Vector solitons in two-component Bose-Einstein condensates with tunable interactions and harmonic potential

Xiao-Fei Zhang,[1,2] Xing-Hua Hu,[1] Xun-Xu Liu,[1] and W. M. Liu[1]
[1]*Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*
[2]*College of Science, Honghe University, Mengzi 661100, China*

(Received 16 December 2008; revised manuscript received 24 January 2009; published 31 March 2009)

We present a family of exact vector-soliton solutions for the coupled nonlinear Schrödinger equations with tunable interactions and harmonic potential, and then apply the model to investigate the dynamics of solitons and collisions between two orthogonal solitons in the case with equal interaction parameters. Our results show that the exact vector-soliton solutions can be obtained with arbitrary tunable interactions as long as a proper harmonic potential is applied. The dynamics of solitons can be controlled by the Feshbach resonance and the collisions are essentially elastic and do not depend on the initial conditions.

DOI: 10.1103/PhysRevA.79.033630 PACS number(s): 03.75.Lm, 03.75.Mn, 03.75.Kk

I. INTRODUCTION

The soliton, as a fundamental excitation of the atomic matter waves, has been studied intensively and attracted more and more attention in Bose-Einstein condensates (BECs) [1–9]. Experimental realization of BECs, in which two (or more) internal states or different atoms can be populated, has stimulated great interests in vector solitons [10–16]. Vector solitons in the form of "dark-dark" [17], "bright-bright" [18], and "bright-dark" solitons [19] are studied. Comparing with the single-component BECs, the extra internal degrees of freedom introduced by multiple components give rise to more rich phenomena and complex dynamics, such as soliton trains, soliton pairs, multidomain walls, and multimode collective excitations [20–22]. Besides, a new type of soliton, symbiotic solitons, has been found in two-component BEC system [23,24]. Recently, experimental observation of heteronuclear Feshbach molecules from ^{87}Rb and ^{85}Rb gases and tunable miscibility in a dual-component Bose-Einstein condensates has been realized in [25]. Theoretically, the growth of a dual species Bose-Einstein condensate is simulated by using a Gross-Pitaevskii (GP) equation with an additional grain term giving rise to the growth in [26].

The properties of the nonlinear excitation are determined by the interaction between atoms, which is typically characterized by the *s*-wave scattering length (SL). A tunable interaction suggests very interesting studies of the many-body behavior of condensate system. Recently experiments have demonstrated that "tuning" of the effective SL, including a possibility to change its sign, can be achieved by using the so-called Feshbach resonance ([27] and references therein) with a tunable time-dependent magnetic field $B(t)$. This exciting technical progress offers a great opportunity for manipulation of atomic matter waves and nonlinear excitations in multicomponent BECs.

As is well known, the nonlinear Schrödinger equation without external potential supports solitonic solutions. However, the coupled GP equations, which are a combination of the nonlinear Schrödinger equation and external potential term, are nonintegrable. Much of the recent study about the analytical solutions has focused on particular situations and thus based on specific assumptions. For example, the relative interaction strengths for inter- and intracomponent collisions are presumed to be equal [28–30]. Motivated by our previous study on single-component BECs, the aim of this paper is to study vector solitons and their dynamics in two-component BECs with tunable interaction parameters and harmonic trapping potential. We extend our previous study on single-component BECs to the two-component case and obtain the integrability conditions for the exact vector-soliton solutions [6]. We also investigate the dynamics of a single soliton and collisions between two orthogonal solitons in the symmetric vector-soliton case. Our results show that the dynamics of a single soliton can be controlled by Feshbach resonance, and the collisions between two orthogonal solitons are essentially elastic and do not depend on the initial separation length, velocities, and the amplitudes of the modulation.

II. MODEL

Two interacting dilute Bose condensates can be well described by the zero-temperature mean-field theory, in which the collisions between the condensate atoms and the thermal cloud are neglected. Considering a two-component BECs each of mass m trapped in a quasi-one-dimensional (1D) harmonic potential, the evolution of this system is govern by a pair of coupled dimensionless GP equations,

$$i\frac{\partial \psi_1}{\partial t} = \left(-\frac{\partial^2}{\partial x^2} + V_1 + g_{11}|\psi_1|^2 + g_{12}|\psi_2|^2\right)\psi_1,$$

$$i\frac{\partial \psi_2}{\partial t} = \left(-\frac{\partial^2}{\partial x^2} + V_2 + g_{22}|\psi_2|^2 + g_{21}|\psi_1|^2\right)\psi_2, \tag{1}$$

where ψ_i denotes the macroscopic wave functions of the ith component, with the normalization conditions $\int_{-\infty}^{\infty}|\psi_1|^2 dx = 1$ and $\int_{-\infty}^{\infty}|\psi_2|^2 dx = N_2/N_1$. Here we do not allow for the components to transform into each other and $N_1 = N_2$, so the number of atoms is conserved for both components. The external harmonic potential can be written as $V_i = \lambda_i^2(t)x^2 = \omega_i^2(t)x^2/\omega_{i\perp}^2$, where ω_i and $\omega_\perp$ are the angular frequencies in the axial and radial directions. Time t and coordinate x are the temporal and spatial coordinates measured in units $2/\omega_\perp$

and $\ell_\perp=\sqrt{\hbar/m\omega_\perp}$, where $\ell_\perp$ is linear oscillator length in the transverse direction. The interactions between atoms are described by a self-interaction $g_{ii}=4a_{ii}N_i/\ell_\perp$ and the interaction between different components $g_{12}=g_{21}=4a_{ij}N_i/\ell_\perp$, where a_{ii} are the scattering lengths of component i and a_{12} is that between components 1 and 2. In the present paper, we assume $m_1=m_2$, $\omega_1=\omega_2=\omega$, and define variables $\sigma_1=g_{12}^2-g_{11}g_{22}$, $\sigma_2=g_{12}-g_{11}$, and $\sigma_3=g_{12}-g_{22}$ for the facilitation of expression.

III. VECTOR SOLITONS

Since we are interested in the exact vector-soliton solutions, we first consider the integrability of Eq. (1). The simplest case is that all the interactions take exactly the same form. Then we can directly extend our previous study on single-component BECs to the two-component case and obtain a similar integrability conditions for the existence of the exact symmetric vector solitons [6]. However, in typical BEC experiments, the strength of the interactions is always unequal to each other; but fortunately, we can employ the extend mapping deformation method, which dose not depend on the integrability, to obtain the exact asymmetric vector-soliton solutions and determine the regions where those solutions are existing.

A. Symmetric vector solitons

When the interactions take equal values, Eq. (1) can be simply written as

$$i\frac{\partial\psi_i}{\partial t}=\left[-\frac{\partial^2}{\partial x^2}+\lambda^2(t)x^2+g(t)(|\psi_i|^2+|\psi_{(3-i)}|^2)\right]\psi_i. \quad (2)$$

In order to obtain the exact analytical solutions of Eq. (2), we need to get an integrability condition for these equations—that is, if we allow the axial frequency of the harmonic potential to become also time dependent, i.e., $\lambda=\lambda(t)$, and require it to satisfy the following integrability relation with the interaction parameter $g(t)$:

$$-\frac{1}{2g(t)}\frac{d^2g(t)}{dt^2}+\frac{1}{g^2(t)}\left(\frac{dg(t)}{dt}\right)^2+2\lambda^2=0. \quad (3)$$

Then by using the following transformation [31]:

$$\psi_i=\frac{1}{\sqrt{g(t)\ell(t)}}\phi_i(X,T)\exp\{i[f(t)x^2]\}, \quad (4)$$

where ϕ_i is an arbitrary function with spatial and temporal variables $X=x/\ell(t)$ and $T_t=\ell^{-2}$, which together with $f(t)$, $\lambda(t)$ is determined by

$$f_t+4f^2+\lambda^2=0, \quad \ell_t-4f\ell=0. \quad (5)$$

With the above transformations, Eq. (2) can be reduced to the standard coupled nonlinear Schrödinger (NLS) equations

$$i\phi_{iT}=-\phi_{iXX}+\sigma(|\phi_1|^2+|\phi_2|^2)\phi_i, \quad (6)$$

where $\sigma=\pm1$ corresponds to the cases of positive and negative scattering lengths. These coupled equations are nothing but the well-known integrable model proposed by Manakov [32] or the defocusing-defocusing NLS equations. Since the exact solutions for these systems have been well studied, the exact vector-soliton solutions for Eq. (2) can be easily obtained by combining the well-known solutions for the Manakov system or the defocusing-defocusing NLS equations through Eq. (4).

The virtue of the transformation is that the variable ϕ_i satisfies the integrable Manakov system or the defocusing-defocusing NLS equations while the varying of the scattering length and external potential appear in the remains of Eq. (4) and variables X and T. It is interesting to observe that the amplitude and width of the bright-bright vector solitons vary with function $g(t)$ according to Eq. (4), which offers us also a possibility to change the soliton's parameters in a controllable manner by choosing a proper external potential to satisfy the integrability condition.

B. Asymmetric vector solitons

We now turn to Eq. (1) with a more general situation, with unequal interaction parameters $g_{11}(t)\neq g_{22}(t)\neq g_{12}(t)$. To carry out the analysis we start from the constant amplitude solutions of Eq. (1) without external potential,

$$\psi_i(x,t)=A_i\exp(i\Omega_i t),$$

$$\Omega_i=g_{ii}|A_i|^2+g_{i,3-i}|A_{3-i}|^2. \quad (7)$$

The instability of this coupled constant amplitude solutions with small perturbations is discussed in [23]. When a harmonic trapping potential is added along the longitudinal direction, the exact vector-soliton solutions only exist under specific assumptions. Does the introduction of the external potential and arbitrary interaction parameters make any qualitative change in the behavior of the integrability conditions? The answer is yes and we find that the integrability conditions in this case have more constrains on the soliton's parameters. In the following, we take advantage of the extend mapping deformation method, which does not depend on the integrability, to obtain the exact vector-soliton solutions for the general equations (1).

Expressing the order parameters in terms of their modulus and phases, i.e., $\psi_i(x,t)=\sqrt{n_i}\exp(i\Delta_i)$, and then separating real and imaginary parts, we obtain a set of coupled nonlinear equations for n_i and Δ_i,

$$\sqrt{n}_{ixx}-\sqrt{n_i}\Delta_{it}-\sqrt{n_i}\Delta_{ix}^2-g_{12}n_{(3-i)}\sqrt{n_i}$$

$$-g_{ii}(\sqrt{n_i})^3-\lambda^2x^2\sqrt{n_i}=0,$$

$$\sqrt{n}_{it}+\sqrt{n_i}\Delta_{ixx}+2\sqrt{n}_{ix}\Delta_{ix}=0, \quad (8)$$

where $i=1,2$. By setting $\sqrt{n_i}=A_0(t)+A_i(t)\phi(\xi)$, $\xi=p_1(t)x+p_2(t)$, and $\Delta_i=k_i(t)+\Gamma_i(t)x+\Lambda_i(t)x^2$ and using the auxiliary equation $\phi'^2=c_0+c_2\phi^2+c_4\phi^4$, we derive a set of overdetermined partial differential equations with respect to $p_1(t)$, $p_2(t)$, $k_i(t)$, $\Gamma_i(t)$, $\Lambda_i(t)$, $A_0(t)$, $A_1(t)$, and $A_2(t)$. Finally, solving these equations, we can obtain the following constrains for the existence of the exact vector-soliton solutions:

$$g_{11}A_1^2(t) + g_{12}A_2^2(t) = g_{22}A_2^2(t) + g_{12}A_1^2(t),$$

$$\frac{-\partial^2 p_1(t)}{\partial t^2}p_1(t) + 2\left(\frac{\partial p_1(t)}{\partial t}\right)^2 + 4p_1^2(t)\lambda^2 = 0,$$

$$p_1^2(t) - g_{11}p_1(t) - g_{12}\frac{\sigma_2}{\sigma_3}p_1(t) = 0, \tag{9}$$

where λ is the trapping frequency. Equation (9) implies that the exact vector-soliton solutions exist with many constrains. For example, the first equation of Eq. (9) imposes a constrain that the amplitudes of the vector solitons must satisfy $A_1^2/A_2^2=(g_{12}-g_{22})/(g_{12}-g_{11})$; in particular, the others impose a relation between the scattering length and trapping potential which reads

$$\frac{g_{12}-g_{22}}{g_{12}^2-g_{11}g_{22}} = \frac{a_1\sin(2\lambda t) - a_2\cos(2\lambda t)}{2\lambda}. \tag{10}$$

In the case of all the interaction parameters $g_{ij}<0$, Eq. (1) has bright-bright vector solitons

$$\psi_{1B} = C_1\sqrt{-a(t)}\,\mathrm{sech}[\xi(x,t)]\exp[iv_{1B}(x,t)],$$

$$\psi_{2B} = C_2\sqrt{-\sigma a(t)}\,\mathrm{sech}[\xi(x,t)]\exp[iv_{2B}(x,t)], \tag{11}$$

where

$$v_{iB} = -\frac{a'(t)}{4a(t)}x^2 + C_2 a(t)x + (C_1^2 - C_2^2)\int a(t)^2 dt + C_{i+2},$$

$$\xi(x,t) = C_1 a(t)x - 2C_1C_2\int a(t)^2 dt + C_5,$$

$$a(t) = \frac{\sigma_1}{\sigma_3} < 0, \quad \sigma = \frac{\sigma_2}{\sigma_3} > 0. \tag{12}$$

When $a(t)>0$ and $\sigma(t)>0$, Eq. (1) has dark-dark vector solitons

$$\psi_{1D} = D_1\sqrt{a(t)}\,\tanh[\xi(x,t)]\exp[iv_{1D}(x,t)],$$

$$\psi_{2D} = D_2\sqrt{\sigma a(t)}\,\tanh[\xi(x,t)]\exp[iv_{2D}(x,t)], \tag{13}$$

where $\xi(x,t)$, $a(t)$, and $\sigma(t)$ are the same as Eq. (12), but the phase functions are given by

$$v_{iD} = -\frac{a'(t)}{4a(t)}x^2 + D_3 a(t)x - (2D_1^2 + D_2^2)\int a(t)^2 dt + D_{i+3}. \tag{14}$$

In Eqs. (11) and (13), the coefficients C_1, C_2, C_3, C_4, C_5, D_1, D_2, D_3, D_4, and D_5 are arbitrary constants. Based on the above results, we can conclude the following conditions for the existence of the exact vector-soliton solutions:

$$\Delta_1 > 0, \quad \Delta_2 < 0: \quad \text{bright-bright (BB)},$$

$$\Delta_1 < 0, \quad \Delta_2 > 0: \quad \text{dark-dark (DD)} \tag{15}$$

with $\Delta_1 \equiv -\sigma_3/\sigma_1$ and $\Delta_2 \equiv \sigma_2/\sigma_1$. As long as Eqs. (9) and (15) are satisfied, we can obtain the exact vector-soliton solutions, either bright-bright or dark-dark, for an arbitrary periodic time dependence of the scattering length since we can choose an appropriate λ to satisfy the above equations. It is relevant to mention that all interaction parameters g_{ii} and g_{ij} can be functions of time t in our cases.

TABLE I. Table for the existence of dark-dark solitons vs intercomponent interaction g_{12}. + (−) indicates the repulsive (attractive) atom-atom interaction in single component. The single-component interaction parameters g_{11} and g_{22} are assumed to be unvaried and $g_{11}>g_{22}$.

Parameters	g_{11}	g_{22}	g_{12}
Case 1	+	+	$g_{12}>g_{11}$, $-\sqrt{g_{11}g_{22}}<g_{12}<g_{22}$
Case 2	−	−	$g_{12}>\sqrt{g_{11}g_{22}}$
Case 3	+	−	$g_{12}>g_{11}$

Now, we conclude the regions for the existence of these vector solitons. In the case of bright-bright vector solitons, we consider a simple case where one of the intracomponent interaction parameters g_{11} is time dependent while others are time independent. According to the integrability condition, the bright-bright vector solitons can exist in two regions: (i) $g_{12}<g_{11}<0$ (here we assume that $0>g_{22}>g_{12}$) and (ii) $g_{11}<g_{12}<0$ (here we assume that $0>g_{12}>g_{22}$). In real experiments, we consider a two-component condensates with $N_1=N_2=5000$, $a_{12}=-2$ nm, $a_{22}=-1$ nm, $\omega_\perp=2\pi\times 100$ Hz, and $\omega_i=2\pi\times 2$ Hz. Then the nonlinearities can be taken as $g_{12}=-40$, $g_{22}=-20$, and $-40<g_{11}<0$.

Table I shows the regions where the dark-dark vector solitons can exist when the intercomponent interaction parameter g_{ij} is time dependent and intracomponent ones g_{ii} are time independent. Case 1 has been extensively studied in [25,33,34]. In cases 2 and 3, it is interesting to find that dark soliton can be formed in a condensate with self-attractive atom-atom interaction. The common sense is that the dark solitons only exist in a condensate with self-repulsive atom-atom interaction; however, in the two-component BECs, the repulsive interaction coming from the second component or the intercomponent interaction induces an effective repulsive interaction in the self-attractive one. This leads to the formation of dark solitons in the component with self-attractive interaction. It is clear that the vector solitons in case 2 are different from the Manakov solitons, where vector solitons are formed by all the nonlinear coefficients cooperation. In our case, the mutual repulsive interaction between the two components solely supports the existence of such dark-dark vector solitons. At last, we would like to point out that the formation of this type of vector solitons has a threshold value in g_{12}, which indicates that the intercomponent repulsive interaction must be strong to induce an effective repulsive interaction in the self-attractive ones.

Furthermore, when a linear gain term, which contributes to the growth of each component during the simultaneous evaporative cooling, is added to the coupled GP equations, our methods will also allow us to obtain the exact vector-soliton solutions, but with a more complicated integrability condition. These solutions may help us to better understand the nonlinear dynamics of the condensates and the dynamical pattern formation during growth of a dual-component BECs, which has been experimentally realized in [25] and theoretically investigated in [26]. This needs a further investigation and is beyond the scope of the present paper.

IV. DYNAMICS AND COLLISIONS OF THE SYMMETRIC VECTOR SOLITONS

One extension of application of the transformation used in Sec. III A is to study the dynamics of a single soliton and the collisions between two orthogonal solitons. In real experiments, the s-wave scattering length can be changed with linear, exponential, and periodic time dependence of the magnetic field via Feshbach resonance. As mentioned above, once a function of the scattering length is given, we can choose a proper external potential according to Eq. (3) to satisfy the integrability condition. In the following, we focus on the most natural periodic time dependence of the scattering length.

A. Dynamics of a single soliton

To better understand the influence of Feshbach resonance on soliton's parameters, we study the dynamics of a single solitons with Feshbach management. As discussed in [35,36], a periodic scattering length is obtained by means of an ac magnetic field, which allows one to create a self-confined two-dimensional (2D) BEC without the magnetic trap. We suppose that the intercomponent interaction parameter g_{12} is modulated periodically as

$$\bar{g}_{12} = \pm g_{12}[1 + m \sin(\omega t)], \tag{16}$$

where m and ω are the amplitude and frequency of the modulation. The symmetric vector solitons can be realized in a two-component system constituting from different hyperfine spin states of the same atom, such as ^{87}Rb with two hyperfine spin states which can be chosen as $|F=1, m_f=-1\rangle$ and $|F=2, m_f=1\rangle$. In this case, the trapping potential has the same shape for these two states, which is in accordance with our previous assumption. We choose the atom number $N_1 = N_2 = 10^4$, and $a_{11} = a_{22} = a_{12} = -5.36[1+m\sin(\omega t)]$ nm. According to Eq. (3), the axial frequency of the harmonic potential should be

$$\omega_1^2(t) = -\frac{m\omega^2\omega_\perp^2}{4[1+m\sin(\omega t)]^2}[\sin(\omega t) + m + m\cos^2(\omega t)]. \tag{17}$$

We now investigate how the amplitude of the ac drive can be used to control the parameters of the vector solitons. When the amplitude is small, $m=0.1$, this leads to a small periodically modulation of the scattering length and the trapping frequency. As shown in Fig. 1, we observe that the amplitude and width of the vector solitons vary periodically with time. When the amplitude takes a bigger value, the vector solitons will have a larger oscillation in amplitudes and widths. A natural extension are the cases where the magnetic field varies exponentially and linearly with time; in these cases, the amplitude and width of the vector solitons are also proportional and inversely proportional to the scattering length, respectively.

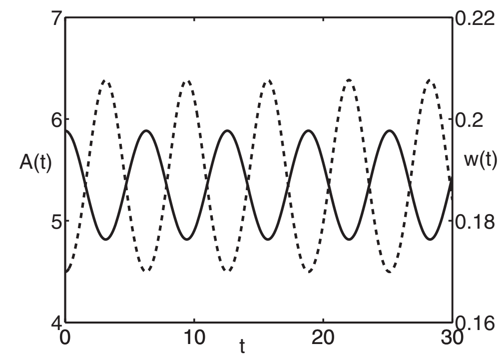

FIG. 1. The variation of amplitude $A(t)$ and width $W(t)$ of the vector solitons when the scattering length varies periodically with time t. The parameters are given as follows: $\ell(0)=g(0)=1$, $\omega=1$, $m=0.1$, $a_{11}=a_{22}=a_{21}=-5.36[1+m\cos(\omega t)]$ nm, and the initial amplitude $A(0)$ and width $W(0)$ are taken equal to 1. The amplitude and width are measured in units $4aN/\ell_\perp$. Time t and coordinate x are the temporal and spatial coordinates measured in units $2/\omega_\perp$ and $\ell_\perp = \sqrt{\hbar/m\omega_\perp}$.

B. Collisions between solitons

Finally, we study the collision between solitons. Interactions between solitons are fascinating since in many aspects solitons interact like particles: they pass through one another and accomplish elastic collision. As shown in Eq. (6), the nonlinearity appears solely through the invariant combination $(|\psi_1|^2+|\psi_2|^2)$. The isotopic invariance ensures that once we obtain a soliton solution for one component, the other one can be obtain in an obvious way,

$$\begin{Bmatrix} \psi_1(x,t) \\ \psi_2(x,t) \end{Bmatrix} = \begin{Bmatrix} \cos(\theta) \\ \sin(\theta) \end{Bmatrix} \begin{Bmatrix} \exp(-i\mu t) \\ \exp(-i\mu t) \end{Bmatrix} \phi(x) \tag{18}$$

with $0 \le \theta \le \pi/2$. Furthermore, the general vector solitons with unequal chemical potentials are also possible. We consider a special case with equal chemical potential between

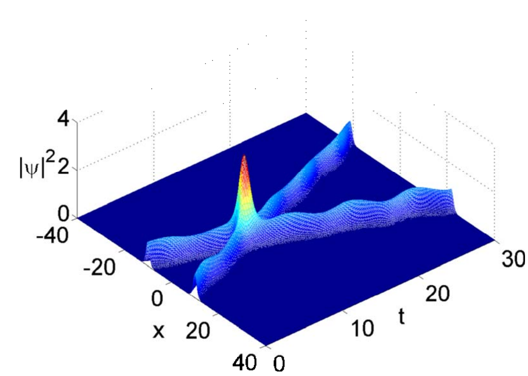

FIG. 2. (Color online) The interaction between two orthogonal solitons when the scattering length varies periodically with time t. The parameters are given as follows: $x_0=20$, $v=1$, $m=0.1$, $a_{11}=a_{22}=a_{21}=-[1+m\cos(\omega t)]$ nm, $\omega=1$, and the other parameters are the same as in Fig. 1.

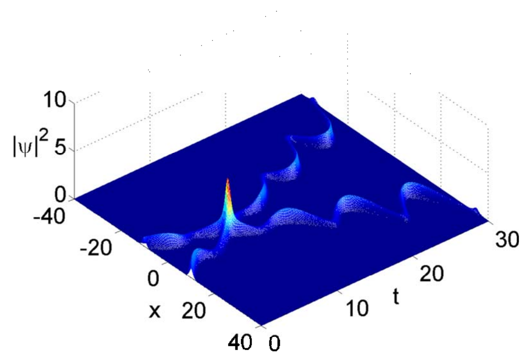

FIG. 3. (Color online) Elastic collision between two orthogonal solitons with a larger oscillation of the soliton's parameters when the modulation is increased from $m=0.1$ to $m=0.5$. The other parameters are the same as in Fig. 2.

two orthogonal solitons which reads $\theta=0$ and $\theta=\pi/2$ and set the initial velocities as $\pm v$, respectively. In this case, the initial conditions at $t=0$ are given by

$$\begin{Bmatrix} \psi_1^{(0)}(x) \\ \psi_2^{(0)}(x) \end{Bmatrix} = \begin{Bmatrix} \phi_1(x-x_0/2) \\ \phi_2(x+x_0/2) \end{Bmatrix}, \tag{19}$$

where x_0 is the initial separation. Figure 2 shows the interaction between two orthogonal solitons when the scattering length varies periodically with time. Our results show that the parameters of the two colliding solitons do not change after collision, which remarkably indicates no energy exchange between the two solitons. Shown in Fig. 3 is the collision scenario when the amplitude of the modulation has a large value as $m=0.5$. It demonstrates that the intrinsic oscillations and elastic collision do not depend on the amplitude of the modulation. The only difference between the two cases is that the parameters of the two solitons, such as amplitude, width, and velocity, have a larger oscillation in the latter. Another interesting issue is whether the initial velocity and separation will destroy the intrinsic elastic collision. Our results show that the outcome does not depend on the initial velocity and separation except for a large enough value of x_0.

The elastic collisions between solitons originate from the integrable Manakov system which is obtained with the condition $g(|\varphi_1|^2+|\varphi_2|^2)\ll 1$. These collision scenarios are similar to the cases in nonlinear optics where vector solitons collide with each other elastically, except that their polarization may change after collision (if the vector solitons have the same or orthogonal polarization, such change will not occur). While, if the condition $g(|\varphi_1|^2+|\varphi_2|^2)\ll 1$ is not satisfied, a more accurate coupled nonpolynomial Schrödinger equations, which are nonintegrable, can be applied to investigate the inelastic collision between solitons. The various inelastic collision scenarios in BEC system and fiber optics were studied in [29,37], which will help us to better understand the physics behind the equations.

V. CONCLUSIONS

In summary, we have obtained the exact vector solitons in the frame of the coupled GP mean-field theory. When the interactions take equal values, we reduce the coupled GP equations to the Manakov system or the defocusing-defocusing NLS equations by another transformation and obtain the exact vector-soliton solutions. Based on this model, we investigate the dynamics of a single soliton and the collisions between two orthogonal solitons. The results show that the soliton's parameters, such as amplitude and width, can be controlled by the Feshbach resonance, and the collisions are essentially elastic and do not depend on the initial conditions. Moreover, we obtain the integrability conditions for a general coupled GP equations with unequal interactions and get the exact vector-soliton solutions. It shows that the dark-dark solitons can be formed in two-component attractive BECs for a sufficiently repulsive intercomponent interaction, which induces an effective repulsive interaction between bosons of the same type. We hope that the results in our paper may be helpful for the experimental realization of such solitons in BECs and may help us understand the properties of the Bose condensate mixtures.

ACKNOWLEDGMENTS

We would like to express our sincere thanks to Professor W. D. Li for helpful discussions. This work was supported by NSFC under Grants No. 60525417, No. 10740420252, and No. 10874235 and by NKBRSFC under Grants No. 2005CB724508, No. 2006CB921400, and No. 2009CB930704.

[1] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999).
[2] K. E. Strecker, G. B. Partridge, A. G. Truscoff, and R. G. Hulet, Nature (London) **417**, 150 (2002).
[3] L. Khaykovich, F. Schreck, G. Ferrari, T. Bourdel, J. Cubizolles, L. D. Carr, Y. Castin, and C. Salomon, Science **296**, 1290 (2002).
[4] C. Becker, S. Stellmer, P. Soltan-Panahi, S. Dörscher, M. Baumert, Eva-Maria Richter, Jochen Kronjäger, K. Bongs, and K. Sengstock, Nat. Phys. **4**, 496 (2008).
[5] E. Timmermans, P. Tommasini, M. Hussein, and A. Kerman, Phys. Rep. **315**, 199 (1999).
[6] X. F. Zhang, Q. Yang, J. F. Zhang, X. Z. Chen, and W. M. Liu, Phys. Rev. A **77**, 023613 (2008); B. Li, X. F. Zhang, Y. Q. Li, Y. Chen, and W. M. Liu, *ibid.* **78**, 023608 (2008).
[7] S. Inouye, M. R. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper-Kurn, and W. Ketterle, Nature (London) **392**, 151 (1998).
[8] E. A. Donley, N. R. Claussen, S. L. Cornish, J. L. Roberts, E. A. Cornell, and C. E. Wieman, Nature (London) **412**, 295 (2001).
[9] A. Simoni, F. Ferlaino, G. Roati, G. Modugno, and M. Ingus-

cio, Phys. Rev. Lett. **90**, 163202 (2003).
[10] G. Thalhammer, G. Barontini, L. De Sarlo, J. Catani, F. Minardi, and M. Inguscio, Phys. Rev. Lett. **100**, 210402 (2008).
[11] C. J. Myatt, E. A. Burt, R. W. Ghrist, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. **78**, 586 (1997).
[12] D. S. Hall, M. R. Matthews, J. R. Ensher, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **81**, 1539 (1998).
[13] D. M. Stamper-Kurn, M. R. Andrews, A. P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, and W. Ketterle, Phys. Rev. Lett. **80**, 2027 (1998).
[14] T. S. Raju, P. K. Panigrahi, and K. Porsezian, Phys. Rev. A **71**, 035601 (2005).
[15] G. Modugno, G. Ferrari, G. Roati, R. J. Brecha, A. Simoni, and M. Inguscio, Science **294**, 1320 (2001).
[16] H. A. Cruz, V. A. Brazhnyi, V. V. Konotop, G. L. Alfimov, and M. Salerno, Phys. Rev. A **76**, 013603 (2007).
[17] A. P. Sheppard and Y. S. Kivshar, Phys. Rev. E **55**, 4773 (1997).
[18] D. J. Kaup and B. A. Malomed, Phys. Rev. A **48**, 599 (1993).
[19] Th. Busch and J. R. Anglin, Phys. Rev. Lett. **87**, 010401 (2001).
[20] Víctor M. Pérez-García and Vadym Vekslerchik, Phys. Rev. E **67**, 061804 (2003); G. D. Montesinos, V. M. Pérez-García, and H. Michinel, Phys. Rev. Lett. **92**, 133901 (2004).
[21] V. A. Brazhnyi and V. V. Konotop, Phys. Rev. E **72**, 026616 (2005); P. G. Kevrekidis, H. Susanto, R. Carretero-Gonzalez, B. A. Malomed, and D. J. Frantzeskakis, *ibid.* **72**, 066604 (2005).
[22] K. Kasamatsu and M. Tsubota, Phys. Rev. Lett. **93**, 100402 (2004); Boris A. Malomed, H. E. Nistazakis, D. J. Frantzeskakis, and P. G. Kevrekidis, Phys. Rev. A **70**, 043616 (2004); J. K. Xue, G. Q. Li, A. X. Zhang, and P. Peng, Phys. Rev. E **77**, 016606 (2008).
[23] Víctor M. Pérez-García and Juan Belmonte Beitia, Phys. Rev. A **72**, 033620 (2005).
[24] S. K. Adhikari, Phys. Lett. A **346**, 179 (2005).
[25] S. B. Papp and C. E. Wieman, Phys. Rev. Lett. **97**, 180404 (2006); S. B. Papp, J. M. Pino, and C. E. Wieman, *ibid.* **101**, 040402 (2008).
[26] S. Ronen, J. L. Bohn, L. E. Halmo, and M. Edwards, Phys. Rev. A **78**, 053613 (2008).
[27] P. G. Kevrekidis, G. Theocharis, D. J. Frantzeskakis, and B. A. Malomed, Phys. Rev. Lett. **90**, 230401 (2003).
[28] L. Li, B. A. Malomed, D. Mihalache, and W. M. Liu, Phys. Rev. E **73**, 066610 (2006).
[29] L. Salasnich and B. A. Malomed, Phys. Rev. A **74**, 053610 (2006).
[30] P. Öhberg and L. Santos, J. Phys. B **34**, 4721 (2001).
[31] C. Sulem and P. L. Sulem, *The Nonlinear Schrödinger Equation* (Springer-Verlag, New York, 1999).
[32] R. Radhakrishnan and M. Lakshmanan, J. Phys. A **28**, 2683 (1995); S. V. Manakov, Sov. Phys. JETP **38**, 248 (1974).
[33] P. Öhberg and L. Santos, Phys. Rev. Lett. **86**, 2918 (2001).
[34] Z. Dutton and C. W. Clark, Phys. Rev. A **71**, 063618 (2005).
[35] F. Kh. Abdullaev, J. G. Caputo, R. A. Kraenkel, and B. A. Malomed, Phys. Rev. A **67**, 013605 (2003).
[36] H. Saito and M. Ueda, Phys. Rev. Lett. **90**, 040403 (2003).
[37] B. A. Malomed and S. Wabnitz, Opt. Lett. **16**, 1388 (1991); Boris A. Malomed, J. Opt. Soc. Am. B **9**, 2075 (1992).