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Resonant plasmon-soliton interaction

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We describe an effective resonant interaction between two localized wave modes of different nature: a plasmon polariton at a metal surface and a self-focusing beam (spatial soliton) in a nonlinear dielectric medium. Propagating in the same direction, they represent an exotic coupled-waveguide system, where the resonant interaction is controlled by the soliton amplitude. This nonlinear system manifests hybridized plasmon-soliton eigenmodes, mutual conversion, and nonadiabatic switching, which offer exciting opportunities for manipulation of plasmons via spatial solitons.

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I. INTRODUCTION

Plasmonics is an important and quickly developing area of modern physics which offers promising applications in nano-optics and electronics $[1-4]$ $[1-4]$ $[1-4]$. It deals with the so-called surface-plasmon polaritons $[5]$ $[5]$ $[5]$, i.e., collective oscillations of the electromagnetic field and electrons which propagate along a metal-dielectric surface and decay exponentially away from the surface.

Plasmons are characterized by frequency ω and propagation constant along the interface $k_p = k_p(\omega) > k$ $(k = \sqrt{\epsilon_0 \omega/c}$ is the wave number in the dielectric medium with the permittivity ε_0). Since $k_p > k$, plasmons can only interact resonantly with evanescent electromagnetic waves in the dielectric medium $[1-4]$ $[1-4]$ $[1-4]$. Accordingly, there are two main methods for excitations of plasmons: (i) via the evanescent wave generated at the total internal reflection $[6]$ $[6]$ $[6]$ and (ii) via a periodic structure producing evanescent modes $[4,7]$ $[4,7]$ $[4,7]$ $[4,7]$. The first method can be modified if there is a dielectric waveguiding layer parallel to the metal-dielectric interface. Propagating modes of the waveguide have evanescent tails outside the waveguide. They can interact resonantly with plasmons at the metal surface that is used for diagnosis of layered dielectric films $\lceil 8 \rceil$ $\lceil 8 \rceil$ $\lceil 8 \rceil$. The plasmon channel can also be regarded as an effective waveguide, so that this configuration can be assimilated to the problem of two coupled waveguides.

In this Rapid Communication, we propose another way of resonant interaction of plasmons with electromagnetic waves. Instead of exploiting various inhomogeneities to generate evanescent modes, we introduce a *nonlinear* dielectric medium. A nonlinear dielectric admits self-focusing solu-tions (spatial solitons) [[9](#page-3-6)] which may propagate parallel to the metal-dielectric interface and, similarly to the modes of the dielectric waveguide, also have evanescent tails and propagation constant $k_s > k$ [[10](#page-3-7)]. A "quasiwaveguide" is formed by the soliton profile which modifies the effective dielectric constant of the nonlinear medium. Hence, the soliton-plasmon configuration couples effective linear and nonlinear waveguides, as shown in Fig. [1,](#page-0-0) and an interaction between the plasmon and soliton may occur at certain resonant parameters. In contrast to previous studies in nonlinear plasmonics, where the effects of nonlinearity on plasmon modes have been considered (see, e.g., $[11]$ $[11]$ $[11]$), here we examine the interaction between two spatially separated modes: soliton and *linear* plasmon. This system is essentially selfinfluencing—the coupling is controlled by the soliton *amplitude* rather than by the configuration parameters.

II. MODEL

To create a model describing the plasmon-soliton system (Fig. [1](#page-0-0)), we adopt several simplifying assumptions. First, the waves propagate along the *z* coordinate and the wave electric field lies in the (x, z) plane, so that the *y* coordinate can be eliminated from further consideration. Second, we assume that the dielectric nonlinearity is localized around a certain distance $x = d$ from the metal surface $x = 0$, so that it practically does not affect the plasmon field. The distance *d* is assumed to be larger than the characteristic widths of the plasmon and soliton fields, which guarantees an exponentially small overlapping of their tails. Hence, the coupling is weak and can be treated perturbatively.

Uncoupled plasmon and soliton fields Ψ_p and Ψ_s [[12](#page-3-9)] can be represented as $\Psi_p(x, z) = c_p(z) \psi_p(x)$ and $\Psi_s(x, z)$ $= c_s(z) \psi_s(x, |c_s|)$, where $c_{p,s}$ are the *z*-dependent amplitudes and $\psi_{p,s}$ are the transverse profiles of the fields with the normalizations $\psi_p(0) = 1$ and $\psi_s(d) = 1$. The transverse profile of the plasmon ψ_p represents two exponents decaying away from <u>the metal-dielectric interface [[1,](#page-3-0)[2](#page-3-10)]</u> $[\psi_p = \exp(-\kappa_p x)$ with $k_p = \sqrt{k_p^2 - k^2}$ in the dielectric, at *x* > 0], whereas the soliton

FIG. 1. (Color online) Plasmon-soliton system as two coupled waveguides.

profile is given by [[10](#page-3-7)] $\psi_s = \operatorname{sech}[\kappa_s(x-d)]$ with κ_s $= k\sqrt{\gamma/2} |c_s|$ (Fig. [1](#page-0-0)). Here γ is the parameter of nonlinearity of the medium $[13]$ $[13]$ $[13]$. We emphasize that the transverse profile of the soliton depends on its amplitude $|c_s|$. In what follows, we assume that the amplitude varies slowly enough, so that one can use quasistationary adiabatic approximation considering $|c_{s}|$ as a local parameter of the problem.

Thus, the total wave field in the problem Ψ can be represented via the ansatz,

$$
\Psi(x,z) = c_p(z)\psi_p(x) + c_s(z)\psi_s(x,|c_s|). \tag{1}
$$

In the zero approximation, when the plasmon-soliton coupling is negligible, amplitudes c_p and c_s are independent and obey, respectively, linear and nonlinear oscillator equations. In the first-order approximation, the plasmon and soliton fields become linearly coupled due to the spatial overlapping of Ψ_p and Ψ_s and the transverse inhomogeneity of the medium along *x* (i.e., the contrast of dielectric indices at the metal-dielectric interface $\varepsilon_m \neq \varepsilon_0$). This results in the coupled oscillator equations for the amplitudes c_p and c_s ,

$$
c_p'' + \beta_p^2 c_p = q(|c_s|)c_s, \quad c_s'' + \beta_s^2(|c_s|)c_s = q(|c_s|)c_p. \tag{2}
$$

Here the prime stands for the derivative with respect to the dimensionless coordinate $\zeta = kz$, $\beta_p = k_p / k > 1$ and $\beta_s = k_s / k \approx 1 + \gamma |c_s|^2 / 4$ (we assume that $\gamma |c_s|^2 \le 1$) are the plasmon and soliton dimensionless propagation constants, and $q \leq 1$ is the coupling coefficient.

To determine the coupling coefficient *q*, note that the right-hand side of the first Eq. ([2](#page-1-0)), i.e., $q(|c_s|)c_s$, represents an external source that excites plasmons at the metal surface. Hence, up to a numerical factor, it should be equal to the soliton field at the metal surface $\Psi_s|_{x=0} = c_s \psi_s|_{x=0}$. This yields the estimation,

$$
q(|c_s|) \sim \psi_s|_{x=0} \simeq \exp(-k\sqrt{\gamma/2}|c_s|d), \tag{3}
$$

which is adopted below. Note that the weak-coupling approximation fails at small soliton amplitudes.

Equations (1) (1) (1) – (3) (3) (3) represent the model describing the plasmon-soliton interaction. These equations are reminiscent of the system of two weakly coupled nonlinear waveguides $[14]$ $[14]$ $[14]$. However, there are two essential peculiarities in the plasmon-soliton equations: (i) only one subsystem is nonlinear; (ii) the coupling coefficient depends on the soliton amplitude.

An effective plasmon-soliton interaction and energy exchange occurs only near the resonance $\beta_p = \beta_s$ which is achieved at the soliton amplitude $|c_s|_{\text{res}} = \sqrt{4(\beta_p-1)/\gamma}$. Solitons with significantly larger or smaller amplitude are uncoupled from the plasmon. In the vicinity of resonance $|\beta_p - \beta_s|$ ≤ β_p , Eq. ([2](#page-1-0)) can be simplified. Making substitution $c_{p,s}(\zeta) = C_{p,s}(\zeta) \exp(i\zeta)$ and assuming that new amplitudes $C_{p,s}$ vary slowly $(|C'_{p,s}| \leq |C_{p,s}|)$ and $C''_{p,s}$ can be neglected), we arrive at the following equations:

$$
-i\begin{pmatrix}C_p\\C_s\end{pmatrix}' = \begin{pmatrix}\nu_p & -q(|C_s|)/2\\-q(|C_s|)/2 & \nu_s(|C_s|)\end{pmatrix}\begin{pmatrix}C_p\\C_s\end{pmatrix}.
$$
 (4)

Here

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$$
\nu_p \equiv \beta_p - 1 \ll 1, \quad \nu_s \equiv \beta_s - 1 = \gamma |C_s|^2 / 4 \ll 1 \tag{5}
$$

are the small deviations of the dimensionless propagation constants from unity. The first inequality (5) (5) (5) implies plasmons in the long-wave region of their spectrum close to the light cone, whereas the second inequality indicates the weakness of nonlinearity.

Equation ([4](#page-1-4)) has the form of a vector nondiffractive nonlinear Schrödinger equation with a nondiagonal Hamiltonian typical in quantum two-level systems $[15]$ $[15]$ $[15]$. It possesses the integral of motion $|C_p|^2 + |C_s|^2$ = const, which is associated with the conservation of the total energy under the plasmonsoliton interaction.

III. EIGENMODES

Modes of Eq. ([4](#page-1-4)) are obtained via substitution $C_{p,s}(\zeta)$ $=A_{p,s}$ exp($iv\zeta$). This yields the characteristic equation determining the eigenvalues,

$$
\nu^{\pm}(|C_s|) = \frac{\nu_p + \nu_s(|C_s|) \pm \sqrt{[\nu_p - \nu_s(|C_s|)]^2 + q^2(|C_s|)}}{2},\qquad(6)
$$

and the ratios of the plasmon and soliton amplitudes determining the eigenvectors,

$$
\alpha^{\pm}(|C_s|) = \frac{A_p}{A_s} = -\frac{q(|C_s|)}{2[\nu^{\pm}(|C_s|) - \nu_p]}.
$$
(7)

Unlike linear systems, here the eigenvectors cannot be normalized arbitrarily because the right-hand side of Eq. (7) (7) (7) depends on the amplitude $|C_s|=|A_s|$.

Equations (6) (6) (6) and (7) (7) (7) describe the collective (hybridized) plasmon-soliton modes appearing due to the coupling. They have a standard form typical for a linear two-level problem with close eigenvalues and small coupling $[15]$ $[15]$ $[15]$. In our case, the problem is essentially nonlinear and the properties of the solution depend on the soliton *amplitude* $|C_s| = |A_s|$. In other words, the soliton amplitude plays the role of the *driving parameter*.

Figure [2](#page-2-0) shows the eigenvalues ν^{\pm} [Eq. ([6](#page-1-6))], together with ν_p and ν_s , amplitude ratios α^{\pm} [Eq. ([7](#page-1-5))], and field profiles $|\Psi(x)|^2$ [Eq. ([1](#page-1-1))] of the collective plasmon-soliton modes as dependent on the soliton amplitude $|C_s|$. In the interaction region $|v_p - v_s| \leq q_{\text{res}}$, the eigenvalues avoid crossing <u>(the</u> minimal frequency gap $q_{\text{res}} \equiv q(|C_s|_{\text{res}})$ $= \exp(-kd\sqrt{2\nu_p})$ is achieved at resonance) and eigenmodes are collective excitations with $|A_p| \sim |A_s|$. At resonance $\nu_s = \nu_p$, the two modes show their closest intensity distributions $(B \text{ and } E)$ corresponding to the symmetric and antisymmetric combinations of plasmon and soliton states with equal amplitudes. Away from the interaction region $|v_p - v_s|$ ≥ q_{res} , the eigenvalues ν^{\pm} tend to the asymptotes ν_p and ν_s representing pure plasmon and soliton states. Accordingly, the energy is concentrated basically in either the plasmon (D, C) or the soliton (A, F) channel. When passing through the interaction region along the same branch in Fig. $1(a)$ $1(a)$, the eigenmodes exchange the amplitudes [Fig. $2(b)$ $2(b)$] and a near-soliton mode transmutes into the plasmon one and vice versa. This offers various possibilities for the energy conversion between the plasmon and soliton channels.

RESONANT PLASMON-SOLITON INTERACTION

IV. DYNAMICS AND DISSIPATION

First, we consider the case when the soliton channel is excited at the input with a near-resonance amplitude $C_s(0)$,

$$
C_p(0) = 0, \quad |C_s(0) - |C_s|_{\text{res}}| \ll |C_s|_{\text{res}}.
$$
 (8)

Pure soliton is not an eigenmode, and initial condition (8) (8) (8) excites a mixture of "+" and "−" eigenmodes with close frequencies. In a linear problem, this would lead to a superposition of two modes and a harmonic low-frequency beating. But in the plasmon-soliton system, the superposition principle is invalid, and the solution of Eq. (4) (4) (4) with Eq. (8) (8) (8) represents a self-modulated nonlinear beating causing a periodic mutual conversion of energy between the soliton and plasmon channels, (Fig. [3](#page-2-2)). The maximal conversion amplitude is reached at a certain value $C_s(0)_{\text{max}} > |C_s|_{\text{res}}$. It corresponds to the maximal beating period $\Delta \zeta \sim 2\pi/q_{\text{res}}$ and it drops abruptly for higher $C_s(0)$. The beating amplitude decreases with the resonant coupling q_{res} decrease [Figs. [3](#page-2-2)(a) and $3(b)$ $3(b)$].

Second, the soliton-plasmon conversion occurs under manipulations with the soliton amplitude—the driving parameter of the system. One could expect that slow changes in the soliton amplitude will result in an adiabatic transformations of the eigenmodes along the "+" or "−" dispersion curves [Fig. $2(a)$ $2(a)$], and the soliton will metamorphose into plasmon when passing over the resonance. For instance, the soliton is excited with an off-resonance amplitude corresponding to the zone *A* in Fig. [2,](#page-2-0) i.e.,

$$
C_p(0) = 0, \quad C_s(0) - |C_s|_{\text{res}} \sim |C_s|_{\text{res}}.
$$
 (9)

These initial conditions approximately match the "+" eigenmode. Then, we introduce a small absorption in the medium which makes the soliton amplitude slowly decreasing along ζ , $|C_s| = |C_s|(\zeta)$. The losses in the plasmon and soliton channels are described by adding small imaginary parts $\sigma_{p,s}$ to their propagation constants,

$$
\nu_{p,s} \to \tilde{\nu}_{p,s} = \nu_{p,s} + i\sigma_{p,s}, \quad |\sigma_{p,s}| \ll q_{\text{res}}.\tag{10}
$$

The weak dissipation (10) (10) (10) does not affect the local characteristics of the waves, which change adiabatically with $|C_s(\zeta)|$ according to the eigenmodes ([6](#page-1-6)) and ([7](#page-1-5)).

With Eq. (10) (10) (10) , this leads to the transformation of states as $A \rightarrow B \rightarrow C$ (Fig. [2](#page-2-0)), i.e., almost 100% conversion of energy from the soliton to the plasmon channel. This would be a perfect switch, but the nonadiabatic Landau-Zener transitions between the "+" and "−" eigenmodes may appear in the vicinity of resonance $|v_p - v_s| \leq q_{\text{res}}$ [[15,](#page-3-13)[16](#page-3-14)]. The probability of the Landau-Zener transitions can be estimated using the parameter of adiabaticity μ [[17](#page-3-15)],

$$
\mu = |v^{\pm'}|/|v^+ - v^-|^2. \tag{11}
$$

If $\mu \ll 1$, the "+" and "-" eigenmodes evolve independently according to the adiabatic evolution. On the contrary, if μ ≥ 1 , the wave undergoes diabatic evolution and practically all the energy stored in the "+" eigenmode is converted into the "−" one when passing the resonance. This means the transformation of states as $A \rightarrow B + E \rightarrow F$ (Fig. [2](#page-2-0)), i.e., the nonadiabatic transition keeps the energy in the soliton chan-

FIG. 2. (Color online) Collective modes in the plasmon-soliton system vs the dimensionless soliton amplitude. (a) Eigenvalues v^{\pm} Eq. ([7](#page-1-5)) (bold curves); eigenvalues of uncoupled modes ν_p and ν_s (dashed lines). (b) Amplitude ratios α^{\pm} Eq. ([8](#page-2-1)). [(c) and (d)] Profiles of the total field $|\Psi(x)|^2$ [Eq. ([1](#page-1-1))], corresponding to the points *A*-*F* at curves in (a) and (b). The plasmon tails at $x < 0$ are not depicted, since they are determined by the properties of a particular metal. Parameters are $\nu_p = 0.2$ and $kd = 6$ ($q_{res} \approx 0.02$).

nel. In the intermediate case $\mu \sim 1$, the output wave will present a mixture of the "+" and "−" modes: *A*→*C*+*F*.

Surprisingly, the adiabatic parameter (11) (11) (11) cannot be made small in the plasmon-soliton system, no matter how weak the dissipation is. The point is that the soliton amplitude $|C_s|$ dramatically changes in the vicinity of resonance due to the soliton-to-plasmon transmutation along the eigenmode, and the adiabatic regime is never achieved. This is a purely nonlinear effect—the driving parameter is a part of the solution. Numerical simulations of Eq. (4) (4) (4) with Eqs. (9) (9) (9) – (11) (11) (11) confirm this conclusion (Fig. [4](#page-3-16)). In the vicinity of resonance, μ_{res} ≥ 1 , which results in a partial soliton-to-plasmon switching and the Landau-Zener transition with the consequent nonlin-

FIG. 3. (Color online) Nonlinear beating of the plasmon (lower curves) and soliton (upper curves) dimensionless amplitudes after the excitation of the soliton channel near resonance. Equations ([4](#page-1-4)) and ([8](#page-2-1)) are solved numerically with $\nu_p=0.2$ and values of $\gamma |C_s(0)|^2/4$ as indicated in the boxes. Values of *kd* correspond to (a) $q_{\text{res}} \approx 0.02$ and (b) $q_{\text{res}} \approx 0.003$.

ear beating between "+" and "−" eigenmodes. Smaller values of the coupling q_{res} result in higher μ_{res} and smaller parts of the energy transferred to the plasmon.

V. DISCUSSION

To summarize, we have shown the possibility of resonant interaction between a plasmon polariton at a metal surface and a parallel self-focusing beam in a nonlinear dielectric. A simple two-level model reveals hybridized plasmon-soliton eigenmodes and their complex nonlinear dynamics which offers plasmon excitation and control using spatial solitons.

An effective soliton-to-plasmon coupling and energy conversion can be achieved in the far-infrared frequency range (the wavelength in vacuum is $\lambda_0 \sim 5-10 \mu$ m), by using a gold interface with a nonlinear dielectric (e.g., a chalcogenide glass, $\varepsilon_0 \sim 5-10$). In this range, plasmons are characterized by the propagation constant close to unity

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FIG. 4. (Color online) Adiabatic dynamics in the plasmon-soliton system with small dissipation [Eqs. ([4](#page-1-4)) and ([9](#page-2-5))–([11](#page-2-4))] $\nu_p = 0.2$ and $kd = 6$. Initial soliton (9) (9) (9) attenuates slowly until it reaches the resonant amplitude. Then, only a part of the energy is abruptly transferred to the plasmon channel $(\mu_{\text{res}} \ge 1)$ with a subsequent beating in the mix of "+" and "-" eigenmodes (cf. Fig. [3](#page-2-2)). The value of the adiabatic parameter $\mu(\zeta)$ (thin dotted line) indicates the effectiveness of the energy exchange between the eigenmodes.

 $\nu_p \sim 10^{-3}$ and relatively small dissipation $\sigma_p \sim 0.1$. Solitons with $\nu_s = \gamma |c_s|^2 / 4 \sim 10^{-3}$ can be produced with typical Kerr nonlinearities. Transverse evanescent tails of plasmon and soliton are determined by $\kappa_p \sim \kappa_s \sim 0.1$ μ m⁻¹, and propagation of the soliton at the distance $d \sim 20-40 \mu m$ $(kd \sim 50-100)$ from the metal surface yields the coupling coefficient $q_{res} \sim 10^{-1} - 10^{-2}$. Although the plasmon dissipation is large enough $\sigma_p \gtrsim q_{\text{res}}$ [[18](#page-3-17)], we have found, by numerical simulations of Eq. (4) (4) (4) with Eq. (10) (10) (10) , that our conclusions on the plasmon-soliton coupling and dynamics $[e.g.,]$ Fig. $4(b)$ $4(b)$] remain qualitatively true at these realistic parameters.

ACKNOWLEDGMENTS

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