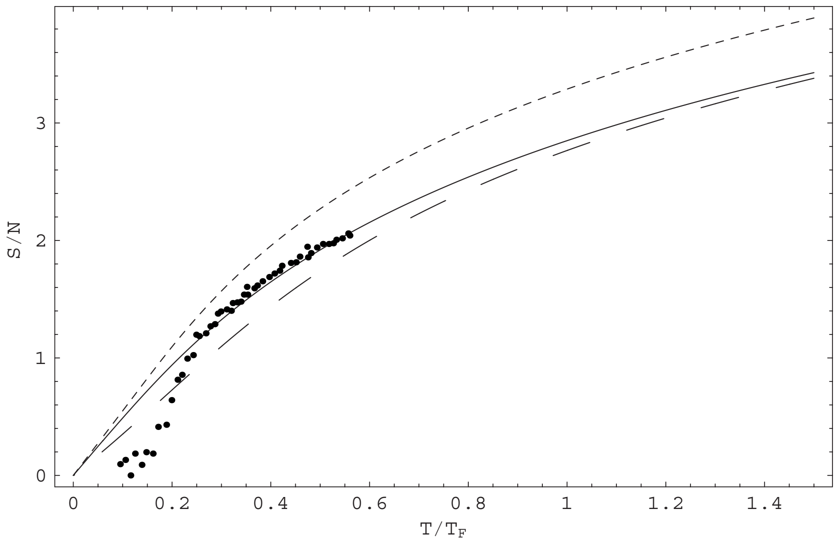

FIG. 3. Entropy per particle versus the rescaled temperature. The line styles are similar to those in Fig. 1. The Monte Carlo simulation result is extracted from Ref. [10].

approximation result is more consistent with the Monte Carlo calculations of $m^*/m \sim 1.04-1.09$ [16,17].

V. CONCLUSION

In terms of the quasilinear approximation method and generalized exclusion statistics model, the internal energies, chemical potentials, and entropies of a unitary Fermi gas have been analyzed in detail. The two different approximations give similar behavior for the internal energies and chemical potentials of a unitary Fermi gas.

The entropy is an important characteristic quantity in statistical mechanics. The entropy obtained by the quasilinear approximation is higher than that of the ideal noninteracting fermion gas. In the Boltzmann regime, the entropy curve given by the generalized exclusion statistics gets closer toward and almost overlaps with that of the ideal Fermi gas. The entropy given by the quasilinear approximation deviates from that of the ideal Fermi gas and the ratio of entropy to that of the ideal Fermi gas approaches a constant.

According to the quasiparticle viewpoint of the Landau Fermi-liquid theory, the slope of entropy per particle determines the effective fermion mass in the low-temperature strongly degenerate region. The numerical analysis demonstrates that the generalized exclusion statistics model gives $m^*/m \approx 0.70 < 1$. The developed quasilinear approximation predicts $m^*/m \approx 1.11 > 1$, which is closer to the updated Monte Carlo investigations.

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