Spectroscopy of superradiant scattering from an array of Bose-Einstein condensates

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We theoretically study the superradiant gain and the direction of this coherent radiant for an array of Bose-Einstein condensates in an optical lattice. We find that the density grating is formed to amplify the scattering light with the phase-matching condition. The scattering spectroscopy in the momentum space can provide a method to measure the overlap of wave functions of neighboring sites, which is related to the inner-site and intersite coherences.

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I. INTRODUCTION

Superradiance from a Bose-Einstein condensation (BEC) offers the opportunity to study the novel physics associated with cooperative scattering of light in ultracold atomic systems. A series of experiments [1–8] and related theories [9–14] have sparked related interests in quantum information [6,7], collective instability [15,16], high-precision measurement [17], and coherent atom optics [2,18].

In a typical BEC superradiant experiment, the pattern of recoiling atoms in the absorption image provides information about the atomic momentum distribution [2–8], where the moving atoms and the static BEC form a matter wave grating. At the same time, the scattering optical spectroscopy shows the gain process with time [1]. To enhance the scattering light signal, the optical cavity is applied in a similar experimental setting, usually called the collective atomic recoil lasing (CARL) [15,16,19], where the atoms are forced to maintain in the density grating by the optical lattice (OL) in the cavity. Different from these cases, here we consider the scattering superradiance from an array of BECs.

OLs, created by pairs of off-resonance counterpropagating laser beams, offer new opportunities to investigate quantum information processing and strongly correlated quantum matter [20]. The periodical potential in an optical lattice forms an atomic density grating; hence to study the superradiance in this array the coherence of atoms from both the same site and neighboring sites has to be considered. Therefore superradiance has the potential to become a method to detect the coherence of atoms in an optical potential. This is different from the interaction of light and BEC in an OL trap without the atom recoiling [21], where the intersite atomic coherence was considered and the inner-site coherence is neglected.

To study the superradiance in an optical lattice, there are several issues to be addressed with regard to the theory about the superradiance from BEC [9]. First the frequency of an optical lattice, usually of several kHz, is much larger than that of the magnetic trap (tens to hundreds Hz), and the effects of trap is usually neglected because its frequency is much smaller than the recoil frequency. Secondly, we need to calculate the gain in special emission angles. In the case of a magnetic trap, the atomic cloud experiences the maximum gain when the mode is along its long axis. This direction is selected for the least width of momentum to get the maximum gain [9]. The long axis and the direction with least width of momentum are the same in a magnetic trap but not necessarily true in an OL trap. Lastly and importantly, since the light scattering depends on the coherence of atoms from different sites, the interference of scattered light results in amplification at some special frequencies and suppression at others. It could provide us a method to obtain the information of the atoms in OL. The density grating formed by optical lattice and by moving and static atoms gives two criterions for the optical amplification. Hence, similar to the coherent-enhanced imaging where Raman superradiance is used to probe the spatial coherence of BEC in a magnetic trap [18], the scattering spectroscopy provides information about the cooperative radiation of inner-site and intersite atoms.

II. GAIN FOR THE CONDENSATE IN THE TRAP

We consider the model that the atomic cloud is prepared in an optical lattice, as shown in Fig. 1. The optical lattice is placed along the x axis with M sites centered at origin. The lattice constant is $a_0=\lambda/2$, the length of condensate is $L=Ma_0$, and the pumping laser incident with wave vector $\mathbf{k}_0$ propagates along its short axis y.

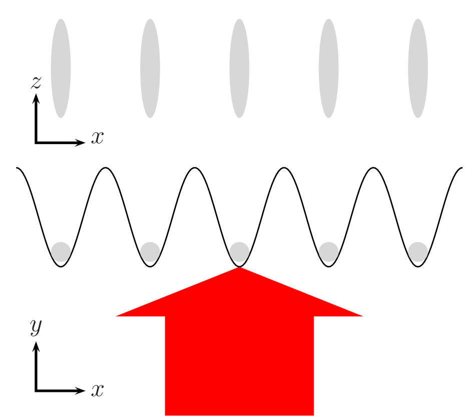

FIG. 1. (Color online) The system sketch. For a cigar-shaped condensate, the optical lattice is formed along x axis with M sites. The pumping laser is propagating along y axis. Moreover in the x-z plane, the trap length in z direction is bigger than that in x direction.

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In order to investigate the superradiant gain, we adiabatically eliminate the excited state of the far-off resonant pump laser and use the rotating wave approximation. Therefore the effective Hamiltonian about the coupling between the atomic and electromagnetic fields can be written as [9]

$$\hat{H}=\hat{H}_a+\hat{H}_p+\hat{H}_i, \tag{1}$$

where the $\hat{H}_a=\int d^3\mathbf{r}\hat{\Psi}^\dagger(\mathbf{r})[\hat{p}^2/2m+\hat{V}(\mathbf{r})]\hat{\Psi}(\mathbf{r})$ is the atomic Hamiltonian, for the classical potential of OL is $V(\mathbf{r})=V_0\cos(x/a_0)$ with the lattice depth V_0. $\hat{H}_p=\int d^3\mathbf{k}\hbar\omega_k\hat{b}^\dagger(\mathbf{k})\hat{b}(\mathbf{k})$ is the Hamiltonian of photon, and the interaction Hamiltonian is

$$\hat{H}_i=\int d^3\mathbf{k}d^3\mathbf{r}[\hbar g(\mathbf{k})\hat{\Psi}^\dagger(\mathbf{r})\hat{\mathbf{b}}^\dagger(\mathbf{k})e^{i(\mathbf{k}_0-\mathbf{k})\cdot\mathbf{r}}\hat{\Psi}(\mathbf{r})+\text{H.c.}], \tag{2}$$

where $\hat{\Psi}(\mathbf{r})$ is atomic field operator and $\hat{\mathbf{b}}^\dagger(\mathbf{k})$ is the annihilation operator of a photon in mode $\mathbf{k}$ in the frame rotating at the pump frequency ω_0. Here the photon energy is $\omega_{\mathbf{k}}=c|\mathbf{k}|-\omega_0$ and $g(\mathbf{k})$ is the coupling coefficient for scattering between the pump and vacuum modes. The interaction between atoms is neglected because it is too small in the time scale considered here.

The atomic field operator can be decomposed to the different side modes as

$$\hat{\Psi}(\mathbf{r},t)=\sum_{\mathbf{q}}\psi_0(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}e^{-i\mu t}\hat{c}_{\mathbf{q}}(t), \tag{3}$$

where the operator $\hat{c}_{\mathbf{q}}$ refers to the recoiling atom scattering with the $\mathbf{q}$ mode light, $\psi_0(\mathbf{r})$ is the ground-state wave function of the condensate with the chemical potential μ. When the OL potential is weak enough, we could deal this case as in a magnetic trap [9]. When it is extremely strong, we are able to approximate the potential as the spatial replication of harmonic trap $V(\mathbf{r})=(V_0/a_0^2)x^2$, decompose the operator according to the eigenstate of the trap,

$$\hat{c}_{\mathbf{q}}=\sum_{n\neq 0}\langle\psi_n|\psi_0e^{i\mathbf{q}\cdot\mathbf{r}}\rangle\hat{c}_n, \tag{4}$$

and define the coefficient

$$A_n\equiv\langle\psi_n|\psi_0e^{i\mathbf{q}\cdot\mathbf{r}}\rangle=\sqrt{P(n,\lambda)}, \tag{5}$$

where ψ_n is the eigenfunction of nth level of the trap, the $P(n,\lambda)$ is the Poisson distribution with parameter $\lambda=(q\sqrt{\hbar/2m\omega_T})^2=\omega_r/\omega_T$, and $\omega_r=\hbar q^2/2m$ is the recoiling frequency. With the first-order side modes considered, the interaction Hamiltonian becomes

$$\hat{H}_i=\sum_{\mathbf{q}\neq 0}\sum_{n\neq 0}\int d^3\mathbf{k}[\hbar g(\mathbf{k})\rho_{\mathbf{q}}(\mathbf{k})A_n\hat{c}_n^\dagger\hat{b}^\dagger(\mathbf{k})\hat{c}_0+\text{H.c.}], \tag{6}$$

where

$$\rho_{\mathbf{q}}(\mathbf{k})=\int d^3\mathbf{r}|\psi_0(\mathbf{r})|^2\exp[-i(\mathbf{k}-\mathbf{k}_0+\mathbf{q})\cdot\mathbf{r}] \tag{7}$$

is the Fourier transform of the ground-state density distribution centered at $\mathbf{k}_0-\mathbf{q}$.

As the experiment shows that there are just several side modes dominating the whole scattering process, in order to simplify this problem, we just take one side mode $\mathbf{q}$ into consideration. Under the Born-Markov approximation, the optical field could be obtained that

$$\hat{b}(t)=\hat{b}(0)e^{-i\omega_k t}+\sum_{n\neq 0}g(\mathbf{k})\rho_{\mathbf{q}}(\mathbf{k})A_n\hat{c}_n^\dagger\hat{c}_0\delta(\omega_k). \tag{8}$$

Inserting Eq. (8) into the dynamic equation of atomic field, $\dot{c}_n=[H,c_n]/i\hbar$, we obtain its evolution equation,

$$\frac{d}{dt}\hat{c}_n=A_n\frac{G_{\mathbf{q}}}{2}\frac{\hat{c}_0^\dagger\hat{c}_0}{N}\sum_{m\neq 0}A_m\hat{c}_m+\hat{f}^\dagger(t)-i\omega_n\hat{c}_n, \tag{9}$$

and the BEC's gain with the $\mathbf{q}$ mode,

$$G_{\mathbf{q}}=N\frac{g^2}{k_0^2}\int d^3k|\rho_{\mathbf{q}}(k)|^2\delta(|\mathbf{k}|-k_0), \tag{10}$$

where we assumed that $g(\mathbf{k})$ is isotropic in the k_x-k_z plane. The first term on the right-hand side of Eq. (9) is the gain from the condensate. The second term is the quantum fluctuation, which as discussed in Ref. [9] does not affect the superradiant behavior in long time, and we just take it as an initial seed. The last term on the right-hand side of Eq. (9) is the energy term.

In order to compare the two cases of pumping the condensate with and without the external potential, we need to discuss Eq. (9) by the mean-field approximation by replacing the field operator $\hat{c}_n$ by a c-number c_n. With the transformation $\tilde{c}_n=c_n\exp(-i\omega_n t)$, Eq. (9) becomes

$$\frac{d}{dt}\tilde{c}_n=A_n\frac{G_{\mathbf{q}}}{2}\frac{\tilde{c}_0^*\tilde{c}_0}{N}\sum_{m\neq 0}A_m\tilde{c}_me^{i(n-m)\omega_T t}. \tag{11}$$

For the case that the potential is turned off when pumping the condensate by the laser, we assume that there is just only one level n satisfying the condition $A_n=1$, with eigenenergy $\omega_n=p^2/2m=\omega_r$, and for the other $A_n=0$. Thus on the right-hand side of Eq. (11) the magnitude of the phase factor $e^{i(n-m)\omega_T t}$ is always equal to one. This form is consistent with Eq. (8) in Ref. [9]. The gain of atomic number is therefore $G_{\mathbf{q}}$. For the case that the external potential exists, the parameters A_n are centered at ω_r/ω_T with a standard deviation $\sqrt{\omega_r/\omega_T}$, and we just need to consider these trap levels within $2\sqrt{\omega_r/\omega_T}$ which make $A_n\neq 0$. The phase factor $e^{i(n-m)\omega_T t}$ is different for different levels. These factors could be approximated by calculating the average phase difference $\omega_T\sqrt{\omega_r/\omega_T}dt=\sqrt{\omega_T\omega_r}dt$ in a during time dt. Thus the different phases of different levels lead to the sum in the right-hand side of Eq. (11) smaller than that in the case with the same phase, as in the previous case. This results in the gain loss given by

$$G'_{\mathbf{q}}-G_{\mathbf{q}}\propto-\frac{\sqrt{\omega_T\omega_r}}{N}. \tag{12}$$

A typical value of the gain is $G_{\mathbf{q}}=4\times 10^4$ for $I=100\ \text{mW/cm}^2$ and $N=10^6$, which is close to the frequency of the atom in the optical lattice trap. Thus when ω_T is large enough to dephase the coherence of condensate and side mode, the gain of light is suppressed. Since the effect of the trap is just a shift in the gain, in the following we mainly discuss the gain without the trap $G_{\mathbf{q}}$.

III. GAIN FROM AN ARRAY OF CONDENSATES IN RELEASED TRAP

Now we consider the case that the pump beam is immediately incident after switching off the potential so the external trapping potential could be neglected. The wave function of the ground state for the ith site is the Wannier function $w_i(\mathbf{r})$, which can be approximated by the Gaussian function $\exp(-\Sigma_{j=1}^{3} r_j^2/2\sigma_j^2)$ with the half width of the wave function σ_j in j direction ($j=x,y,z$). Thus the ground-state wave function in the optical lattice is given by $\psi_0(\mathbf{r})=C_{\mathrm{nor}}\Sigma_i w_i(\mathbf{r})$, where $C_{\mathrm{nor}}=(\int d^3\mathbf{r}|\Sigma_{i=1}^{M} w_i(\mathbf{r})|^2)^{1/2}$ is the normalization factor. We assume $\sigma_z \gg \sigma_x$ and $\sigma_z \gg \sigma_y$. For a single site, the maximum gain is the z direction because the photon could experience the most atomic amplification [9]. Here, we need to carefully discuss the gain for the whole atomic cloud. Since the M sites are placed along x axis with lattice spacing a_0, then the atomic density can be expressed as

$$|\psi_0(\mathbf{r})|^2 = C_{\mathrm{nor}}^2\left[\sum_{i=1}^{M}|w_i(\mathbf{r})|^2 + 2\sum_{i=1}^{M-1} w_i(\mathbf{r})w_{i+1}(\mathbf{r})\right], \quad (13)$$

where the first term on the right-hand side describes the atomic density of site i. The second term is the overlapping between neighboring sites, which is considered only when the wave function of one site is wide enough to overlap its neighbors. The Fourier transformation of the density at $\mathbf{q}=\mathbf{k}_0$ is

$$\rho_{\mathbf{k}_0}(k) = C_{\mathrm{nor}}^2 \exp\left(-\sum_{j=1}^{3}\frac{\sigma_j^2 k_j^2}{4}\right)\frac{\sin\left(\frac{Ma_0}{2}k_x\right)}{\sin\left(\frac{a_0}{2}k_x\right)} \times\left[1+\exp\left(-\frac{a_0^2}{4\sigma_x^2}\right)\right]. \quad (14)$$

We proceed to calculate the superradiant gain according to Eq. (10). The sampling factor in $\rho_{\mathbf{q}}(\mathbf{k})$, $\sin(\frac{Ma_0}{2}k_x)/\sin(\frac{a_0}{2}k_x)$, as shown in solid line of Fig. 2, gives the density profile a sampling, namely, $\rho_{\mathbf{q}}(\mathbf{k})$ is not zero only in the regions with half width $2\pi/Ma_0$ and separation distance $2\pi/a_0$ between neighboring regions. Moreover, the Gaussian factor of $\rho_{\mathbf{q}}(\mathbf{k})$, $\exp(-\Sigma_{j=1}^{3}\frac{\sigma_j^2 k_j^2}{4})$, is centered at $\mathbf{k}_0-\mathbf{q}$ and spreads in k_i direction, which is significant in the region $|k_i-k_0+q|<1/\sigma_i$, as shown in the dashed and dot-dashed lines in Fig. 2. The Gaussian factor is wider when the width in the single-site wave function is smaller. We assume that $k_0 \gg 1/\sigma_j, k_0 \gg 1/a_0$, which means that the atomic momentum is narrow enough so that all its components could contribute to the optical amplification [9]. Moreover in the region of $\rho_{\mathbf{q}}(\mathbf{k})$ where the Gaussian factor is significant, we can approximate the surface of the sphere $|\mathbf{k}|=|\mathbf{k}_0|$ near $\mathbf{k}_0$

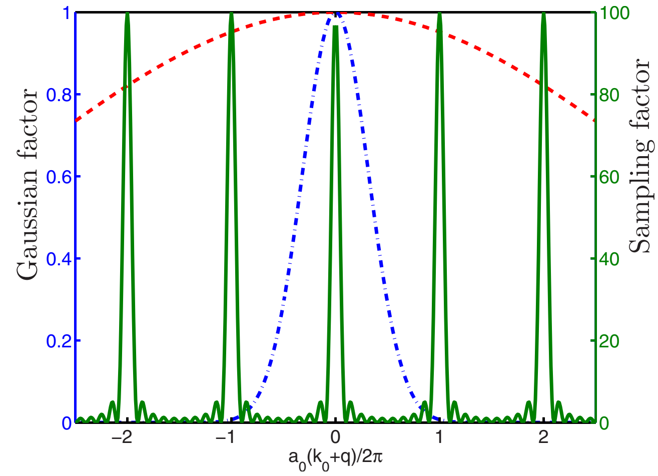

FIG. 2. (Color online) The sampling and the Gaussian factor. The solid line is the sampling factor for $M=10$, which is the result of interference between different sites. The Gaussian factor is the result of single-site amplification of light, which is drawn in dash line for $\sigma_x=0.1a_0$ and in dash dotted line for $\sigma_x=a_0$.

as a plane tangent to the sphere, and the integral in Eq. (10) is equivalent to be the integral on this plane. We consider $\mathbf{q}=\mathbf{k}_0+k_0\hat{\theta}$, where $\hat{\theta}=\cos\theta\hat{\mathbf{k}}_x+\sin\theta\hat{\mathbf{k}}_z$ is a unit vector in the k_x-k_z plane.

Here we mainly consider the maximum gain of light in two extreme directions: $\theta=0$ in z direction and $\theta=\pi/2$ in x direction. For $\theta=\pi/2$, the gain is

$$G_x = G_0\frac{M^2}{\sigma_z}, \quad (15)$$

where

$$G_0 = \frac{g^2}{k_0^2}C_{\mathrm{nor}}^2\frac{2\pi}{\sigma_y}\left[1+\exp\left(-\frac{a^2}{4\sigma_x^2}\right)\right] \quad (16)$$

is related to the normalization factor C_{nor} with the width in y-direction σ_y and the factor $[1+\exp(-\frac{a^2}{4\sigma_x^2})]$ reflecting the coherence of neighboring sites. Equation (15) shows that the gain G_x is proportional to the number of site squared, M^2, which is the result of cooperative radiation.

When $\theta=0$, since the nonzero region of δ function in Eq. (10) is a plane parallel to the k_x-k_y plane, the width of $\rho_{\mathbf{q}}(k)$ will affect the result of $G_{\mathbf{q}}$. Hence, we need to discuss two conditions: $\sigma_x < a_0$ and $\sigma_x \geq a_0$. For the case that $\sigma_x < a_0, \rho_{\mathbf{q}}(\mathbf{k})$ has a_0/σ_x sidebands, after summing all these sidebands which are with half width $1/Ma_0$, we get the superradiant gain given by

$$G_z = G_0 M^2 \frac{a_0}{\sigma_x}\frac{1}{Ma_0} = G_0\frac{M}{\sigma_x}. \quad (17)$$

For the case that $\sigma_x \geq a_0$, there is just one nonzero region of $\rho_{\mathbf{q}}(\mathbf{k})$, thus, the superradiant gain is

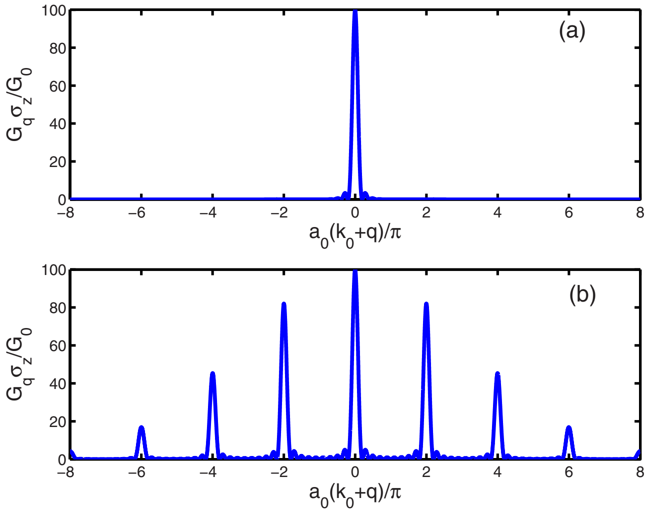

FIG. 3. (Color online) The gain with a factor σ_z/G_0, in x direction for the (a) $\sigma_x = a_0$ and (b) $\sigma_x = 0.1a_0$. The gain has satellites when the width of single-site wave function is large. When it is narrow, the satellites disappear. Here, $M=10$.

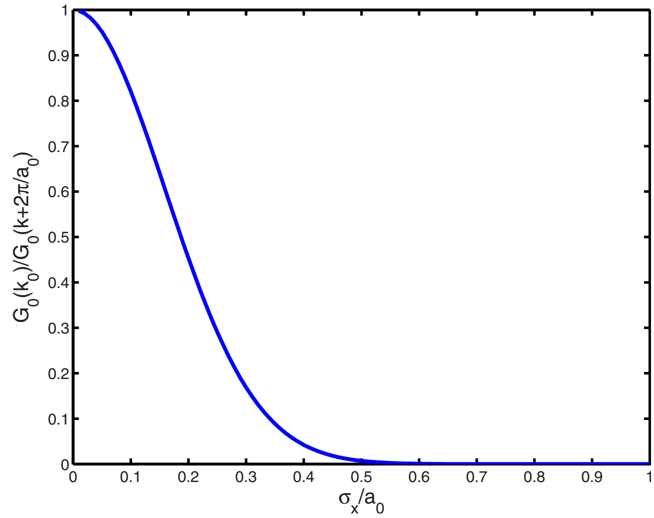

FIG. 4. (Color online) The ratio of the maximum gain to the second maximum gain in x direction versus the one-site wave-packet width. Here, $M=10$.

$$G_z = G_0 M^2 \frac{1}{M a_0} = G_0 \frac{M}{a_0}. \tag{18}$$

In both cases, the gain G_z is proportional to M due to the incoherent sum of different sites.

When $\sigma_x < a_0$, the gain ratio for the two extreme direction is that $\frac{G_x}{G_z} = \frac{M a_0}{\sigma_z} = L/\sigma_z$ which is the aspect ratio, consistent with the theory without OL trap. When $\sigma_x > a_0$, the gain ratio becomes $\frac{G_x}{G_z} = \frac{M\sigma_x}{\sigma_z}$ which is the effective length ratio in this two directions. It should be noted that this theory is sound under the condition that $k_0 \gg 1/\sigma_x$. A typical experimental value is $k_0 = 2\pi/780$ nm, satisfying $\sigma_x \gg 1/k_0$. If we need the gain in z direction larger than that in x direction, we need $\sigma_z \gg M/k_0$, which is hard to be realized in experiment. Thus the radiation usually takes place in x direction.

IV. SPECTROSCOPY OF SUPERRADIANT SCATTERING

In Sec. III, we understand that the gain of light is usually propagating along x axis. Considering $\mathbf{q} = q\hat{\mathbf{x}}$ in the x direction, for the different $\mathbf{q}$ the gain can be expressed as

$$G_{\mathbf{q}} = \frac{G_0}{\sigma_z} \exp\left[-\frac{\sigma_x^2 (k_0+q)^2}{2} \right] \frac{\sin^2 \dfrac{M a_0 (k_0+q)}{2}}{\sin^2 \dfrac{a_0(k_0+q)}{2}}. \tag{19}$$

In this equation, we know that the maximum gains emerge at $\frac{a_0(k_0+q)}{2} = n\pi$. In other words, the gain has maximum around $k_0 + 2n\pi/a_0$ with separation $2\pi/a_0$.

The spectroscopy for different width of single-site wave function is plotted in Fig. 3. As shown in subfigure (a), when $\sigma_x \geq a_0$, there is only one peak in the gain. Subfigure (b) shows that when the $\sigma_x \ll a_0$, there are sidebands. The radiant light has sidebands when wave functions of neighboring sites are not overlapped. The reason is that atoms in different sites are pumped by the same phase light and become the same phase dipole. The radiant light which is propagating along the lattice has a phase difference in neighboring sites, $a_0(\mathbf{k} - \mathbf{k}_0) \cdot \hat{\mathbf{x}}$. Thus radiant lights with different frequencies will have different gains by the averaging over the whole lattice. The constructive interference will single out the frequency component satisfying the condition $a_0(\mathbf{k} - \mathbf{k}_0) \cdot \hat{\mathbf{x}} = 2n\pi$ to amplify, and other components will be suppressed due to the destructive interference of M sites.

For the larger width of single-site wave function, the gain of sidebands is smaller. Thus by measuring the sideband gain could give us a method to obtain the information about the width σ_x. Figure 4 shows the ratio of maximum gain to the second maximum gain versus σ_x. By the spectroscopical measuring, we could obtain the information of the width of wave packet, which is relevant to the potential quantum phase transition.

V. DISCUSSION AND CONCLUSIONS

In the BEC superradiant experiment [1], a photon is scattered by an atom in the BEC which acquires the recoil momentum. The moving atoms and the static BEC form a matter wave grating which enhances the same direction scattering. Due to the mode competition, the highly directional emissions of light are along the long axis of the condensate. Considering the self-amplification only exists within the coherence time of the system, the stability of the relative phase between different atomic matter waves determines the coherence time of the matter wave in the superradiance scattering. On the other hand, the coherence is preserved as the relative phase of the probe beam and pump light field in the CARL, where the atoms are forced to maintain in this grating formed by these two light fields.

Different to these experimental schemes, here we extend the theory of superradiance of BEC [10] to the case of an OL trap where an array of atoms forms a density grating. The superradiance gain is calculated in the quantum theory. In this theory, we consider inner-site and intersite coherences of atoms. Only the scattering light satisfying the condition $(\mathbf{k}-\mathbf{k}_0)\cdot a_0\hat{\mathbf{x}}=2n\pi$ will be singled out for amplification and the other components will be suppressed to different extent. Together with the grating formed by the static and moving condensates, both gratings give frequency selection rules. It is similar to a diode laser with internal and external cavities. Only the light resonant to both cavities will be amplified.

The motion of recoiling atoms in the high-frequency OL trap causes the loss of coherence and the loss of the optical gain proportional to $\sqrt{\omega_T\omega_r}$. In a typical magnetic trap, the trap frequency is smaller, and the loss can be neglected. It can inhibit the collective radiation when the trap frequency is high enough. By calculating the ratio of optical gain in the two extreme direction, we show that the gain is proportional to the length that the light travels in the condensate.

Depending on the lattice depth, the wave function of a single-site overlaps differently with its neighboring sites. When the OL potential is low enough, wave functions of neighboring sites fully overlap, similar to a condensate in the magnetic trap. When the OL potential is high enough, wave functions of neighboring sites are separated. The different overlapping results in the different scattering spectroscopy. Thus the spectroscopy provides us with a new method to detect the coherence of different sites. Moreover, unlike the time-of-flight method used in detecting quantum phase transition [20], the spectroscopy method is a nondestructive method. More understanding of this mechanism can be helpful in understanding self-organization, especially how the long-range order arising in the self-synchronization process, such as in the phase transition between the superfluid (SF) and Mott-insulator (MI).

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[1] S. Inouye, A. P. Chikkatur, D. M. Stamper-Kurn, J. Stenger, D. E. Pritchard, and W. Ketterle, Science **285**, 571 (1999).

[2] D. Schneble, Y. Torii, M. Boyd, E. W. Streed, D. E. Pritchard, and W. Ketterle, Science **300**, 475 (2003).

[3] M. Kozuma, Y. Suzuki, Y. Torii, T. Sugiura, T. Kuga, E. W. Hagley, and L. Deng, Science **286**, 2309 (1999).

[4] Y. Yoshikawa, T. Sugiura, Y. Torii, and T. Kuga, Phys. Rev. A **69**, 041603(R) (2004).

[5] D. Schneble, G. K. Campbell, E. W. Streed, M. Boyd, D. E. Pritchard, and W. Ketterle, Phys. Rev. A **69**, 041601(R) (2004).

[6] K. M. R. van der Stam, R. Meppelink, J. M. Vogels, J. W. Thomsen, and P. van der Straten, e-print arXiv:0707.1465.

[7] N. Bar-Gill, E. E. Rowen, and N. Davidson, Phys. Rev. A **76**, 043603 (2007).

[8] F. Yang, X. J. Zhou, J. Li, Y. K. Chen, L. Xia, and X. Z. Chen, Phys. Rev. A **78**, 043611 (2008); J. Li, X. J. Zhou, F. Yang, and X. Z. Chen, Phys. Lett. A **372**, 4750 (2008).

[9] M. G. Moore and P. Meystre, Phys. Rev. Lett. **83**, 5202 (1999).

[10] O. Zobay and G. M. Nikolopoulos, Phys. Rev. A **73**, 013620 (2006); O. Zobay and G. M. Nikolopoulos, *ibid.* **72**, 041604(R) (2005).

[11] H. Pu, W. Zhang, and P. Meystre, Phys. Rev. Lett. **91**, 150407 (2003).

[12] O. E. Mustecaplioglu and L. You, Phys. Rev. A **62**, 063615 (2000).

[13] G. R. M. Robb, N. Poivella, and R. Bonifacio, J. Opt. B: Quantum Semiclassical Opt. **7**, 93 (2005).

[14] H. Uys and P. Meystre, Phys. Rev. A **75**, 033805 (2007).

[15] S. Slama, S. Bux, G. Krenz, C. Zimmermann, and P. W. Courteille, Phys. Rev. Lett. **98**, 053603 (2007).

[16] S. Slama, G. Krenz, S. Bux, C. Zimmermann, and P. W. Courteille, Phys. Rev. A **75**, 063620 (2007).

[17] D. J. Wineland, J. J. Bollinger, W. M. Itano, F. L. Moore, and D. J. Heinzen, Phys. Rev. A **46**, R6797 (1992).

[18] L. E. Sadler, J. M. Higbie, S. R. Leslie, M. Vengalattore, and D. M. Stamper-Kurn, Phys. Rev. Lett. **98**, 110401 (2007).

[19] R. Bonifacio and L. De Salvo, Nucl. Instrum. Methods Phys. Res. A **341**, 360 (1994).

[20] M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch, and I. Bloch, Nature (London) **415**, 39 (2002).

[21] I. B. Mekhov, C. Maschler, and H. Ritsch, Phys. Rev. Lett. **98**, 100402 (2007).