High-fidelity atomic-state teleportation protocol with non-maximally-entangled states

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We propose a protocol of the long-distance atomic-state teleportation via cavity decay, which allows for high-fidelity teleportation even with currently available optical cavities. The protocol is based on the scheme proposed by Bose *et al.* [Phys. Rev. Lett. **83**, 5158 (1999)] but with one important modification: it employs non-maximally-entangled states instead of maximally entangled states.

I. INTRODUCTION

Recent years witnessed considerable progress both in theoretical and experimental quantum information science. The long-range goal in the field is the realization of quantum networks composed of many nodes and channels. The present status of the research in the field has been reviewed in [1]. The nodes of the quantum network require quantum systems that can store quantum information for sufficiently long time and quantum channels which should allow for fast transfer of quantum information between the nodes. A single atom (or ion) can be considered as a perfect quantum memory—qubit can be stored in atomic states even for 10 s [2]. Thus, trapped atoms are candidates for being components of quantum registers or nodes of quantum networks. Fast connections between the nodes can be realized with photonic qubits which are the best carriers of quantum information. To transfer quantum information stored in one node to another node through the photonic channel, it is necessary to have effective methods for mapping atomic states into field states and back [3–7]. A number of schemes for creating entanglement and performing quantum teleportation has been proposed [8–19]. Next step would be to accomplish the long-distance atomic-state teleportation mediated by photons, but this task appears to be very challenging and has not been experimentally achieved yet.

A pretty simple way to complete a long-distance teleportation of atomic states mediated by photons was proposed by Bose *et al.* [11]. Some modifications of this protocol can also be found in [20–22]. The teleportation scheme of Bose *et al.* [11] consist of two atom-cavity systems, a 50:50 beam splitter, and two detectors as depicted in Fig. 1. With this device the teleportation can be carried out by just performing the joint detection of both cavities fields if, before detection, the sender (Alice) maps the state of her atom onto the field state of her cavity, and the receiver (Bob) creates the maximally entangled state of his atom and his cavity field. Recent progress in technology allows for such state mapping [6,7] and performing the joint detection [23]. Creation of the maximally entangled state of the atom-cavity system also should be possible with the current technology. However, the Bose *et al.* protocol [11] is hardly feasible because the fidelity of state mapping is drastically reduced by large damping values of the currently available cavities.

In this paper we propose a modification of the Bose *et al.* scheme [11] consisting in exploiting, instead of the maxi-

mally entangled state, a non-maximally-entangled state with the amplitudes chosen in such a way that the damping factors introduced by the state mapping are fully compensated for. With this modification of the protocol, it should be possible to achieve high teleportation fidelities even with currently available cavities. The price we have to pay for the higher fidelities is a lower probability of success.

II. TELEPORTATION PROTOCOL WITH NON-MAXIMALLY-ENTANGLED STATES

First, let us present the main idea in a simplified way—comparing it to the standard teleportation protocol [24,25]. In the standard teleportation protocol Alice has unknown to her (and to Bob) qubit $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ and one qubit of the Einstein-Podolsky-Rosen (EPR) pair. The second qubit of the EPR pair is on Bob's site. Suppose, however, that we have the situation depicted in Fig. 1. The state to be teleported is initially stored in the Alice atom and next is mapped using the laser to the cavity field qubit, but the mapping is not
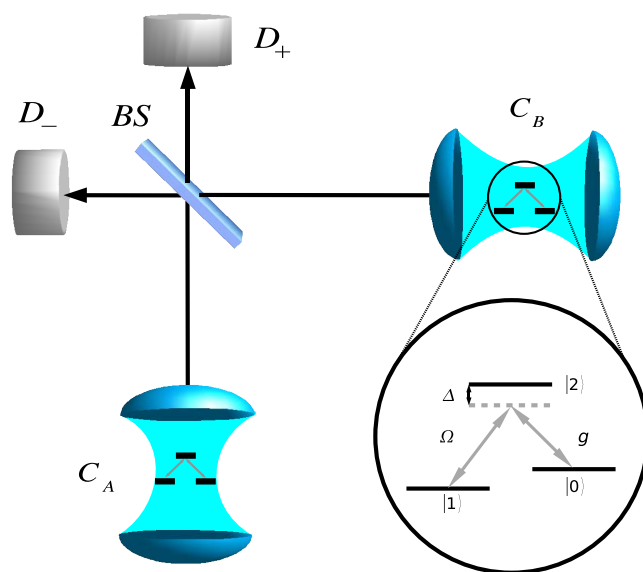

FIG. 1. (Color online) The teleportation device and level scheme of the Λ atom interacting with the classical laser field with coupling strength Ω and with the quantized cavity mode with the coupling strength g. Both fields are detuned from the corresponding transition frequencies by Δ.
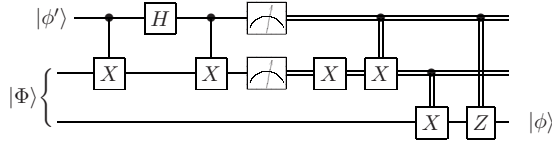
FIG. 2. Modified teleportation circuit.

perfect, and the initial state is slightly distorted. Let the state of the Alice cavity field takes the form

$$|\phi'\rangle = \mathcal{N}(\alpha|0\rangle + \zeta\beta|1\rangle), \quad (1)$$

where $\mathcal{N} = 1/\sqrt{|\alpha|^2 + |\zeta|^2|\beta|^2}$ is the normalization factor and ζ is a parameter that measures to what degree the original state has been distorted. If there is no distortion $\zeta = 1$, and the state is just the original state. It is important that the parameter ζ does not depend on the original state (it does not depend on α and β) but depends solely on the mapping procedure which is known for both parties of the protocol. Both parties can agree on the details of the procedure in advance. Now the question arises: can we use our knowledge of ζ to improve the fidelity of the teleported state?

The standard teleportation protocol would teleport the distorted state [Eq. (1)] to Bob. However, if we choose the non-maximally-entangled state, instead of the maximally entangled state, in the teleportation protocol, we can correct the imperfections introduced by the mapping procedure by using a slightly modified teleportation protocol. The teleportation circuit for this protocol is illustrated in Fig. 2.

The first qubit is the Alice cavity field qubit, the state of which is initially the state $|\phi'\rangle$ given by Eq. (1), and the state $|\Phi\rangle$ is the non-maximally-entangled state given by

$$|\Phi\rangle = a|10\rangle + b|01\rangle. \quad (2)$$

The overall initial state is thus

$$|\Psi_0\rangle = |\phi'\rangle \otimes |\Phi\rangle = \mathcal{N}(\alpha|0\rangle + \zeta\beta|1\rangle) \otimes (a|10\rangle + b|01\rangle)$$

$$= \mathcal{N}[\alpha|0\rangle(a|10\rangle + b|01\rangle) + \zeta\beta|1\rangle(a|10\rangle + b|01\rangle)]. \quad (3)$$

It is easy to show using the circuit from Fig. 2 that just before the measurements, the state is given by

$$|\Psi_1\rangle = \frac{\mathcal{N}}{\sqrt{2}}[|00\rangle(b\alpha|1\rangle + a\zeta\beta|0\rangle) + |11\rangle(b\alpha|1\rangle - a\zeta\beta|0\rangle) + |01\rangle$$

$$\times (a\alpha|0\rangle + b\zeta\beta|1\rangle) + |10\rangle(a\alpha|0\rangle - b\zeta\beta|1\rangle)]. \quad (4)$$

Now, we see that if we prepare the non-maximally-entangled state [Eq. (2)] in such a way that $a = \zeta b$ we obtain

$$|\Psi_1\rangle = \frac{\mathcal{N}b}{\sqrt{2}}\{[|00\rangle(\alpha|1\rangle + \zeta^2\beta|0\rangle) + |11\rangle(\alpha|1\rangle - \zeta^2\beta|0\rangle)]$$

$$+ \zeta[|01\rangle(\alpha|0\rangle + \beta|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle)]\}. \quad (5)$$

When Alice performs the measurement on her two qubits, there are two cases when only one of the detectors registers a photon, and the state is projected either to $|01\rangle$ or $|10\rangle$. Since we assume that the beam splitter is used in the measuring apparatus, only the two outcomes are considered as successful because the beam splitter can only distinguish two states

from the Bell basis. The other two outcomes are rejected as unsuccessful. Alice next communicate to Bob, using the classical channel, the results of her measurement (two classical bits), and Bob applying the postmeasurement operations shown in Fig. 2, can recover the original Alice's state $|\phi\rangle$ with the perfect fidelity.

Of course, the teleportation scheme depicted in Fig. 2 works perfectly well as the standard teleportation protocol when the measuring device can distinguish all four Bell states, the original undistorted state $|\phi\rangle$ is initially on the first qubit ($\zeta = 1$), and the shared entangled state $|\Phi\rangle$ is the maximally entangled state ($a = b = 1/\sqrt{2}$).

III. PHYSICAL MODEL

In the first stage of teleportation protocol, when Alice has to map the initial state of her atom $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ onto the field state of her cavity and when Bob has to create an entangled state of his atom and his cavity field, the most important role in the teleportation protocol play the two atom-cavity systems. Let us first describe them in more detail. Alice and Bob can change the state of their own atom-cavity system by switching their lasers on. When the laser illuminates the atom trapped inside the cavity then the evolution of the atom-cavity system is governed by the effective non-Hermitian Hamiltonian ($\hbar = 1$ here and in the following),

$$H = (\Delta - i\gamma)\sigma_{22} + (\Omega\sigma_{21} + ga\sigma_{20} + \text{H.c.}) - i\kappa a^\dagger a, \quad (6)$$

where $\sigma_{ij} \equiv |i\rangle\langle j|$ denote the atomic flip operators and a denotes the annihilation operator of the cavity field mode. One mirror in each cavity is partially transparent to allow for the joint measurement of the fields leaking out from both cavities. Of course, the transparency of the mirror leads to a damping of the cavity field mode. We assume that photons leak out of the cavity at a rate 2κ. For simplicity, we neglect the spontaneous decay rate of the excited atomic state γ. This approximation is valid if conditions $\Delta \gg g$, Ω, γ and $\gamma g^2/\Delta^2, \gamma\Omega^2/\Delta^2 \ll \kappa$ are fulfilled [26]. We can further simplify Hamiltonian (6) assuming that $\Omega = g$. Then, after adiabatic elimination of the excited atomic state, the Hamiltonian takes the form

$$H = -\delta\sigma_{11} - \delta a^\dagger a\sigma_{00} - (\delta a\sigma_{10} + \text{H.c.}) - i\kappa a^\dagger a, \quad (7)$$

where $\delta = g^2/\Delta$. Using Hamiltonian (7) one can easily get analytical expressions describing evolution of the initial quantum states $|0\rangle_{\text{atom}}|0\rangle_{\text{mode}}$ and $|1\rangle_{\text{atom}}|0\rangle_{\text{mode}}$. First of the states experiences no dynamics because there is no operator in Hamiltonian (7) which can change this state. The evolution of the second state is given by

$$e^{-iHt}|10\rangle = e^{i\delta t}e^{-\kappa t/2}[ia(t)|01\rangle + b(t)|10\rangle], \quad (8)$$

where we abbreviate the atom-cavity state $|j\rangle_{\text{atom}} \otimes |n\rangle_{\text{mode}}$ to $|jn\rangle$ and we use

$$a(t) = \frac{2\delta}{\Omega_\kappa} \sin\left(\frac{\Omega_\kappa t}{2}\right),$$

$$b(t) = \cos\left(\frac{\Omega_\kappa t}{2}\right) + \frac{\kappa}{\Omega_\kappa} \sin\left(\frac{\Omega_\kappa t}{2}\right), \tag{9}$$

where $\Omega_\kappa = \sqrt{4\delta^2 - \kappa^2}$. If the laser is turned off ($\Omega = 0$) then the Hamiltonian takes the form $H = -\delta a^\dagger a \sigma_{00} - i\kappa a^\dagger a$ and then the time evolution of the system can be obtained using the relations

$$e^{-iHt}|10\rangle = |10\rangle,$$

$$e^{-iHt}|01\rangle = e^{i\delta t}e^{-\kappa t}|01\rangle. \tag{10}$$

Equations (10) are needed to describe evolution of the device state during the second stage, in which the joint measurement of both cavities fields is performed. At this stage of the protocol the most important role play the detectors D_+ and D_- together with the beam splitter BS. Registration of the photon emission by one of the detectors corresponds to the action of the collapse operator on the joint state of Alice's and Bob's systems. The collapse operator has the form

$$C = \sqrt{\kappa}(a_A + i\epsilon a_B), \tag{11}$$

where ϵ is 1 for photon detection in D_+ and -1 for photon detection in D_-.

IV. TELEPORTATION VIA CAVITY DECAY WITH NON-MAXIMALLY-ENTANGLED STATES

Now, we can analyze the modified teleportation protocol which makes it possible to compensate fully for the destructive effect of cavity decay and, as we believe, it could be realized even with currently available cavities. The teleportation protocol consists of three stages, so it is as simple as the original teleportation protocol of Bose *et al.* [11]. The three stages are (A) the preparation stage, (B) the detection stage, and (C) the recovery stage. At the beginning of the protocol Alice's atom is prepared in a state, which is unknown for Alice. Bob's atom is prepared in the state $|1\rangle_{\text{atom}}$. Initially the field modes of both cavities are empty, so the states of both atom-cavity systems are given by

$$|\psi\rangle_A = |\phi\rangle_{\text{atom}} \otimes |0\rangle_{\text{mode}} = \alpha|00\rangle_A + \beta|10\rangle_A, \tag{12}$$

$$|\psi\rangle_B = |10\rangle_B. \tag{13}$$

As we have mentioned above, Alice has to map the state stored in her atom onto the field state of her cavity in the preparation stage. She can do it by just turning her laser on for the time $t_A = (2/\Omega_\kappa)[\pi - \arctan(\Omega_\kappa/\kappa)]$ [11,27]. After this operation her atom-cavity system is found to be in the state

$$|\tilde{\psi}\rangle_A = \alpha|00\rangle_A + ie^{i\delta t_A}e^{-\kappa t_A/2}\beta|01\rangle_A. \tag{14}$$

It is seen that the state mapping is done although it is imperfect because of the damping factor $e^{-\kappa t_A/2}$. We cannot avoid this damping factor, but we can show that it is possible to compensate for it. To this aim, in the modified teleportation protocol, Bob creates a non-maximally-entangled state instead of creating maximally entangled state as it is done in the standard teleportation protocol. He turns his laser on for time t_B changing his system state to

$$|\tilde{\psi}\rangle_B = e^{-\kappa t_B/2}[ia(t_B)|01\rangle_B + b(t_B)|10\rangle_B]. \tag{15}$$

The expression for t_B will be given later. Now, we have to derive the expression for probability that the first stage is successful. The preparation stage will succeed only under the absence of photon detection event. Probabilities that no collapse occurs during Alice's and Bob's operations are given by the squared norms of the state vectors (14) and (15), respectively. They are given by

$$P_A = |\alpha|^2 + e^{-\kappa t_A}|\beta|^2,$$

$$P_B = e^{-\kappa t_B}[|a(t_B)|^2 + |b(t_B)|^2]. \tag{16}$$

Alice and Bob complete their actions in the same instant of time. Then they turn the lasers off and the detection stage starts. Alice during the second stage just waits for a finite time $t_D \gg \kappa^{-1}$ registering events of photon detection. This stage and the whole teleportation protocol is successful when Alice registers one, and only one, photon. In other cases, when Alice registers no photon or when she registers two photons, the initial Alice's state is destroyed. Until the time of photon detection t_j the evolution of the state of both atom-cavity systems is given by (10), and at time t_j both systems states are described by

$$|\tilde{\psi}(t_j)\rangle_A = \frac{1}{\sqrt{P_A}}(ie^{i\delta(t_A + t_j)}e^{-\kappa(t_A + 2t_j)/2}\beta|01\rangle_A + \alpha|00\rangle_A), \tag{17}$$

$$|\tilde{\psi}(t_j)\rangle_B = \frac{e^{-\kappa t_B/2}}{\sqrt{P_B}}[ia(t_B)e^{i\delta t_j}e^{-\kappa t_j}|01\rangle_B + b(t_B)|10\rangle_B]. \tag{18}$$

The probability of no photon emission before time t_j is given by $P_D(t_j) = P_A(t_j)P_B(t_j)$, where

$$P_A(t_j) = \frac{|\alpha|^2 + e^{-\kappa(t_A + 2t_j)}|\beta|^2}{|\alpha|^2 + e^{-\kappa t_A}|\beta|^2},$$

$$P_B(t_j) = \frac{|a(t_B)|^2 e^{-\kappa 2t_j} + |b(t_B)|^2}{|a(t_B)|^2 + |b(t_B)|^2}. \tag{19}$$

At time t_j one of the detectors registers a photon emission, what corresponds to the change in the joint state of both atom-cavity systems. After the collapse the joint state is given by $|\tilde{\phi}(t_j)\rangle = C|\psi(t_j)\rangle_A \otimes |\psi(t_j)\rangle_B$. The probability that the collapse occurs in the time interval t_j to $t_j + dt$ can be calculated from $2\langle\tilde{\phi}(t_j)|\tilde{\phi}(t_j)\rangle dt$. After the collapse we have to normalize the state $|\tilde{\phi}(t_j)\rangle \rightarrow |\phi(t_j)\rangle$ and then the evolution of the joint state can again be determined using Eq. (10), and at the end of the detection stage, at time t_D, the state is given by $|\tilde{\phi}(t_D)\rangle = \exp[-iH(t_D - t_j)]|\phi(t_j)\rangle$. For $t_D \gg \kappa^{-1}$ the normalized joint state can be very well approximated by $|\phi(t_D)\rangle = |00\rangle_A \otimes |\psi_B(t_D)\rangle$, where
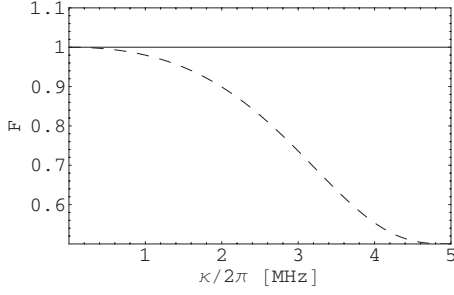
FIG. 3. The teleportation fidelity as a function of cavity decay rate κ for the modified protocol (solid line) and the original protocol (dashed line) for $(\Delta,g)/(2\pi)=(100,16)$ MHz.

$$|\psi_B(t_D)\rangle = \frac{e^{i\delta t_A}e^{-\kappa t_A/2}\beta b(t_B)|10\rangle_B + \epsilon\alpha i a(t_B)|00\rangle_B}{\sqrt{e^{-\kappa t_A}|\beta|^2|b(t_B)|^2 + |\alpha|^2|a(t_B)|^2}}. \quad (20)$$

From Eq. (20) it is seen that the unwanted damping factor $e^{-\kappa t_A/2}$ disappears if the condition

$$e^{-\kappa t_A/2}b(t_B) = a(t_B) \quad (21)$$

is satisfied. We can now give the expression for the time t_B;

$$t_B = \frac{2}{\Omega_\kappa}\left[\arctan\left(\frac{\Omega_\kappa e^{-\kappa t_A/2}}{2\delta - e^{-\kappa t_A/2}\kappa}\right) + n\pi\right]. \quad (22)$$

The time given by expression (22) is the key parameter, which must be known to Bob to create the non-maximally-entangled state (2). Time t_B is the function of t_A, so both these times must be known to Bob. Note, however, that neither time t_B nor time t_A depend on the amplitudes of the teleported state. If in the preparation stage Bob turns his laser on for time t_B then, after the detection stage, the state of his system becomes identical to the initial unknown Alice's state except for the phase factor

$$|\psi_B(t_D)\rangle = \alpha|00\rangle_B - i\epsilon e^{i\delta t_A}\beta|10\rangle_B. \quad (23)$$

Fortunately, the phase factor can be removed using the Zeeman evolution [11], what Bob performs in the last stage after receiving classical information about Alice's measurement. At the end of the whole protocol Bob has the original Alice's state stored in his atom $|\phi\rangle = \alpha|0\rangle_{\text{atom}} + \beta|1\rangle_{\text{atom}}$. It turns out that the fidelity of this teleportation protocol can be close to unity even for realistic cavity decay rates. Figure 3 shows teleportation fidelities of both protocols (the modified with non-maximally-entangled state and the Bose *et al.* protocol) as functions of the cavity decay rate.

One can see that for real cavity decay rate $\kappa/2\pi = 3.8$ MHz [6] the fidelity of teleported state is still equal unity while the fidelity of the original protocol does not exceed the value 2/3. This result is quite impressive but one can easily note that the high fidelity is not for free. Since P_A depends on the damping factor, the probability that the teleportation protocol will be successful is lowered by the increasing cavity decay rates. Let us now estimate the probability of success for currently available cavities. The probability that all stages of the protocol will succeed has the following form:
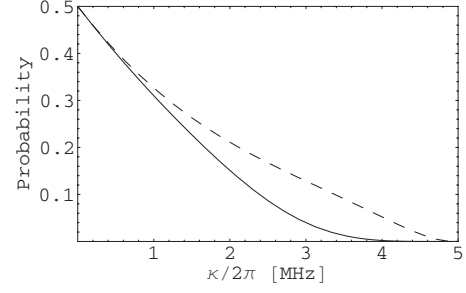

FIG. 4. Probability of success versus cavity decay rate κ for the modified protocol (solid line) and the original protocol (dashed line) for $(\Delta,g)/(2\pi)=(100,16)$ MHz.

$$P_{\text{suc}} = P_A P_B \int_0^{t_D} P_A(t_j)P_B(t_j)\langle\tilde{\phi}(t_D)|\tilde{\phi}(t_D)\rangle 2\langle\tilde{\phi}(t_j)|\tilde{\phi}(t_j)\rangle dt_j.$$

$$(24)$$

For $t_D \gg \kappa^{-1}$, the probability of success can be very well approximated by

$$P_{\text{suc}} = e^{-\kappa t_B}a(t_B)^2. \quad (25)$$

We can use this simple formula to estimate the value of the success probability for the experimental parameters of Ref. [6], i.e., we take $(g,\kappa)/2\pi = (16,3.8)$ MHz. The protocol requires, however, bigger values of the detuning than that of Ref. [6], so we take $\Delta/2\pi = 100$ MHz. With this set of parameters we get the probability of success of about 0.005, and this is the price we have to pay for getting fidelity close to unity. In Fig. 4 we plot the probability of success as a function of the cavity decay rate κ for the modified protocol and compare it to the corresponding dependence for the original protocol of Bose *et al.* [11] for the parameter values $(\Delta,g)/(2\pi)=(100,16)$ MHz. It is seen that the probability of success for the modified protocol goes to zero faster than that for the original protocol, but it still has considerable values for realistic decay rates.

Let us now take into account the important imperfection which is present in all real setups, i.e., finite detection efficiency. This imperfection is caused by absorption in the mirrors, photon losses during the propagation between the cavities and the detectors, and by nonunity detectors efficiency. In Ref. [28] the overall detection efficiency is only $\eta=0.05$. Therefore, with such efficiency only a small fraction of all successful runs will be detected. Moreover, the case of two photons emissions will be erroneously counted as a successful case if only one photon is detected. Of course, this effect would lead to lowering of the fidelity. The two-photon case is also very important if detectors cannot distinguish a single photon from two photons, since both emitted photons are always collected by the same detector [23], it is not possible to reject such unsuccessful runs. If we want to estimate the real values for the teleportation fidelity and the success probability, we have to include the efficiency η and the two-photon case in our calculations. The probability that two photons will be emitted from the cavities during the teleportation protocol and only one of them will be detected in the detection stage is given by

$$P_{2\text{em}}(\eta) = |\beta|^2 e^{-\kappa t_A}\eta(1 - \eta\xi), \qquad (26)$$

where $\xi = 1$ for photon-number-resolving detectors and $\xi = 1 - P_{\text{suc}}$ for conventional photon detectors. This probability depends on the modulus of the amplitude β which is in general unknown. Hence, it is necessary to compute the average probability of two photon emissions taken over all possible input states. Such an average probability takes the form

$$\bar{P}_{2\text{em}}(\eta) = e^{-\kappa t_A}\eta(1 - \eta\xi)/2. \qquad (27)$$

The average probability that the measurement will indicate success is then given by

$$\bar{P}_{\text{suc}}(\eta) = \eta P_{\text{suc}} + \bar{P}_{2\text{em}}(\eta). \qquad (28)$$

In the case of two-photon emissions Bob's atom is in the state $|0\rangle$. If we cannot reject all runs in which two photons were emitted then the final state of Bob's atom is a mixture of $|0\rangle$ and $|\phi\rangle$, i.e.,

$$\rho = \frac{\eta P_{\text{suc}}|\phi\rangle\langle\phi| + P_{2\text{em}}(\eta)|0\rangle\langle0|}{\eta P_{\text{suc}} + P_{2\text{em}}(\eta)}. \qquad (29)$$

We can calculate the average fidelity using the density matrix ρ. The average fidelity of the teleportation protocol is given by

$$\bar{F}(\eta) = 1/2 + P_{\text{suc}}/B - (P_{\text{suc}}/B)^2\ln(1 + B/P_{\text{suc}}), \qquad (30)$$

where $B = \exp(-\kappa t_A)(1 - \eta\xi)$. It is now evident that there is one more important feature of the protocol. Expressions (28) and (30) indicate that for large cavity decay rates, it is almost irrelevant if the detectors can distinguish a single photon from two photons. For sufficiently large cavity decay rates P_{suc} is small, and therefore, ξ for conventional detectors is close to unity. For example, the parameter's regime $(\Delta, \Omega, g, \kappa)/(2\pi) = (100, 16, 16, 3.8)$ MHz leads to $\xi = 0.995$. Hence, possible implementations of the protocol with currently available cavities do not require detectors with the single-photon resolution.

V. NUMERICAL RESULTS

The analysis of experimental feasibility of this protocol requires taking into account another imperfection, which is the spontaneous emission from the atom. We have done it using numerical calculations. In the following we present some details of the calculations. We have calculated the average fidelity and the average probability of success using Hamiltonian (6) and the quantum trajectories method [29,30]. Unfortunately, the evolution of the atom-cavity system is different than that described by Eq. (8) for the parameters of Ref. [6]: the population of the excited state $|2\rangle$ cannot be neglected and the periodic behavior of the system is lost because of the damping present in the system. Nevertheless, we can choose parameters close to that of Ref. [6] for which the average fidelity of the teleportation protocol is still high. For the well chosen parameters, times t_A and t_B should not be too long as compared to κ^{-1} and γ^{-1}. If we want to satisfy this condition, we have to set Δ to be small enough.
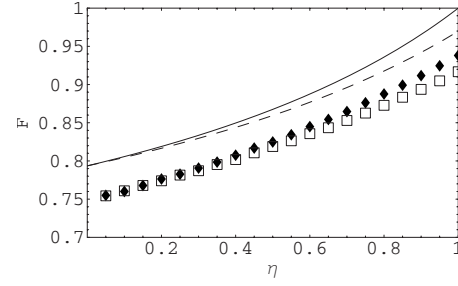

FIG. 5. The average fidelity of teleportation given by Eq. (30) for photon-number-resolving detectors (solid curve) and for conventional single-photon detectors (dashed curve) as a function of the overall detection efficiency. The diamonds and open squares refer to numerical results for photon-number-resolving and conventional detectors, respectively. The numerical results include spontaneous emission from excited atomic state and dark counts. The parameters are $(\Delta, \Omega, g, \kappa, \gamma)/(2\pi) = (62.5, 16, 16, 4, 2.6)$ MHz.

Then, however, we get considerable population of the excited state $|2\rangle$. Fortunately, this population oscillates, and we can use the fine tuning technique [31] to minimize its effect. Applying this technique we have chosen $(\Delta, \Omega, g, \kappa, \gamma)/(2\pi) = (62.5, 16, 16, 4, 2.6)$ MHz. For these parameters analytical expressions for t_A and t_B are not precise enough, and therefore we have used numerically optimized times $t_A = 0.1058$ μs and $t_B = 0.0131$ μs and not too long detection time $t_D = 4\kappa^{-1} \approx 0.16$ μs. The detection time $t_D = 4\kappa^{-1}$ is long enough to get a quite high value of the fidelity [11]. We do not set longer times of the detection stage to make the influence of the dark counts on the protocol negligible. For the dark count rate of 50 s^{-1} [32] the mean time between dark counts (10 ms for both detectors) is much larger than the time window for detection in the protocol $t_A + t_D \approx 0.3$ μs. Thus the dark counts can be neglected. Nevertheless, we have taken them into account in our numerical calculations. Results obtained from quantum trajectory approach are presented in Figs. 5 and 6.

In order to analyze the experimental feasibility of the protocol and abilities of improving the fidelity, we plot in Fig. 5 the average fidelity as a function of the overall detection
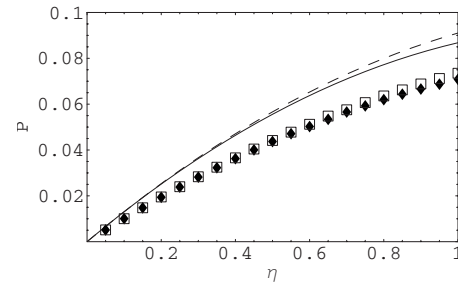

FIG. 6. The average probability of success vs the overall detection efficiency. For detectors, which are able to distinguish a single photon from two photons analytical results given by Eq. (28) are presented by the solid curve and numerical results are presented by diamonds. The dashed curve and open squares correspond to analytical and numerical results, respectively, for conventional single-photon detectors. The parameters are $(\Delta, \Omega, g, \kappa, \gamma)/(2\pi) = (62.5, 16, 16, 4, 2.6)$ MHz.

efficiency η. As it is evident from the figure, the average fidelity tends to 0.794 with decreasing η. So, the average fidelity significantly exceeds the value 2/3 even for the real overall detection efficiency $\eta=0.05$. Of course, the spontaneous decay rate $\gamma/2\pi=2.6$ MHz and dark counts of 50 s^{-1} reduce the average fidelity, but it is still well above the limit of 2/3. It is important because the average fidelity of the teleportation based on classical resources only cannot exceed this limit [33,34]. Note that the protocol makes it possible to achieve values of the fidelity much higher than 0.794. In principle, we can obtain the fidelity even close to unity, but it would require better than currently available overall detection efficiencies. The effect of the overall detection inefficiency on the teleportation protocol is much stronger than that of other imperfections present in real experimental setups. Also, the probability of success is lowered by nonideal overall detection efficiency, as it is evident from Fig. 6. The probability of success tends to zero with decreasing η. For the currently available efficiency of 0.05, the success rate has the value of 0.005, which means that it takes on average hundreds of runs to get successful teleportation. Such small probability of success means that this teleportation protocol will not have commercial applications for currently available cavities and detectors. However, this probability is big enough to perform long-distance teleportation of atomic states and test it. With present day technology 2000 trials of protocol that consists state mapping stage and detection stage last 360 ms [6] only. Therefore all data required can be collected in a reasonable time.

From Figs. 5 and 6 it is also seen that the expensive photon-number-resolving detectors are not necessary for the parameters used in our computations. For η close to unity, there is only a small difference between the fidelity obtained with the assumption that the detectors have the ability to distinguish a single photon from two photons and the fidelity obtained with assumption that the detectors have not such ability. For the real overall detection efficiency $\eta=0.05$ the difference is indistinguishable.

VI. EXPERIMENTAL FEASIBILITY OF THE PROTOCOL

Finally, we shortly discuss the realizability of our teleportation protocol. As mentioned above, almost all parameters used in our computations are feasible with current technology. The only parameter the value of which may be demanding for present technology is the detuning. In our numerical calculations we have chosen $\Delta/(2\pi)=62.5$ MHz, which is the value six times greater than that of Ref. [6]. Moreover, so far we have assumed that the laser pulses have rectangular shapes. This assumption makes it possible to examine the proposed teleportation protocol analytically and numerically. However, the shortest rising time of such pulse has duration 100 ns [6,35]. Therefore, real pulses that are approximately rectangular cannot be shorter than 1 μs. The pulses duration times used in our numerical calculations are much shorter: $t_A=0.1058$ μs and $t_B=0.0131$ μs. So, it is rather unrealistic to implement experimentally the protocol in its present form. Nevertheless, the protocol can be easily adapted for using other shapes of laser pulses. All what is actually needed to complete this teleportation protocol is the ability to perform the state mapping

$$\alpha|00\rangle + \beta|10\rangle \rightarrow \alpha|00\rangle + e^{-\kappa t/2}\beta|01\rangle, \tag{31}$$

and the ability to generate the non-maximally-entangled state

$$|10\rangle \rightarrow a(t)|01\rangle + b(t)|10\rangle, \tag{32}$$

with small $|a(t)|^2$. First of these operations have already been demonstrated experimentally [6]. The second operation can be achieved with short Gaussian pulses.

VII. CONCLUSIONS

In conclusion, we have presented a modified protocol that, in principle, should allow for atomic-state teleportation via cavity decay using currently available optical cavities. We have shown that the destructive influence of large cavity decay on the fidelity of teleported state can be minimized by using in the teleportation protocol the non-maximally-entangled state instead of the maximally entangled state. This happens despite the fact that both of them separately lead to lowering of the teleportation fidelity [36]. Advantage of using non-maximally-entangled states has been indicated also for other quantum information protocols [37].

We have also shown that there are two other distinguishing features of the protocol presented here which make it easier to implement experimentally. First is the possibility of using conventional single-photon detectors instead of the photon-number-resolving detectors. Second is the average fidelity exceeding the limit 2/3 even for very small values of the overall detection efficiency. However, the high fidelities of teleported states for real cavities can be achieved with the protocol at the expense of accepting low success rates.

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