Quantum-information transfer in a coupled resonator waveguide

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We propose an efficient scheme for the implementation of quantum information transfer in a one-dimensional coupled resonator waveguide. We show that, based on the effective long-range dipole-dipole interactions between the atoms mediated by the cavity modes, Raman transitions between the atoms trapped in different nodes can take place. Quantum information could be transferred directly between the opposite ends of the coupled waveguide without involving the intermediate nodes via either Raman transitions or the stimulated Raman adiabatic passages. Since this scheme, in principle, is a one-step protocol, it may provide useful applications in quantum communications.

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I. INTRODUCTION

The transfer of quantum states from one place to another is an important goal in the field of quantum information science for distributing and processing information [1,2]. To accomplish this task, several approaches have been employed. For long distance quantum communications, optical systems such as cavity QED system [3,4] are used to transfer states from one node to another through photons transmitting in a fiber [5,6]. For the case of short distance quantum communications, spin chains are proposed [7]. In spin chains, single spin addressing is difficult because the spatial separation between neighboring spins is very small. Thus the control over the couplings between the spins or over individual spins is very hard to achieve. Therefore, these protocols based on spin chains have some drawbacks, which impair the performance for quantum information transfer (QIT). Recently coupled resonator waveguide has attracted great interests [8–14]. We have proposed a protocol for generating atomic cluster states using coupled resonators for one-way quantum computation [15]. Coupled resonator waveguide has the advantage of easily addressing individual lattice sites with optical lasers. Furthermore, the atoms trapped in the resonators can have relatively long-lived atomic levels for encoding quantum information. Therefore, it is desirable to develop a technique for implementing short distance quantum communications in a coupled resonator waveguide.

In this work, we propose a scheme for the implementation of QIT between three-state atoms trapped in a one-dimensional coupled resonator waveguide. We first demonstrate that the coupled system can be reduced to an effective Λ configuration which supports Raman transitions between the first atom and the end one. Then we utilize this protocol to implement short distance quantum communications. This proposal exploits the effective long-range dipole-dipole interactions mediated by the cavity modes between the atoms. The nonlocal interactions combined with lasers are utilized to induce Raman transitions between the atoms trapped in the two ends of the waveguide via the exchange of virtual cavity photons. Quantum states can be transferred directly from the first node to the end one within the one-dimensional coupled resonator waveguide, through either Raman transitions or the stimulated Raman adiabatic passages (STIRAPs) [16]. This proposal for QIT in a network using coupled resonators should provide very interesting applications in the field of quantum information processing, such as entanglement distribution, teleportation [17], and distributed quantum computation [18]. Experimentally this protocol could be realized with the state-of-the-art technology.

II. QUANTUM-INFORMATION TRANSFER IN A ONE-DIMENSIONAL COUPLED RESONATOR WAVEGUIDE

Consider a one-dimensional coupled resonator waveguide consisting of N nodes, as sketched in Fig. 1. The coupled resonator waveguide can be realized in a wide range of physical systems, such as nanocavities in photonic crystals [19], and superconducting transmission line resonators [20]. To implement QIT, each node consists of a cavity and a trapped three-state atom. Each atom has the level structure of a three-state system with two lower states $|0\rangle_j$ and $|1\rangle_j$ ($j=1,2,\ldots,N$) for storage of one qubit of quantum information, and an upper state $|e\rangle_j$. The cavity mode is far detuned

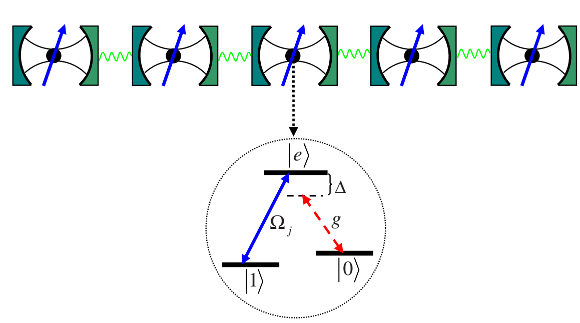

FIG. 1. (Color online) Schematic diagram of a one-dimensional coupled resonator waveguide consisting of N nodes and three-state atoms trapped in each resonator. The transition $|0\rangle \leftrightarrow |e\rangle$ is strongly detuned from the cavity modes, which induces a long-range interaction between the atoms.

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from the atomic transition $|0\rangle_j \leftrightarrow |e\rangle_j$ (transition frequency ω_0) with the coupling constant g and detuning Δ. The transition $|1\rangle_j \leftrightarrow |e\rangle_j$ (transition frequency ω_1) in each atom is driven resonantly with lasers (frequency $\omega_{Lj}=\omega_1$) with Rabi frequencies Ω_j.

We consider each trapped atom interacting with the cavity fields and lasers. The Hamiltonian that describes the photons in the cavity modes is [8–13]

$$\hat{H}_c = \omega_c \sum_{j=1}^{N} a_j^\dagger a_j + J_c \sum_{j=1}^{N} (a_j^\dagger a_{j+1} + a_j a_{j+1}^\dagger), \tag{1}$$

where a_j is annihilation operator for the photon in cavity j and J_c is the hopping rate of photons between neighboring cavities. For convenience we introduce the notation $\mathbf{J} = (uj,0,0)$ to denote the position of the jth site where u is the length of the one-dimensional crystal cell. If the periodic boundary conditions are considered, $\hat{H}_c$ can be diagonalized through the Fourier transformation. Then we obtain $\hat{H}_c = \Sigma_k \omega_k a_k^\dagger a_k$, where $\omega_k = \omega_c + 2J_c \cos k$ and $k=(2\pi m)/(Nu)$ for $m=0,1,\ldots,N-1$. Under the rotating wave and dipole approximations, the interaction between the atoms and cavity fields is

$$\hat{H}_{ac} = \sum_{j=1}^{N} g(a_j^\dagger |0\rangle_j \langle e| + a_j |e\rangle_j \langle 0|), \tag{2}$$

and the interaction between the atoms and lasers reads

$$\hat{H}_L = \sum_{j=1}^{N} (\Omega_j e^{-i\omega_1 t} |e\rangle_j \langle 1| + \text{H.c.}). \tag{3}$$

Here we add a laser to each resonator for generality, but in the following when we discuss how to implement QIT, we in fact only require the lasers added to the first and the last cavities be switched on. In the interaction picture the Hamiltonian that governs the coupled system is

$$\hat{H}_I = \sum_{j=1}^{N} \left[\Omega_j |e\rangle_j \langle 1| + \sum_k g/\sqrt{N} |0\rangle_j \langle e| a_k^\dagger e^{i\mathbf{k}\cdot\mathbf{J}+i\delta_k t} + \text{H.c.} \right], \tag{4}$$

with $\delta_k = \omega_k - \omega_0$. To further reduce the model, we assume $\delta_k \gg g$ (for all k), then we can adiabatically eliminate the photons from the above description [21,22]. By considering the terms up to second order and dropping the fast oscillating terms, we obtain the following effective Hamiltonian:

$$\hat{H}_{\text{eff}} = \sum_{j=1}^{N} \left[J_0 |e\rangle_j \langle e| + \left(\Omega_j |e\rangle_j \langle 1| + \sum_{l=1}^{N} J_l |0\rangle_j \langle e| \otimes |e\rangle_{j+l} \langle 0| + \text{H.c.} \right) \right], \tag{5}$$

with $J_0 = \Sigma_k [g^2/(N\delta_k)]$, $J_l = \Sigma_k [g^2 e^{ikl}/(N\delta_k)]$, and the conventions $|e\rangle_{N+i}\langle 0| \equiv 0$, $(i=1,2,\ldots,N)$. The first term corresponds to the level shift for each atom, the second term describes the interactions between atoms and lasers, and the last term represents the effective dipole coupling of trapped atoms induced by cavity modes.

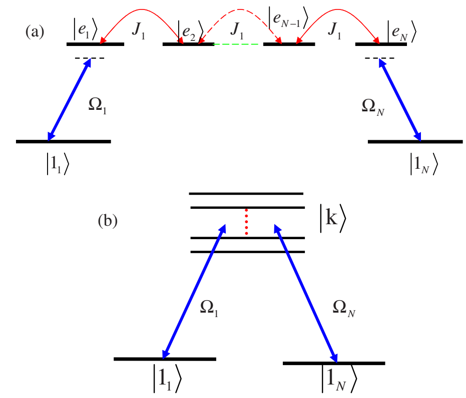

FIG. 2. (Color online) (a) Schematic diagram of the system under the interaction Hamiltonian (5). (b) Coupling configuration corresponding to (a) in a new basis $\{|\mathbf{1}_1\rangle, |\mathbf{k}\rangle \ldots, |\mathbf{1}_N\rangle\}$.

We introduce the states $|\mathbf{1}_i\rangle = |000\ldots 1_i \ldots 000\rangle$ and $|\mathbf{e}_i\rangle = |000\ldots e_i \ldots 000\rangle$, which denote that the atom at the ith site has been flipped to the state $|1\rangle$ and $|e\rangle$ while other atoms stay in $|0\rangle$. Assuming that only the lasers Ω_1 and Ω_N are switched on and the system initially stays in $|\mathbf{1}_1\rangle$, then the coupling scheme can be schematically illustrated in Fig. 2(a). To gain more insight into this coupling configuration, we turn to a new basis $\{|\mathbf{1}_1\rangle, |\mathbf{k}\rangle \ldots, |\mathbf{1}_N\rangle\}$, where $|\mathbf{k}\rangle = 1/\sqrt{N}\Sigma_{j=1}^{N} e^{ikj} |\mathbf{e}_j\rangle$. Then we could diagonalize the effective dipole coupling $V_d = \Sigma_{j=1}^{N} [J_0 |e\rangle_j \langle e| + \Sigma_{l=1}^{N} (J_l |0\rangle_j \langle e| \otimes |e\rangle_{j+l} \langle 0| + \text{H.c.})]$ in this subspace. The eigenstates of V_d are $\{|\mathbf{k}\rangle, k = (2\pi m)/N (m=0,1,\ldots,N-1)\}$. The eigenvalues are given by $E_k = J_0 + \Sigma_{l=1}^{N} 2J_l \cos(kl)$. Thus we can write V_d as $V_d = \Sigma_k E_k |\mathbf{k}\rangle\langle \mathbf{k}|$. In such a case, the effective Hamiltonian $\hat{H}_{\text{eff}}$ can be rewritten in the subspace $\{|\mathbf{1}_1\rangle, |\mathbf{k}\rangle \ldots, |\mathbf{1}_N\rangle\}$ as

$$\hat{H}_{\text{eff}} = \sum_k [E_k |\mathbf{k}\rangle\langle \mathbf{k}| + (\Omega_{1k} |\mathbf{k}\rangle\langle \mathbf{1}_1| + \Omega_{2k} |\mathbf{k}\rangle\langle \mathbf{1}_N| + \text{H.c.})], \tag{6}$$

with $\Omega_{1k} = \langle \mathbf{k}|\hat{H}_{\text{eff}}|\mathbf{1}_1\rangle = \Omega_1 e^{ik}/\sqrt{N}$ and $\Omega_{2k} = \langle \mathbf{k}|\hat{H}_{\text{eff}}|\mathbf{1}_N\rangle = \Omega_N e^{iNk}/\sqrt{N}$. The schematic diagram of this coupling configuration in this new basis is shown in Fig. 2(b), from which we see that Hamiltonian (6) describes an effective Λ system, with two lower states $|\mathbf{1}_1\rangle$, $|\mathbf{1}_N\rangle$ and several upper states $|\mathbf{k}\rangle$. Under the conditions $E_k \gg \{\Omega_{1k}, \Omega_{2k}\}$, Raman transitions can take place between the states $|\mathbf{1}_1\rangle$ and $|\mathbf{1}_N\rangle$. Through adiabatic elimination of the states $|\mathbf{k}\rangle$, the effective Hamiltonian describing this case is

$$\hat{H}_{\text{eff}} = \Theta_r |\mathbf{1}_N\rangle\langle \mathbf{1}_1| + \text{H.c.}, \tag{7}$$

with $\Theta_r = \Sigma_k \Omega_{1k} \Omega_{2k}^* / E_k$ as the effective Raman transition rate. This Hamiltonian describes direct Raman transitions between the first node and the last one, assisted by the intermediate nodes through virtual photon exchanges.

We now discuss how to implement QIT in this one-dimensional coupled resonator waveguide. We assume that the state sender Alice is located at the first node and the state receiver Bob is located at the end of the waveguide. Alice wants to transfer an unknown state to Bob through this waveguide. To start the protocol, Alice places the atom at the first site in the arbitrary unknown state $\alpha|0\rangle_1+\beta|1\rangle_1$, while the atoms in other nodes are prepared in the state $|0\rangle$. We can describe the state of the whole system at this instant as $|\Psi(0)\rangle=\alpha|\mathbf{0}\rangle+\beta|\mathbf{1}_1\rangle$, with $|\mathbf{0}\rangle=|000\ldots0\rangle$. Then under the interaction of Eq. (7), the state vector at the time t is

$$|\Psi(t)\rangle = \alpha|\mathbf{0}\rangle + \beta[\cos(\Theta_r t)|\mathbf{1}_1\rangle - i\sin(\Theta_r t)|\mathbf{1}_N\rangle]. \quad (8)$$

At the moment $\Theta_r t_f=\pi/2$ they turn off the couplings and Bob gets the state $|\Psi(t_f)\rangle=\alpha|\mathbf{0}\rangle-i\beta|\mathbf{1}_N\rangle$. If Bob performs a gate operation $U=(1,i)$, he could retrieve the state $\alpha|0\rangle_N+\beta|1\rangle_N$ for the atom N. The procedure completes QIT inside the one-dimensional coupled resonator waveguide, which in principle could be extended to realize short distance quantum communications. Different from the schemes based on spin chains for short distance quantum communications, the principle advantage of this scheme is that, in the coupled resonator waveguide, individual lattice sites can be addressed with optical lasers. Therefore, it is much easier to switch the interactions of the qubit on which the initial state is encoded and the qubit on which the final state is received with the rest of the waveguide in this proposal.

It is noted that QIT can also be implemented through STIRAP techniques [16] with this one-dimensional coupled resonator waveguide. In such a case, we require the lasers to select a resonant transition from the initial state $|\mathbf{1}_1\rangle$ to the final state $|\mathbf{1}_N\rangle$ via an intermediate state such as $|\tilde{\mathbf{k}}\rangle$, while other transition channels are far off resonance. Then the system is reduced to a typical Λ configuration, which supports a dark state involving the two states $|\mathbf{1}_1\rangle$ and $|\mathbf{1}_N\rangle$. Adiabatic passage following the dark state can be implemented by varying the Rabi frequencies slowly. Then an arbitrary unknown state $\alpha|0\rangle_1+\beta|1\rangle_1$ can be transferred directly from the first atom to the end one following the STIRAP.

It is necessary to verify the model through numerical simulations. We consider the case of QIT in three coupled resonators. The system is initially prepared in the state $\frac{1}{\sqrt{2}}(|0\rangle_1+|1\rangle_1)|0\rangle_2|0\rangle_3$. Employing a quantum master equation approach, we have simulated the dynamics of the system through the Monte Carlo wave function (MCWF) formalism [23,24]. In Fig. 3 the numerical solutions of the density matrix equations for the full system described by the exact Hamiltonian H are shown together with the dynamics of the system undergoing the effective Hamiltonian (7). Here the parameters are chosen such that they are within the parameter range for which the scheme is valid (discussed in the next paragraph). It is clear that the agreement between the exact and effective models is excellent under the given parameters. The system starts from the state $\frac{1}{\sqrt{2}}(|0\rangle_1+|1\rangle_1)|0\rangle_2|0\rangle_3$. At the time $t=\pi/2\Theta_r$, the first atom evolves into its ground state $|0_1\rangle$ and the third atom evolves into $\frac{1}{\sqrt{2}}(|0\rangle_3-i|1\rangle_3)$. This process completes the procedure for QIT between these two atoms. During this process, the populations of the atomic excited states and the cavity modes keep small.

We now study the performance of this protocol under realistic circumstances and estimate the range of parameters

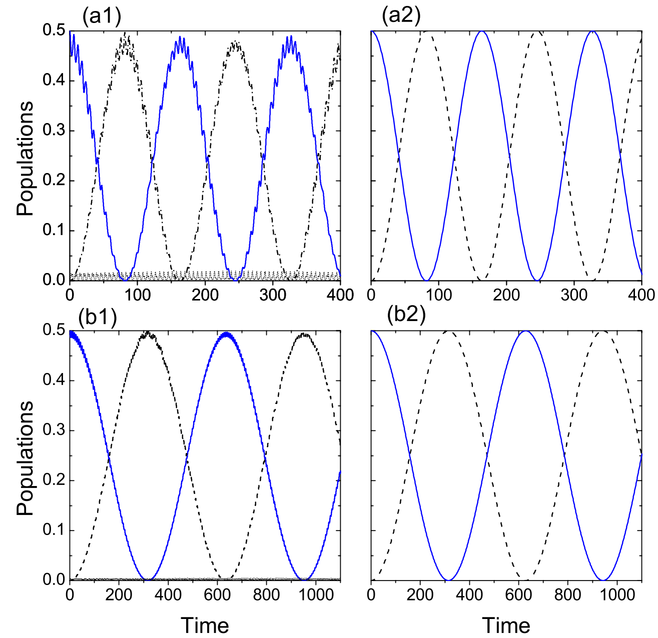

FIG. 3. (Color online) Evolution of the system from both exact calculations of the master equation [(a1) and (b1)] and the solutions for the effective Hamiltonian (7) [(a2) and (b2)]. In all the figures, solid lines represent the population of $|1\rangle_1|0\rangle_2|0\rangle_3$ and dot lines represent the population of $|0\rangle_1|0\rangle_2|1\rangle_3$. The parameters are chosen as $\Delta=10g$, $J_c=0.5g$ for [(a1) and (a2)] $\Omega_1=\Omega_3=0.02g$ and for [(b1) and (b2)] $\Omega_1=\Omega_3=0.01g$. Time is measured in unit of g^{-1}.

implementing optimal QIT. Consider a one-dimensional coupled resonator waveguide consisting of N nodes. Alice wants to transfer an arbitrary quantum state from the first node to Bob who is located at the end. To quantify the performance of QIT, we utilize the fidelity $F=\langle\psi_p|\rho_f|\psi_p\rangle$, where $|\psi_p\rangle$ refers to the perfectly transferred state and ρ_f is the final reduced density matrix of the last atom under realistic circumstances. The fidelity is reduced due to the small probabilities of populating either the atomic excited states or the cavity modes. For this protocol, spontaneous emission from the state $|e\rangle_j$ at a rate γ and cavity decay of photons at a rate κ lead to effective decay rates $\Gamma_E=\Sigma_k|\Omega_k/E_k|^2\gamma$ and $\Gamma_C=\Sigma_k|g/(\sqrt{N}\delta_k)|^2\kappa$, with $\Omega_k=\max(\Omega_{1k},\Omega_{2k})$. Hence to achieve coherent interaction requires that $\{\Gamma_E,\Gamma_C\}<\Theta_r$. These requirements could be satisfied if $\gamma\ll J_C g^2/\Delta^2$ and $\kappa\ll J_C$. Since photons are more likely to tunnel to the next cavity than decay into free space, $\kappa\ll J_C$ should hold in most cases. For the condition $\gamma\ll J_C g^2/\Delta^2$ to hold, cavities with a high ratio g/γ are very good candidates. These two requirements together imply that the cavities should have a high cooperativity factor. To make sure this scheme is valid, we also require that $\Delta\gg g$ and $g^2/\Delta\gg\Omega_i$. Taking into account these probabilities of error, the fidelity is estimated as $F\simeq 1-\Gamma_E t_f-\Gamma_C t_f$, where $t_f=\pi/2\Theta_r$ is the time to complete QIT.

For experimental implementation of QIT in a coupled resonator waveguide, atoms or polar molecules trapped in coupled superconducting stripline microwave resonators [20,25] are promising candidates. It is noted that hybrid devices combining solid state circuits with trapped atoms or molecules have been explored [26]. We choose the param-

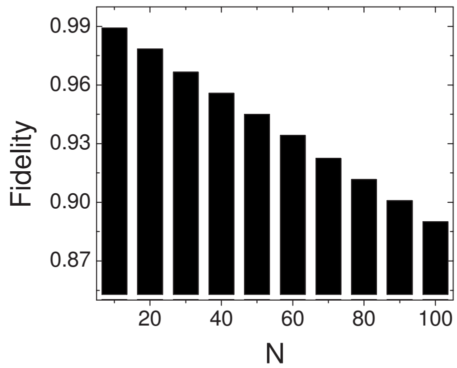

FIG. 4. Fidelity bars for different waveguide lengths N. Parameters are given in the text.

eters as $g \sim 2\pi \times 200$ MHz, $\Delta \sim 2\pi \times 2$ GHz, $J_C \sim 2\pi \times 100$ MHz, $\gamma \sim 2\pi \times 20$ kHz, $\kappa \sim 2\pi \times 50$ kHz [25], and $\Omega \sim 2\pi \times 2$ MHz. Then we can estimate the fidelity of this state transfer channel. In Fig. 4 we display the fidelity for various waveguide lengths N. We see that as the cavity number increases the fidelity decreases. For a waveguide consisted of 100 coupled resonators, the fidelity is about 88% and the time to complete QIT is $t_f \sim 0.01$ μs. Thus the number of cavities should be within 100 to make this scheme efficient. To improve the fidelity and correct the error for QIT in this network, the proposed methods for quantum error correction can be utilized [27].

III. CONCLUSION

We have presented a protocol for the implementation of short distance quantum communications in a one-dimensional coupled resonator waveguide. This protocol utilizes the cavity field induced nonlocal interactions and Raman transitions between trapped atoms at the opposite ends of the waveguide. QIT could take place directly from the first node to the end one without involving the intermediate nodes, which represents an interesting step toward realizing quantum communications.

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